Correlation and Calibration Effects on MIMO Capacity Performance

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Abstract: MIMO wireless systems are studied in this paper with particular emphasis on the achieved performance in terms of achieved capacity, in different operational environments. First, the definition of a MIMO system is presented along with the performance advantages. Then, Shannon's extended capacity formula is discussed and a simplified expression is derived by applying linear transformations. The paper concentrates on the capacity of Rayleigh channels and then studies the case where the signals are submitted to fading correlation, furthermore a study on the effects of calibration errors regarding capacity is presented.

Key-Words: MIMO systems, capacity, Rayleigh channel, fading correlation, ergodic capacity, amplitude and phase mismatch, calibration .

1 Introduction

The MIMO channel is simply defined as the combination of a transmitter, a receiver and a wireless channel which appears to have multiple inputs and multiple outputs, as illustrated in Fig. 1. Practically, such a system is implemented with multiple antennas both at the transmitter and the receiver end.

The innovation introduced by MIMO systems is that they take advantage of the multipath induced by the radio channel, while all the technologies developed up to now had as a goal the diminution of the multipath. Based on this concept, MIMO systems offer two great advantages: First, they provide the wireless link with great capacity, and then they improve the quality of the link by decreasing the average symbol error rate (A-SER).

Due to the fact that wideband applications are increasingly demanded by a growing number of users, MIMO systems present a solution to the problem of effective exploitation of frequency spectrum, which is crucial for all telecommunication systems.

2 Capacity of a mimo channel

We consider the MIMO channel illustrated in Figure 1. In order to study the capacity we represent the channel with a $M_R \times M_T$ matrix

$$\mathbf{H} = \begin{bmatrix} h_{1,1}(\tau,t) & h_{1,2}(\tau,t) & \dots & h_{1,M_T}(\tau,t) \\ h_{2,1}(\tau,t) & h_{2,2}(\tau,t) & \dots & h_{2,M_T}(\tau,t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_R,1}(\tau,t) & h_{M_R,2}(\tau,t) & \dots & h_{M_R,M_T}(\tau,t) \end{bmatrix} \tag{1}$$

The matrix elements are complex numbers that represent the attenuation and the phase shift of the signal that arrives to the receiver with a delay of τ sec. In that case the MIMO system may be described in matrix notation as $\mathbf{y} = \mathbf{H} \otimes \mathbf{s}(t)$ where $\mathbf{s}(t) = [s_1(t)s_2(t)\dots s_{M_T}(t)]^T$ is a $M_T \times 1$ vector which represents the signals transmitted from the M_T transmit antennas and $\mathbf{y}(t) = [y_1(t)y_2(t)\dots y_{M_R}(t)]^T$ is an $M_R \times 1$ vector which represents the signals received from the M_R receive antennas.

The MIMO channel capacity is given by Shannon's extended formula as

$$C = \max_{tr(\mathbf{R}_{ss})\leq p} \log_2 \left[\det(\mathbf{I} + \mathbf{H}\mathbf{R}_{ss}\mathbf{H}^H)\right] \tag{2}$$

its proof could be found in [2]. In equation (2) the matrix $\mathbf{H}^H$ is the transpose conjugate of the channel matrix $\mathbf{H}$, $\mathbf{R}_{ss}$ is the covariance matrix of the transmitted signal vector $\mathbf{s}(t)$ and p is the maximum normalized average transmit power.

2.1 Simplified capacity formula

As we mentioned earlier, we consider a linear MIMO system. As a result, by means of a linear trans-

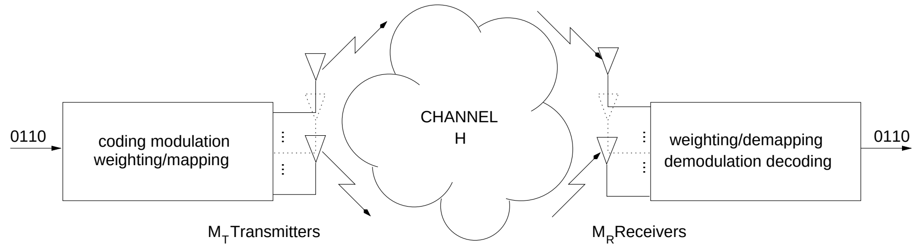


Figure 1: A MIMO system with M_T transmitting antennas and M_R receiving antennas.

formation, the MIMO channel can be transformed into $n = \mathrm{rank}(\mathbf{H})$ uncorrelated single input single output (SISO) subchannels. This transformation leads to a simplified formula for capacity which is presented in equation (3),

$$\sum_{k=1}^{n} \log_2(1 + p_k \varepsilon_k^2) \tag{3}$$

with the power restriction $\sum_{k=1}^{n} p_k \leq p$. In equation (3) the values ε_k^2 are the eigenvalues of the $\mathbf{HH}^H$ matrix and p_k is the power allocated to each subchannel. The transformations involved that leads to (3) can be found in [3].

