Optimization Analysis for the Method of Auxiliary Sources Applied to the Scattering Problem for Dielectric Objects

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Abstract: - In this paper a numerical analysis of electromagnetic scattering from various infinite, dielectric cylindrical objects is presented using the Method of Auxiliary Sources (MAS). Several cross sections which can be considered as perturbed duplicates of the circle were investigated. The computational error associated with the numerical solution of the MAS linear system, as well as the condition number of the pertinent impedance matrix are plotted and the results are discussed. Furthermore, a parametric analysis is conducted in order to examine the effect of various parameters on the accuracy of the solution and the stability of the linear system. The aim of this study is the extraction of useful conclusions on the optimum location of the auxiliary sources for arbitrarily shaped dielectric scatterers.

Key-Words: - Electromagnetic Scattering, Method of Auxiliary Sources, Dielectric Scatterers, Optimization

1 Introduction

The Method of Auxiliary Sources (MAS) [1]-[2] is an alternative approach to more widespread algorithms such as the Moment Method (MoM), for the solution of the forward scattering problem. Numerous applications of MAS [3] have clearly highlighted the merits and advantages of the technique. Specifically, MAS is characterized by algorithmic simplicity and a less time-consuming procedure for the evaluation of the matrix elements compared to MoM, due to a much smaller constant multiplier in the cost estimate, thus rendering the method potentially favourable for electrically large geometries [4].

Nevertheless, the MAS merits cannot be fully exploited, unless the location of the auxiliary sources (AS) is optimized. Otherwise, the technique's performance may range from merely inefficient to completely inaccurate. Recently, rigorous optimization was carried out for infinite cylindrical scatterers with circular cross section, satisfying various boundary conditions, including perfectly conducting (PEC) [5], impedance [6] and dielectric surfaces [7]. The optimization was based on the theory of circulant matrices, which inherently characterizes all MAS implementations associated with circular geometries. Although the analytical results cannot be straightforwardly extended to other shapes, the physical insight gained can serve as a guideline towards the feasibility of such a generalization. Hence, the purpose of this paper is an attempt to optimize MAS for more generic dielectric scatterers, using mostly numerical schemes, based however on the observations made in [5]-[7]. Similar work for PEC scatterers is presented in [8]. Specifically, it is useful to examine the behavior of the numerical MAS solution, as the cross section gradually deviates from the circle.

In Section 2, a brief description of the main results presented in [5]-[7] for circular cross sections is given, whereas a number of novel geometries, serving as perturbations of the circle, are introduced. The numerical error of the linear system solution, along with the matrix condition number is computed and plotted in Section 3. Finally, in Section 4, several conclusions and possible future work are discussed.

2 Theoretical Background

2.1 Circular Geometries

In [7], a simple, benchmark problem with a known solution was chosen, i.e., scattering of a transverse magnetic (TM) plane wave from a dielectric, infinite, circular cylinder characterized by complex permittivity ε_r and relative permeability equal to 1. The MAS linear system was formulated by constructing two fictitious auxiliary surfaces both conformal to the actual surface and invoking appropriate boundary conditions. The impedance matrix (in block form) was inverted via eigenvalue evaluation and subsequent diagonalization. The derivation of exact expressions was thus facilitated for the error, associated with the boundary condition satisfaction. Furthermore, an exact expression for the condition number of the system was also derived. Similar work has been completed for circular, PEC [5] and impedance cylinders [6]. For the dielectric case [7], it was shown that the location of the exterior auxiliary surface does not affect significantly the computational error, meaning that it can be arbitrarily chosen without risking degradation of the method's accuracy. It was also observed that the variation of the computational error due to the effect of the dielectric permittivity, is not significant.

The main conclusion in all these papers was that, irrespective of the scatterer material, the analytical error generated by the discrete nature of MAS generally decreases as the auxiliary surface approaches the center of the cylinder. As an exception to this rule poses a denumerable set of auxiliary surface locations, in the vicinity of which the error bursts abruptly, due to resonances, mathematically associated with the zeroes of Bessel functions. On the other hand, the numerical error, computed by LU decomposition and inversion, agrees with the analytical one only above a certain threshold of the auxiliary surface radius. Below this threshold, it grows erratically, due to severe matrix ill - conditioning. Finally, the condition number was shown to grow exponentially with the number of unknowns.

