Analysis of the Initial Point Separate Surface for the Minimal-Time System Design Process

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Abstract: - The set of the different design strategies for analog system design were analyzed on the basis of the new system design methodology. Optimal position for the design algorithm start point was analyzed to minimize the computer design time. The initial point selection has been done on the basis of the before discovered acceleration effect of the system design process. The geometrical dividing surface was defined and analyzed to obtain the optimal position of the algorithm start point. Numerical results of both passive and active nonlinear electronic circuit design prove the possibility of the optimal selection of the design algorithm start point.

Key-Words: - Quasi optimal algorithm, control theory application, optimal start point selection.

1 Introduction

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement. Besides the traditionally used ideas of sparse matrix techniques and decomposition techniques [1]-[5] some another ways were determine to reduce the total computer design time. The generalized theory for the system design on the basis of the control theory formulation was elaborated in some previous works [6], [7]. This approach serves for the time-optimal design algorithm definition. On the other hand this approach gives the possibility to analyze with a great clearness the design process while moving along the trajectory curve into the design space. A general methodology we named the General Design Strategy (GDS). It includes a set of the different design strategies. The traditional design methodology, that we call Traditional Design Strategy (TDS), and the Modified Traditional Design Strategy (MTDS) are two extreme cases of the GDS, when all the circuit model equations are solved during the design process (TDS) or all he circuit equations are eliminated from the circuit model during the design process (MTDS). It is necessary to introduce the special penalty function for the MTDS [6]. The optimization procedure in this case is realized in broadened optimization space, but as described in [8] for the complex system synthesis a

significant computer time gain can be obtained. The GDS combine all the possibilities using some equations in the system representation as in TDS and some of the so call penalty functions in the objective function as in MTDS. Such possibilities are selected by means of the control function vector U [7], the result is 2^{M} possible design strategies, where M is the number of the system dependent parameters.

The use of GDS allows the appearances of the acceleration effect [9]-[10], as consequence the total computer time diminish, however such effect is highly dependent of initial values for the design process [11]. In order to construct an optimal algorithm for the design process it is necessary to characterize the behavior of the design trajectories and therefore of the time consumption, according to the start point and give a rule for the automatic selection of better initial values. The present work is intended to answer some of the questions associated with the selection of initial conditions needed in the quasi optimal algorithm for design process.

This paper is organized as follows: in section 2 the necessary definitions are made; section 3 is devoted to the separate surface analysis, which is divided in two subsections, one for passive circuits and other one for active circuits; finally conclusions are made in section 4.

2 **Preliminary definitions**

We divide the vector of the system state variables X into two parts: the vector of the independent variables X', $X'=(x_1, x_2, ..., x_K)$, where K is the total number of independent design variables and the vector of the dependent variables X'', $X''=(x_{K+1}, x_{K+2}, ..., x_{K+M})$ where M is the total number of dependent design variables.

The design process for any analog system design can be defined [6] as the problem of the generalized objective function F(X, U) minimization by means of the vector equation:

$$X^{s+1} = X^s + t_s \cdot H^s \tag{1}$$

with the constraints:

$$(1-u_j)g_j(X) = 0, \quad j = 1, 2, ..., M$$
 (2)

where $g_j(X)$ for all j is the circuit model, s is the iterations number, t_s is the iteration parameter, $t_s \in \mathbb{R}^1$, $H \equiv H(X,U)$ is the direction of the generalized objective function F(X,U) decreasing, U is the vector of the special control functions $U = (u_1, u_2, ..., u_m)$, where $u_j \in \Omega$; $\Omega = \{0;1\}$. The complete objective function is defined now as the sum of three parts:

$$F(X,U) = C(X) + D(X) + \frac{1}{e} \sum_{j=1}^{M} u_j g_j(X)$$
 (3)

where C(X) is the initial objective function of the design process, D(X) is an additional term to avoid the different solutions for the different design strategies and to overcome the problem of non simple solution obtain, and the last terms in formula (3) is the additional penalty functions, and e is a weight factor with next property $0 < e \le 1$. This problem formulation permits to redistribute the computer time expense between the problem (2) solve and the optimization procedure (1) for the function F(X,U). The control vector U is the main instrument for the redistribution process in this case.

3 The Separate Surface

The acceleration effect of design process has strong dependency of initial point selection [10]; it may not

appear under certain circumstances. How we can achieve the acceleration effect by means of the initial point variation? To answer this question a non conclusive but extensive study, for passive and active circuits, has been done in [11]. A systematic idea for selecting initial values is a geometrical one, and it is based on the conception of the Separate Surface, which is defined as the surface that divides the total design space in two subspaces. The first subspace includes all points that can produce the acceleration effect and the second one defines the points that can not be used for the acceleration effect obtaining.