Equation (3) indicates that the achieved capacity depends on the distribution of ε_k^2 and on the allocated power p_k. As a result, the MIMO system capacity depends on the algorithm that is used for allocating power to the transmitter's elements

2.2 Shannon's capacity formula without channel knowledge at the transmitter

All the theoretical analysis considers the Channel State Information (CSI) [1] known to the receiver. This consideration stands as the receiver usually performs tracking methods in order to obtain the CSI, while it does not stand for the transmitter case. In case the channel is not known at the transmitter, the signals to be transmitted are equi-powered at the transmit antennas. Referring to Fig.1 the power allocated to each of the M_T elements is $p_k = \frac{p}{M_T}$. In that case the $\mathbf{R}_{ss}$ matrix of equation (2) equals the identity matrix ($\mathbf{R}_{ss} = \mathbf{I}$).

We use the above equations to equations (2) and (3). The capacity expressions that are derived are shown in equations (4), (5).

$$C = \log_2 \left[\det(\mathbf{I} + \frac{p}{M_T}\mathbf{HH}^H) \right] \tag{4}$$

$$\sum_{k=1}^{n} \log_2(1 + \frac{p}{M_T}\varepsilon_k^2) \tag{5}$$

Equation (5) indicates that the capacity of a MIMO channel can be expressed by the sum of the capacities of $n = \mathrm{rank}(\mathbf{H})$ SISO channels, each having power gain ε_k^2 and transmit power p/M_T.

In cases where the CSI is known to transmitter, the power allocation to transmitter elements can be performed based on the waterfiling algorithm [4].

3 Capacity of Rayleigh channels

In this section we present the capacity formula of Rayleigh channel. It should be mentioned that throughout the following analysis the channel is not known at the transmitter and as a result equations (4) and (5) are used for the channel capacity.

When the wireless environment is characterized by strong multipath, the envelope of the received signal follows the Rayleigh distribution. However, the Rayleigh model can not be applied in three cases. First, when the limited number of paths between the transmitter and the receiver prohibit the use of the central limit theorem. Then, in cases that the location of buildings leads to the waveguide phenomenon and finally, in areas near the base station where a line of sight (LOS) component may dominate. For the last case the envelope of the received signal follows the Ricean distribution.

3.1 Channel matrix for Rayleigh fading

The channel matrix $\mathbf{H}$ in equation (1) depends on the channel model. Specifically, when the conditions of the environment permit the use of a Rayleigh model and the antennas of the transmitter and the receiver are sufficiently separated, the elements of the channel matrix $\mathbf{H}$ can be modeled as zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables, with unit variance. The resulting matrix is symbolized $\mathbf{H}_W$ and is referred as spatially white matrix.

The capacity formula under the assumptions of

Rayleigh channel and equal power allocation is:

$$C = \log_2 \left[\det \left(\mathbf{I} + \frac{p}{M_T} \mathbf{H}_W \mathbf{H}_W^H \right) \right] \tag{6}$$

Equation (6) is used in the final section for the simulations concerning the Rayleigh channel.

3.2 Channel matrix for spatial fading correlation

The Rayleigh channel assumes flat fading in the space, time and frequency domain. However, the signal components arriving at the receiver may experience correlation due to the limited distance of the antenna elements. In that case, the use of $\mathbf{H}_W$ as the channel matrix is inappropriate.

The model used in order to take under consideration the aforementioned correlation is described by the equation: $\mathrm{vec}(\mathbf{H}) = \mathbf{R}^{\frac{1}{2}} \mathrm{vec}(\mathbf{H}_W)$
where $\mathrm{vec}(\mathbf{H})$ denotes a vector [1] made by the columns of $\mathbf{H}$ and $\mathbf{R}$ is the $M_R M_T \times M_R M_T$ covariance matrix of the channel.