2.2 Circular-like Geometries

Unfortunately, the analytical tools invoked in [5]-[7] for the circular case are apparently not applicable to generic shapes. The reason is that for any other cross-section, the resulting MAS matrix fails to be circulant, and therefore it is not necessarily amenable to analytical inversion. However, it is expected that small perturbations to the circular geometry should not affect significantly the qualitative behavior of either the error, or of the condition number. It would be very interesting to study both these parameters quantitatively, for geometries close enough to a circle, in order to make observations potentially helpful in a generic MAS optimization. Thus, in this paper, MAS is applied to dielectric scatterers, their cross section being geometrically described by an ellipse, a "daisy" and a super-ellipse. The computational error and the matrix condition number (in the 1-norm) are calculated via a direct numerical method, specifically LU decomposition with partial pivoting.

2.2.1 Ellipse

An ellipse can be considered as a perturbation of a circle with radius a, by defining one of its semi-axes to be equal to a, and the second one equal to $b=\gamma a$, where the positive parameter $\gamma<1$ is the ellipticity. The equation of the ellipse is, of course,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{1}$$

2.2.2 "Daisy"

To obtain a circle-like geometry with a periodically perturbed circumference, the "daisy" is defined by

$$r(\theta)=r_{\min}+r_0\left|\cos(\omega\theta)\right| \tag{2}$$

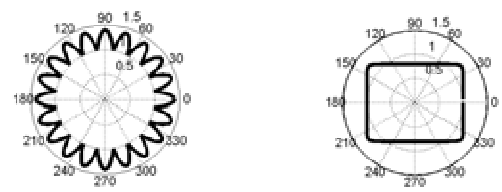
where r_{min} is the minimum radius, r_0 is the maximum perturbation and ω is the angular frequency (see Fig. 1a).

2.2.3 Super-ellipse

A super–ellipse is a generalization of an ellipse, its equation given by

$$\frac{x^\nu}{a^\nu}+\frac{y^\nu}{b^\nu}=1, \nu\in \mathrm{N} \tag{3}$$

For $\nu\to\infty$, the super-ellipse approaches a rectangle (see Fig. 1b).



(a) (b)

Fig. 1: Geometries

3 Numerical Results and Discussion

In all cases examined, the auxiliary surfaces were defined to be conformal (similar) to the scatterer boundary, with a similarity ratio $0<\tau<1$ for the interior surface and $\tau'=2$ is the similarity ratio for the outer surface. The error was defined exactly as in [5]-[7]. The relative electrical permittivity was defined $\varepsilon_r=5$ in all cases.

As a first example, we consider an elliptical scatterer with ellipticity γ=0.8 and large semi-axis a=λ. The plots of the condition number and the numerical error are presented in Figures 2(a) and

2(b) respectively, as a function of τ. The condition number is generally decreasing for increasing τ, assuming a constant low value after τ=0.6. For τ<0.6 the system becomes severely ill - conditioned. The numerical error is not monotonic, and is minimized approximately for τ=0.6. Peaks obviously correspond to resonances.

Next we consider a "daisy" of minimum radius 0.5λ, maximum perturbation 0.025λ and angular frequency ω=10 rad^{-1}. Figure 2(c) show that again the condition number becomes large and unstable for values of τ less than 0.6, whereas the numerical error remains stable for a wider range of values of the parameter τ as is depicted in Fig. 2(d). Peaks of the condition number and the numerical error are present here, too.

As a third example we considered a super-ellipse with ν=4, a=λ and b=0.8λ. The condition number and the numerical error are low and stable for τ>0.5.

To optimize the AS location, and hence the MAS performance, the computational error should evidently be minimized. Unlike the circular cylinder case [5]-[7], the plots in Fig. 2 show that the error does not necessarily decrease for decreasing τ. However, matrix conditioning invariably deteriorates for values of τ less than 0.5, adding numerical noise to the solution, and therefore concealing the actual error behaviour for very small auxiliary surfaces. Like in [5]-[7], it is clear from the plots that the computational error demonstrates an irregular behaviour when the condition number is larger than 10^7 (a threshold associated with FORTRAN's double precision).