The shape of Separate Surface represents the behavior of the design process according to the start point, and it is different for different type of circuits, for that reason the study of separate surface made here is divided in passive circuits and active circuits.

3.1 Passive Circuits Separate Surface

Analysis of separate surface for passive circuits of different nodes is presented in this section for linear and non linear examples. The description for the design process is made briefly in each example; the complete emphasis is put on results related with the separate surface concept.

3.1.1 Example 1

Fig. 1 shows a simple one node passive circuit, where the resistance *r* has an non linear dependency given as: $r = r_1 + b_n V_1$, where r_1 is a constant, and b_n is a non linearity factor.



Fig. 1. Simplest non linear passive circuit.

There are two variables in this circuit, resistor r_2 as independent parameter (K = 1) and voltage V_1 as dependent parameter (M = 1). The vector X has two components: $X = (x_1, x_2)$, where $x_1^2 \equiv r_2$ and $x_2 \equiv V_1$. The main equation for this circuit is given by the Kirchhoff law as follows:

$$g_1(X) \equiv x_2 \left(x_1^2 + r_1 + b_n x_2 \right) - x_1^2 = 0 \qquad (4)$$

The simple objective function can be defined as $C(X) = (x_2 - k_v)^2$, where k_v is the design condition.

Fig. 2 shows the family curves of MTDS, for the linear case, $b_n = 0$. Both variables for MTDS are independent. The initial value of the first variable has been set to 1 ($x_1^0 = 1$), and the second variable (x_2^0) has variation of the initial values generates the family curves.



Fig. 2. ETDM family curves, $b_n = 0$.

For this particular example where X vector has two parameters, the surface that divides design space is more a line than actually a surface. The dark line in Fig. 2 represents a separate line type 1 (SL 1), this is a curve that divides the design trajectories that arrive to the final point F from the left side, from those that arrive from the right side of the plane. The dash line in Fig. 2 shows a separate line type 2 (SL 2) that divides the trajectories in which acceleration effect is present, from those where such an effect does not appears. The SL 2 touch the T point in the imaginary line x_1^F , the final point projection of independent variable, following this line reaches the point F. The SL 2 finishes following the same path that SL 1, from F point to the right of side of the graphic over a MTDS trajectory. The interior enclosed by this curve is a prohibited zone, while in the exterior is a permitted zone for the acceleration effect.

Fig. 3 shows MTDS for the family curves for non linear case, $b_n = 1$. Initial values of the first variable have been set to 0.8, while initial values for x_2 raises the family curves. Separate lines type 1 and 2 have now more enclosed area, however, this phenomenon occurs for the positive values of x_2 , for the negative values of the variable x_2 , instead, the area enclosed



Fig. 3. ETDM family curves, $b_n = 1$.

by separate surfaces type 1 and 2 keeps similar that for thee linear case.

It is possible to say that the values of the parameter x_2 , which correspond to the condition $x_2^0 \leq -1$ guarantee the acceleration effects occurrence, for both, linear and non linear case. We will keep in main this important fact, useful in the quasi optimal algorithm construction.

3.1.2 Example 2

This example corresponds to the five cells passive circuit and it is used to analyze the separate surfaces for the passive circuits from 2 to 5 nodes (Fig. 4). The design process is defined by the equations (1)-(3) with the additional function D(X) given by the formula:

$$D(X) = \sum_{i=1}^{K-1} (x_i - d_i)^2$$
(5)

where the terms d_i are used to obtain the same final point *F* of design for all the different strategies.



Fig. 4. Five cells passive circuit with four non linear elements y_{ni} .



Fig. 5. Separate surface type 1 for passive circuits: a) two nodes, $b_{n1} = 1$ (non linear case); b) three nodes, $b_{n1} = b_{n2} = 0$ (linear case).

We begin our analysis for this example with two first cells of Fig. 4; as result of design process different design trajectories are obtained. Fig. 5 (a) shows a projection of separate surface type 1 (PSS 1), which is compose by two actually possible design trajectories. There are three independent variables fixed to 1, $x_1^0 = x_2^0 = x_3^0 = 1.0$ and two dependent variables $x_4^0 = 1.0$ and $x_5^0 = x_t^0$, where x_t is the "tracer" variable which value variations is used to obtain the MTDS family curves, in order to know the separate surface behavior. There is a non linear element, $y_{n1} = r_0 + b_{n1}(x_5 - x_4)^2$ with $b_{n1} = 1$ (non linear case). From Fig. 5 (a) it is possible to say that acceleration effect is present when $|x_5^0| > 0.38$ if $x_1^0 = x_2^0 = x_3^0 = x_4^0 = 1.0$.