In order to simplify that model, we assume that the reception correlation matrix, $\mathbf{R}_R$, is independent of the transmitting element. The same assumption is made for the transmission correlation matrix,$\mathbf{R}_T$. In this case the channel matrix is given by equation (7).

$$\mathbf{H} = \mathbf{R}_R^{\frac{1}{2}} \mathbf{H}_W \mathbf{R}_T^{\frac{1}{2}} \tag{7}$$

Correlation matrices $\mathbf{R}_T$, $\mathbf{R}_R$ can be calculated using several models. The model that will be used in the following simulations calculates these matrices as a function of the distance,d, between the receiving/transmitting elements and is described in detail in [5].

3.3 Modeling phase and amplitude mismatch

In this section we study the capacity achieved by the MIMO system when amplitude and phase distortion is introduced at the transmitter. The introduced distortion is represented by a $M_T \times M_T$ diagonal matrix

$$\mathbf{C_T} = \begin{bmatrix} C_{1,1}e^{j\theta_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C_{M_T,M_T}e^{j\theta_{M_T}} \end{bmatrix} \tag{8}$$

The amplitude $C_{i,i}$ is real and represents the amplitude distortion induced to the transmitted signal by the i^{th} transmitting chain leading to the i^{th} element. The phase $\theta_{i,i}$ is the corresponding phase distortion.

The method that is used in this paper in order for the amplitude and phase mismatch to be considered in the capacity calculations is based on the followings observations.

The distortion matrix described in (8), is multiplied with the $M_T \times 1$ signal vector that is launched from the transmitter. So the input-output relation, mentioned above, for the MIMO channel may be expressed as $\mathbf{y} = \mathbf{H} \otimes (\mathbf{C_T} \cdot \mathbf{s})$ or under the narrowband assumption $\mathbf{y} = \mathbf{H} \cdot (\mathbf{C_T} \cdot \mathbf{s})$. The last equation can be rewritten as

$$\mathbf{y} = (\mathbf{H} \cdot \mathbf{C_T}) \cdot \mathbf{s} \tag{9}$$

The last equation indicates that the simplest way in order to consider the introduced distortion in our theoretical capacity calculations is by multipling the channel matrix $\mathbf{H}$ with matrix $\mathbf{C_T}$ described in (8). The total channel matrix is then, $\mathbf{H}' = \mathbf{H_W} \cdot \mathbf{C_T}$

The simulation that take place afterwards consider a normalized channel matrix. The normalization is given in (10) and is performed on each realization of the end to end channel.

$$\mathbf{H}^i_{norm} = \mathbf{H} \left[\|\mathbf{H}^i\|_F^2 / M_T M_R \right]^{-\frac{1}{2}} \tag{10}$$

where $\| \bullet \|$ is the Frobenius norm of the channel matrix.

4 Capacity of stochastic channels

Rayleigh channel is stochastic channel and as a result, the capacity of this channel is a random variable. In order to study the capacity of stochastic channels we use two statistical quantities.

The *ergodic capacity* of a MIMO channel is the ensemble average of the information rate over the distribution of the elements of the channel matrix $\mathbf{H}$[6]. In case of no CSI at the transmitter, the ergodic capacity is given by

$$\overline{C} = E \left[\log_2 \left(\det \left(\mathbf{I} + \frac{p}{M_T} \mathbf{H}\mathbf{H}^H \right) \right) \right] \tag{11}$$

Figure 2 illustrates the ergodic capacity for different antenna configurations as a function of the SNR, when the channel is unknown at the transmitter. As expected, the ergodic capacity increases with SNR. In addition, the ergodic capacity of a single input multiple output (SIMO) channel $M_R \times 1$ appears to be greater than the ergodic capacity of a multiple input single output (MISO) $1 \times M_T$. The reason for that is discussed in the following section.

[1] If $\mathbf{H} = [h_1 h_2 ... h_{M_T}]$ is $M_R \times M_T$ then $vec(\mathbf{H}) = [h_1^T h_2^T ... h_{M_T}^T]$ is $M_R M_T \times 1$.

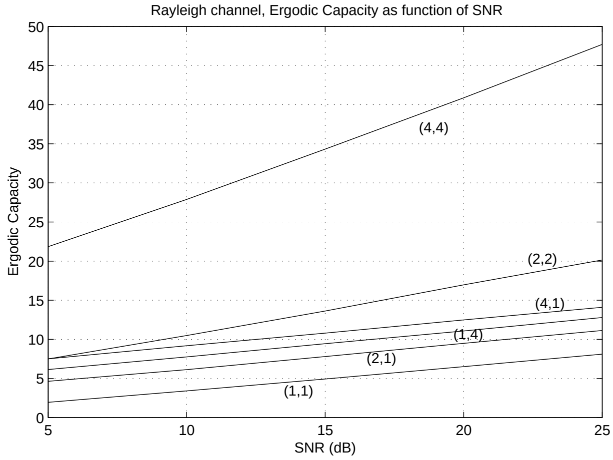


Figure 2: Ergodic capacity for different antenna configurations. The label of each plot line represents the channel ($M_R \times M_T$).