Moreover, the numerical data show that in all cases several peaks are present, from a physical point of view corresponding to resonance effects. Although in the circular case [5]-[7] the location of these peaks was possible to determine analytically in terms of the zeroes of Bessel functions, this is not true for more generic geometries. Approximate values of the resonant values of τ can only be estimated on the basis of perturbation theory, for geometries deviating only a little from the original circular configuration. These values can, in principle, be expressed in terms of the respective τ's for the circle and the order O(ε) of the perturbation.

Finally, given the problems caused by matrix ill - conditioning, it would be beneficial to construct a suitable pre-conditioner applicable to the MAS linear system. Since the optimum pre-conditioner for the circular cylinder is analytically known (it is actually the inverse of the MAS matrix), the pre-conditioner for a geometrical perturbation of the circle may be designed as the inverse of some MAS matrix, associated with a closely related circular configuration. Intense research on this topic is currently in progress.

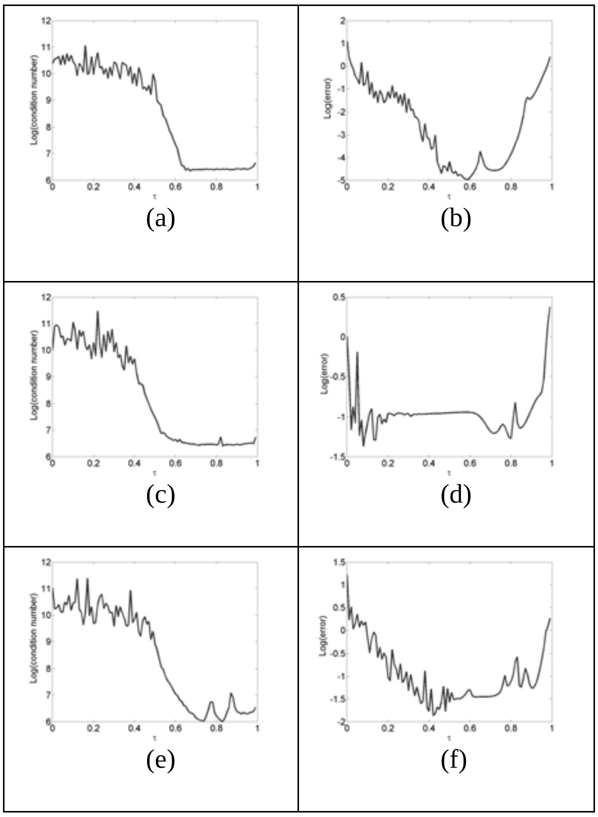


Fig. 2: Condition number and numerical error plots for various geometries. The first column depicts the condition number and the second the numerical error. Plots (a), (b) show the results for the ellipse, while (c), (d) for the "daisy" and (e), (f) for the super-ellipse.

3.1 Parametric analysis

Several simulations were carried out to investigate the effect on the condition number and the numerical error when various parameters, associated with the scatterer's shape or the number of auxiliary sources, vary.

Fig. 3 depicts the condition number and the numerical error for various parameters determining the shape of the scatterer. In Figs. 3(a) and 3(b) it is shown that decreasing values of the ellipticity in the case of an ellipse (departing from a circle as in [5]), results in a slight increase of the condition number for almost all values of the parameter τ and in an even more noticeable increase of the numerical error.

