Performance Analysis of the Ranging Process in Fixed and Mobile Broadband Wireless Access Systems based on OFDMA

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Abstract: - In this letter, we consider a ranging procedure occurred in broadband wireless access systems based on the Wireless MAN-OFDMA (Metropolitan Area Network -Orthogonal Frequency Division Multiple Access). The ranging collision may be occurred when multiple subscribers select the same ranging codes from the given ranging code pool and range the BS by transmitting those identical ranging codes at the same time frame. We propose a mathematical performance model to analyze the probability of ranging collision affected by both the capacity of the ranging code pool and the rate of the MSS arriving to the system and investigate the numerical results of the proposed model. By the proposed model and its numerical results, we can estimate the capacity of ranging code pool to be necessary to BS(Base Station). We compare the analytical results with the simulated results to validate the proposed mathematical model is correct.

Key-Words: - Queueing theory, Performance analysis, Ranging process, OFDMA, Broadband wireless access system

1 Introduction

A ranging procedure is a collection of processes for a Mobile Subscriber Station (MSS) and a Base Station (BS) to maintain the quality of the radio frequency communication link between them. There are four types of ranging procedure in the Wireless MAN-OFDMA, say, Initial Ranging, Periodic Ranging, Bandwidth Requests and CDMA Handoff Ranging[1, 2]. When the MSS performs one of ranging procedure with the BS, the MSS transmits the Ranging Request with a pseudo-noise 144 bit-long Ranging Code to the BS on the uplink Ranging Sub-channel. The Wireless MAN-OFDMA PHY[1, 2] specifies a Ranging Subchannel and a set of special pseudo-noise 144 bit-long Ranging Codes in detail. This ranging code set is divided into four subsets, say, Initial Ranging code set, Periodic Ranging code set, Bandwidth Request Ranging code set, and CDMA Handoff Ranging code set for the corresponding ranging procedures. The BS informs the MSS of the range of codes allocated for each subset by broadcasting the UCD(Uplink Channel Descriptor) Channel Encoding message including those values over the air.

On performing a ranging procedure, an MSS ran-

domly selects one code from the corresponding ranging code subset assigned for the specified ranging procedure. The BS can determine the ranging purpose of the MSS after receiving a ranging code by checking the subset to which the received code belongs.

If two MSSs in the same ranging procedure on Ranging Sub-channel select the identical ranging code, then they experience a collision and fail to range the BS because the BS can not differentiate these identical ranging codes from two MSSs and does not send any acknowledgement to these MSSs. Once an MSS experiences a collision, it continues to select a code until it has no collision. So, the number of ranging codes in each ranging code set should be large enough so that the MSS hardly experience any collision on ranging.

However, the increase of the number of ranging codes in a ranging code set increases the implementation complexity in the BS because the BS requires the higher processing power and takes more time to check whether the received ranging code belongs to the specified ranging code set or not as the number of ranging codes in a specified ranging code set increases. Accordingly, it is better for the implementation if the number of ranging codes in a ranging code set decreases as minimum as possible.

Therefore, it is an critical issue to determine the appropriate number of ranging codes in a ranging code set which satisfies a given ranging collision probability, defined by the probability that at least one MSS experiences a collision.

Cho et al. [3] has proposed a QoS architecture for the IEEE 802.16a MAC protocol and presented a bandwidth allocation and admission control policy for the architecture. They have also evaluated analytically the system performance and verified the analytical results through a simulation. To our best knowledge, however, no work on the minimum required number of ranging codes to satisfy a given collision probability has appeared in the literature. In this letter, we propose a mathematical performance model to analyze the probability of ranging collision affected by both the capacity of the ranging code pool and the rate of the MSS arriving to the system and investigate the numerical results of the proposed model. By the proposed model and its numerical results, we can compute the minimum number of ranging codes to satisfy a given collision probability under various traffic environments. We provide the comparison results between the proposed analytical results and the simulated results to justify the proposed mathematical model.

2 Modeling and Analysis

In this section, we are focusing on a ranging procedure on a Ranging Sub-channel. Here are the assumptions for system modeling. Without loss of generality, these assumptions are reasonable because they follow TDD(Time Division Duplex) of Wireless MAN-OFDMA PHY[1, 2] and general user arrival pattern.

- Time is divided into frames of equal size.
- New MSSs arrive at the system according to a Poisson process with rate λ at frame boundaries.
- At the beginning of a frame an MSS in the system randomly selects a code among *n* codes in the given list to access the network.
- If an MSS selects a code which is not selected by the other MSS(s), then the MSS finished the ranging procedure.
- If two or more MSSs select the same code, then there a collision occurs. An MSS experiencing a collision continues to select a code in the consecutive frames until it has no collision.