The *outage capacity* quantifies the level of capacity performance guaranteed with a certain level of reliability. For example, q% outage capacity, $C_{out,q}$, indicates that the system can achieve minimum capacity level $C_{out,q}$ with probability (100-q)%.

5 Simulations

The figures resulted from the simulations are presented on the next page.

5.1 Rayleigh channel without spatial fading correlation

In this case the channel matrix that it is used for capacity calculations is $\mathbf{H}_W$. This matrix is full-ranked as its elements are independent variables that follow the ZMCSCG distribution. As a result the MIMO channel is transformed into exactly $n = \text{rank}(\mathbf{H}_W) = \min(M_R, M_T)$ SISO subchannels.

Figure 3a indicates that increasing the number of antenna elements leads to a capacity increase. Especially, we notice that the a large capacity increase involves array antennas at both the transmitter and the receiver. For example, an (8,1) MIMO channel supports lower capacity gain than the (2,2) MIMO channel. This is justified through the MIMO system transformation concept mentioned earlier. Specifically, the (8,1) channel gives n=1, while the (2,2) gives n=2, considering now the fact that the independent SISO subchannels that are created are responsible for the information transfer we can justify the result.

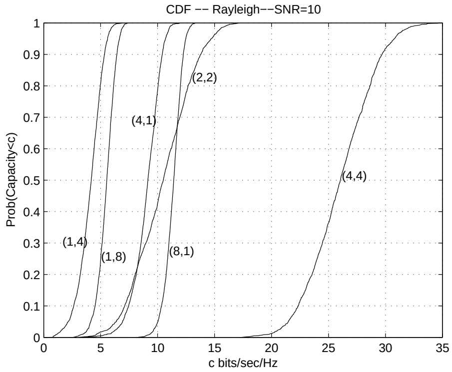
Finally, Figure 3a indicates that the presence of an array antenna at the receiver is more important than the presence of the same array antenna at the transmitter. For example we can notice that the channel (4,1) presents better capacity behaviour in comparison with the channel (1,4). The explanation for this lies in the assumption that the transmitter does not have CSI and as a result it 'equi-powers' the elements regardless of the channel. On the contrary, the receiver is considered to possess this information and as a result it may use its array antenna for optimum combining based on CSI.

5.2 Rayleigh channel with spatial fading correlation

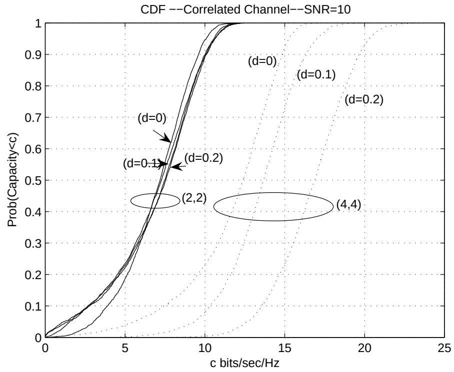
In this case the channel matrix that it is used for capacity calculations is given by equation (7). Figure 3b illustrates the CDFs of capacity for different antennas configurations and uses as a parameter the interelement spacing, d.

First, we can see that as the distance between the antenna elements decreases the capacity decreases too. The reason lies in the increase of correlation with the decrease of the elements' distance. The correlation of the transmitted and received signals causes the decrease of the independent propagation paths and as a result, the decrease of the information transmitted. The independent paths between the transmitter and the receiver are also called effective degrees of freedom (EDOF)[3].

At the same time, we note that the (4,4) MIMO channel presents greater capacity gains compared to the (2,2) channel under the same correlation conditions. This is shown with the two circles drawn at Figure 3b, where the CDFs of (4,4) channel are shifted to the right. As a result, we realize that MIMO systems can diminish the problems caused to capacity by fading correlation in

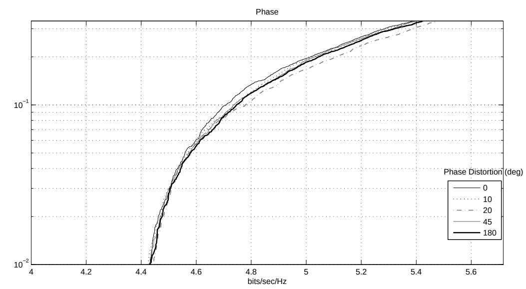


(a) CDFs of capacity for the Rayleigh MIMO channel with a SNR of 10dB.