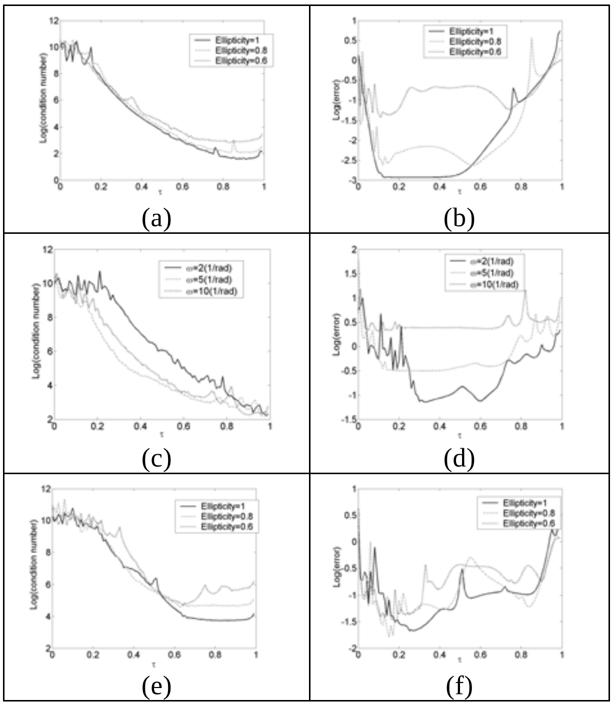


Fig. 3: Comparative condition number and numerical error plots for various geometries and parameters measuring the departure from circular shape. The first column depicts the condition number and the second the numerical error. Plots (a), (b) show the results for the ellipse, (c), (d) for the “daisy” and (e), (f) for the super-ellipse.

Thus, an increasing degree of alteration of the shape of the circle has a slight deteriorating effect on the stability and the accuracy of the algorithm.

Similarly in the case of a “daisy” the increase of the frequency ω (Eq. 2) which yields a more circle-like shape, results in lower numerical error for almost all choices of placement for the auxiliary surface, except for the small values of τ. On the other hand, the condition number is slightly worse due to the less smooth surface of the scatterer.

In the case of the super-ellipse (v=4), the decrease of the ratio b/a (ellipticity of the super-ellipse) has similar effects as in the case of the ellipse.

The effect of the number of auxiliary sources on the condition number and the numerical error is also investigated for the same three scatterers. As can be clearly seen in Fig. 4, the instability of the ensuing numerical system increases but the numerical error is improved when the number of auxiliary sources increases.

4 Conclusions and Future work

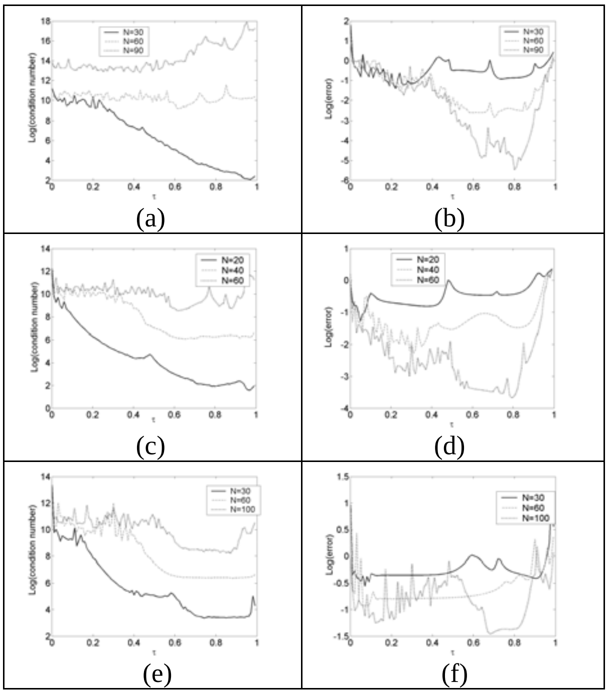


Fig. 4: Comparative condition number and numerical error plots for various geometries and numbers of auxiliary sources. The first column depicts the condition number and the second the numerical error. Plots (a), (b) show the results for the ellipse, (c), (d) for the “daisy” and (e), (f) for the super-ellipse.

In this paper, a numerical analysis of the MAS accuracy and stability for dielectric scatterers associated with geometrical perturbations of a circular cylinder was presented. For all cases, the computational error and the 1-norm condition number of the MAS linear system were calculated. Plots of the condition number are reminiscent of the circular case, slowly increasing for shrinking auxiliary surfaces, and finally becoming highly irregular. Isolated peaks are also present, corresponding to resonance effects related to roots of Bessel functions. On the other hand, plots of the computational error prove that the error does not necessarily decrease for retracting auxiliary surfaces. However, the true, global behavior of the error is not easily discernible, due to severe ill - conditioning of the system for small auxiliary surfaces. Therefore, it is imperative that the system conditioning of a general MAS linear system be reduced, using a suitable pre-conditioner, before an optimization procedure for arbitrary geometries is proposed.

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