In the analysis, we use the following notation: When we have m MSSs, after each MSS selects a code

 $N_1^{(m)}\colon$ the number of codes, each having only one $\ensuremath{\mathsf{MSS}}$

 $N_2^{(m)}\colon$ the number of codes, each having at least two $\rm MSSs$

n: the total number of codes in the list

Now assume that there are m mobile users contending just before a new frame starts. We first compute the distribution of $(N_1^{(m)}, N_2^{(m)})$ for the $m (\geq 1)$ MSSs. To do so, we use induction argument as follows: Initially we consider a single MSS, i.e., m = 1. Then the initial state is $(N_1^{(1)}, N_2^{(1)}) = (1, 0)$ because the single MSS selects a code in the list and no contention occurs. Next, we consider one more MSS, i.e., m = 2. Then from the initial state $(N_1^{(1)}, N_2^{(1)}) = (1, 0)$ we have the following state transitions:

$$P\{(N_1^{(2)}, N_2^{(2)}) = (0, 1) | (N_1^{(1)}, N_2^{(1)}) = (1, 0)\} = \frac{1}{n}$$
$$P\{(N_1^{(2)}, N_2^{(2)}) = (2, 0) | (N_1^{(1)}, N_2^{(1)}) = (1, 0)\} = \frac{n-1}{n}$$

In general, assume that $(N_1^{(m)}, N_2^{(m)}) = (k_1, k_2)$. If we consider one more MSS, the transition probabilities are given by

$$P\{(N_1^{(m+1)}, N_2^{(m+1)}) = (k_1, k_2)|$$

$$(N_1^{(m)}, N_2^{(m)}) = (k_1, k_2)\} = \frac{k_2}{n},$$

$$P\{(N_1^{(m+1)}, N_2^{(m+1)}) = (k_1 - 1, k_2 + 1)|$$

$$(N_1^{(m)}, N_2^{(m)}) = (k_1, k_2)\} = \frac{k_1}{n},$$

$$P\{(N_1^{(m+1)}, N_2^{(m+1)}) = (k_1 + 1, k_2)|(N_1^{(m)}, N_2^{(m)})$$

$$= (k_1, k_2)\} = \frac{n - k_1 - k_2}{n}I\{k_1 + k_2 + 1 \le n\}.$$

All the other transition probabilities are 0. In the third equation $I\{A\}$ is the indicate function, defined by 1 if the event A occurs and 0 otherwise.

Then it can be easily shown that, the distribution of $(N_1^{(m)},N_2^{(m)}),m\geq 1,$ is recursively computed by

$$\begin{aligned} r_m(k_1,k_2) &\stackrel{\Delta}{=} P\{N_1^{(m)} = k_1, N_2^{(m)} = k_2\} \\ = & r_{m-1}(k_1,k_2)\frac{k_2}{n} \\ & + r_{m-1}(k_1-1,k_2)\frac{n-k_1-k_2+1}{n} \\ & \times I\{k_1 \geq 1, k_1+k_2 \leq n\} \\ & + r_{m-1}(k_1+1,k_2-1)\frac{k_1+1}{n}I\{k_2 \geq 1\} \end{aligned}$$

for $m \ge 2, 0 \le k_1 \le n, 0 \le k_2 \le n$ and $k_1 + k_2 \ne 0$, and

$$r_1(1,0) = 1.$$

Note that the necessary condition for $r_m(k_1, k_2)$ to be positive is $k_1 + 2k_2 \le m$. Next, we construct an embedded Markov chain where the *n*-th embedded point is the time epoch at which the *n*th frame starts. Let X_n be the number of MSSs preparing new ranging procedure in the system just after the *n*-th embedded point, that is, those who experience collisions at the beginning of the *n*-th frame.

Let a_k be the probability that there are k new MSSs arriving at the system in a frame, which is given by

$$a_k = \frac{e^{-\lambda d} (\lambda d)^k}{k!}, \qquad k \ge 0.$$

where d is the size of a frame, which is equal to 5×10^{-3} in a WiBro system.

Now to compute the transition probability matrix of $\{X_n\}$, assume that $X_n = k$. Noting that new MSSs arriving during the *n*-th frame participate in the ranging procedure at the beginning of the n + 1st frame, we get the following transition probabilities:

Case 1: when k = 0,

$$X_{n+1} = 0$$

with probability a_0 , and

$$X_{n+1} = i - l$$

with probability $a_i \sum_{j=0}^n r_i(l,j)$ for $i \ge 0$ and $0 \le l \le i$. Case 2: when k > 0,

$$X_{n+1} = k + i - l$$

with probability $a_i \sum_{j=0}^n r_{k+i}(l,j)$ for $i \ge 0$ and $0 \le l \le k+i$.

Let \mathbf{Q} denote the transition probability matrix of the Markov chain X_n . If there exists a stationary probability vector $\mathbf{q} = (q_0, q_1, \cdots)$ of the Markov chain, it satisfies balance equations $\mathbf{q}\mathbf{Q} = \mathbf{q}$ and can be computed numerically [4].