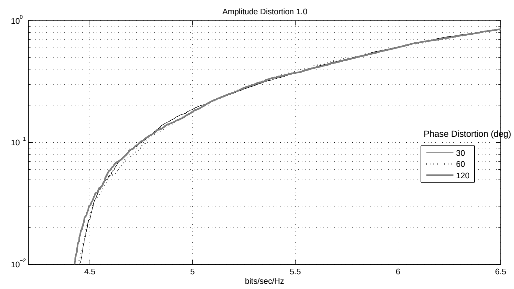


(b) CDFs of capacity for the Rayleigh MIMO channel with spatial fading correlation.

Figure 3: CDFs of Rayleigh channel capacity.



(a) CDF of capacity in the case of a 2x2 channel for different phase distortions.



(b) CDF of capacity in the case of a 2x2 channel for different phase distortions and constant amplitude distortion.

Figure 4: CDFs of capacity in the case of amplitude and phase mismatch.

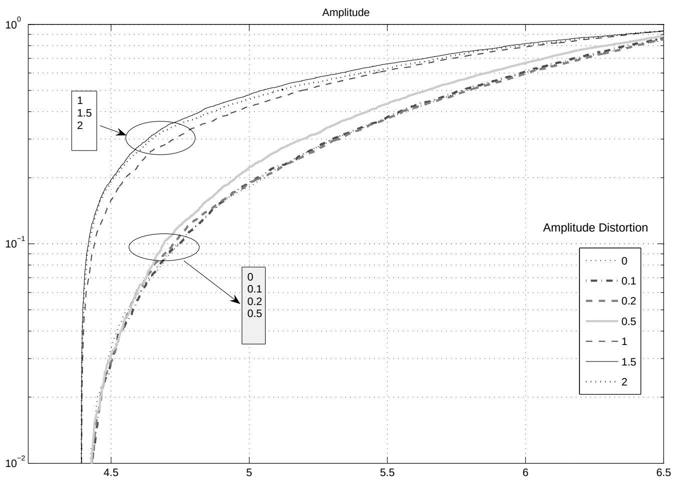


Figure 5: CDF of capacity in the case of a 2x2 channel for different amplitude distortion.

general, and not only the correlation induced to signals due to the interelement spacing.

Finally, we can mention that the more the antenna elements, the more the capacity is affected by spatial correlation. This is expected because of the induction phenomenon.

5.3 MIMO capacity with amplitude and phase mismatch

First, we should mention that the studies regarding the effect of amplitude and phase mismatch on MIMO system is based on the theoretical extended Shannon's capacity formula. The aim here is to study how the calibration errors affect the capacity based on the considered MIMO system implementation scheme.

The effect of calibration distortion might be different for specific MIMO implementation schemes, such as beamforming or diversity and hence, more analysis is currently under way.

Figure 5 indicates that increasing the amplitude distortion factor leads to a capacity decrease. This is more evident when the amplitude distortion increases from 0.5 to 1 which is due to the negative amplitude values that start to appear.

Figures 4a and 4b show how phase distortion influences capacity. The figures imply that phase does not affect the capacity of the MIMO channel. This is justified since we used the general capacity formula for the MIMO channel that does not consider phase dependency for the elements power.

6 Conclusions

This paper presented the key issues related with the MIMO systems. It describes the behaviour of MIMO system capacity under two different cases of operational environments. The conclusions can be summarized as follows.

The capacity of the Rayleigh MIMO channel increases substantially when both the transmitter and the receiver use array antennas. In case of no CSI at the transmitter, the use of an array antenna at the receiver is more important than the use of the same array antenna at the transmitter. In case that the insufficient interelement distance at the transmitter and/or the receiver introduces spatial fading correlation to the Rayleigh MIMO channel the capacity decreases. The problem is diminished with the use of more antenna elements, which, however, cause stronger capacity variability due to spatial fading correlation.

Finally, the paper presented initial results for the effect of calibration distortion (amplitude and phase mismatches) on the achieved capacity and showed that there is stronger dependency on amplitude rather than phase.

7 Acknowledgments

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