Fig. 5(b) shows a projection of separate surface type 1 (PSS 1) for the circuit of Fig. 4 but now using cells 1, 2 and 3, with initial values:

$$x_1^0 = x_2^0 = \dots = x_N^0 = 1.0$$
(6)

where N = 6 and $x_7^0 = x_t^0$. From Fig. 5(b), acceleration effect is present when $x_7^0 > -0.72$ and $x_7^0 > 0.52$ or more compact $|x_7^0| > 0.72$, if the condition (6) is accomplished with $b_{n1} = b_{n2} = 0$ (linear case).

Continuing with the increment of nodes for the separate surface behavior analysis Fig. 6 shows separate surface type 1 and 2 for (a) four nodes and (b) five node passive circuits. Separate surface of Fig. 6(a) corresponds to the circuit of Fig. 4 using cells 1, 2, 3 and 4, there are five independent variables and four dependent variables in this case. Initial conditions satisfies equation (6) with N = 8 and $x_9^0 = x_t^0$. For condition $|x_9^0| > 0.76$ acceleration effect is present.



Fig. 6. Separate surface type 1 and 2 for passive circuits: a) four nodes, $b_{n1} = b_{n2} = b_{n3} = 0$ (linear circuit); b) five nodes, $b_{n1} = \cdots = b_{n4} = 1$ (non linear circuit).

Finally Fig. 6(b) shows separate surface for a non linear passive circuit, with six independent, five dependent variables and four non linear parameters y_{n1}, \dots, y_{n4} , which values satisfies equation (6) with N = 10 and $x_{11}^0 = x_t^0$. Separate surface type 2 appears when $x_6^0 = 0.63$ and $x_6^0 = -0.75$, the area enclosed by corresponding design trajectories is the prohibited region for the acceleration effect of this circuit.

3.2 Active Circuits Separate Surface

An active circuit of three nodes and its equivalent circuit are shown in Fig. 7. There are three independent variables x_1, x_2, x_3 which initial values are set 1, $x_1^0 = x_2^0 = x_3^0 = 1.0$; and three dependent variables x_4, x_5, x_6 .

The variation of the initial value of the dependent variable (x_6^0) generates the MTDS family curves (Fig. 8). On the odder hand, variables x_4 , x_5 are used





Fig. 7. One transistor amplifier: a) three nodes active circuit; b) equivalent circuit using an Ebers-Moll model.

to set up initial conditions for the transistor union voltages, as follows:

$$V_{BEi} = x_4^0 - x_5^0 = 0.3V \tag{7}$$

$$V_{CBi} = x_4^0 - x_6^0 = -0.1V \tag{8}$$

These initial conditions are important for the numerical stability since SPICE model of Bipolar Junction Transistor (BJT) [12] has FV exponential terms and great values of the union voltages in combination with Newton-Raphson algorithm, drives up into unstable design region [13].

The simple objective function that is used to reach the design requirements is given as:

$$C(X) = (x_6 - k_v) + (V_{BE} - r_1)^2 + (V_{BC} - r_2)^2 \quad (9)$$

where the design condition is selected by $k_v = 6.79V$, and terms $\mathbf{r}_1, \mathbf{r}_2$ are the final desired values for transistor's unions. The corresponding MTDS family curves and separate surfaces type 1



Fig. 8. MTDS family curves and projections of separate surfaces type 1 and 2 for one transistor amplifier.

and 2 of design process for the circuit of Fig. 7, are shown in Fig. 8. Separate surface type 1 is defined when $x_6^0 = 8.7$ and $x_6^0 = 0.1$, while separate surface type 2 is defined when $x_6^0 = 8.7$ and $x_6^0 = 2.0$, in both cases if conditions (8) and (9) are satisfied. In consequence the acceleration effect is present for this example, when $x_6^0 < 2.0 \lor x_6^0 > 8.7$, again if the conditions (8) and (9) are satisfied.

This analysis made for one transistor circuit can be repeated for the circuits with more transistors, finding in all analyzed circuits the separate surfaces and therefore the acceleration effect of the design process.

4 Conclusion

One of the main conclusions is that the geometrical idea of separate surface is very useful in selecting initial values of the design process, and it always appears in the circuits we analyzed. There are two types of separate surfaces, each of them divides the design spice, and give us the possibility to now in advance if the acceleration will be possible to obtain. Furthermore this new conception of separate surface is useful in choosing the start point of design process, which is the first step in the quasi optimal algorithm intended for an optimal design of the circuits and systems. Such an algorithm is under development, and results presented here are important in its structure determination.

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