Let $\pi = (\pi_0, \pi_1, \cdots)$ be the number of MSSs preparing ranging procedures at the beginning of an arbitrary frame in the steady state. Then π satisfies

$$\pi_i = \sum_{j=0}^i q_i a_{i-j}, \qquad i \ge 0$$

Then the collision probability that at least one MSS experiences a collision is given by

$$\sum_{k=2}^{n} \pi_k \left(1 - \frac{n(n-1)\cdots(n-k+1)}{n^k} \right) + \sum_{k=n+1}^{\infty} \pi_k.$$

3 Numerical and Simulation results

To investigate the effect of the number of ranging codes and the arrival rate of new MSSs on the ranging collision probability, we compute the ranging collision probability based on our analysis. Since the number of MSSs preparing ranging procedures ranges from 0 to ∞ , we use truncation method in the numerical analysis. We perform also simulations to check the validity of our mathematical analysis. The simulation time is 50000 frames. In all numerical and simulation results, we set the time duration d of a frame to 5×10^{-3} .

In Table. 1, we change the number n of ranging codes from 16 to 64 with fixed arrival rate 100(1/sec). In the numerical analysis, we set the truncation number to 50, that is, the dimension of the transition matrix **Q** is set to 50. As seen in Table 1., the ranging collision probability increases with the decrease of the number of ranging codes as expected, and the simulation results validate that our mathematical analysis is correct.

Table 1. The ranging collision probability vs. the number of ranging codes

\overline{n}	16	32	64
analysis	0.008671	0.004111	0.002003
95 % C.I.(Min)	0.007631	0.003817	0.001868
95 % C.I.(Max)	0.008993	0.004230	0.002118

In Table 2., we change the arrival rate from 50 to 300 with fixed number of ranging codes, equal to 64. As the arrival rate increases, the collision probability increases. In addition, we see that the higher the arrival rate is, the more significant performance degradation the system experiences.

Table 2. The ranging collision probability

vs. the arrival rate				
rate	50	100		
analysis	0.000498	0.002003		
95 % C.I.(Min)	0.000449	0.001868		
95 % C.I.(Max)	0.000567	0.002118		
rate	200	300		
analysis	0.008073	0.018267		
95 % C.I.(Min)	0.007793	0.017647		
95 % C.I.(Max)	0.008591	0.018537		

By investigating Table 1 and 2, we can reach a conclusion that the ranging collision probability is more affected by the variation of arrival rate of the MSS rather than the variation of ranging code number. Table 1 shows that the decrease of the ranging collision probability is almost the inverse of the increase of the number of ranging codes. Accordingly, if the arrival rate of the MSS is to be given at a maximum that BS can accommodate, then we can easily obtain the suitable ranging code number to satisfy the given ranging collision probability.

4 Conclusions

We consider a ranging procedure occurred in broadband wireless access systems based on the Wireless MAN-OFDMA. On ranging the BS, each MSS selects a ranging code randomly from a set of ranging codes. It may experience a collision when another MSS selects the same ranging code at the same time frame. It is generally recognized that MSS experiencing a collision continues selecting a ranging code and ranges the BS again in consecutive frames until it has no collision. Thus, when the ranging collision occurs at a higher rate because of the scarce ranging codes, the system performance like the connection setup delay, the connection drop rate caused by the ranging collision degrade significantly because the connection setup delay and the connection drop rate heavily increase due to the failure of ranging. Moreover, the increase of the number of ranging codes in a ranging code set increases the implementation complexity in the BS.

Therefore, it is meaningful to investigate an algorithm to determine the appropriate number of ranging codes in a ranging code set which satisfies a given ranging collision probability.

In this letter, we propose a mathematical performance model to analyze the probability of ranging collision affected by both the capacity of the ranging code pool and the rate of the MSS arriving to the system and investigate the numerical results of the proposed model.

Our numerical analysis along with simulation results show that the ranging collision probability is more affected by the variation of arrival rate of the MSS rather than the variation of ranging code number.

By the proposed model and its numerical results, we can determine a suitable number of ranging codes in a ranging procedure that guarantees the given collision probability under various traffic environments.

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References

- Draft IEEE Standard for Local and metropolitan area networks, Part 16: Air Interface for Fixed Broadband Wireless Access Systems, IEEE P802.16-REVd/D5-2004, IEEE, May, 2004
- [2] Draft IEEE Standard for Local and metropolitan area networks, Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems, Amendment for Physical and Medium Access Control Layers for Combined Fixed and Mobile Operation in Licensed Bands, IEEE P802.16e/D5a-2004, IEEE, December 2004
- [3] D. H. Cho, J. H. Song, M. S. Kim and K. J. Han, "Performance Analysis of the IEEE 802.16 Wireless Metropolitan Area Network", *Proceedings of the First International Conference on Distributed Frameworks* for Multimedia Applications, 2005.
- [4] C. Tijim, A First Course in Stochastic Models, John Wiley and Sons, New York, 2003.