Time-Frequency Modeling and Estimation of Wireless OFDM Channels

AYDIN AKAN[1] and LUIS F. CHAPARRO[2]
[1] Department of Electrical and Electronics Engineering, Istanbul University
Avcilar, 34320, Istanbul, TURKEY
[2] Department of Electrical and Computer Engineering, University of Pittsburgh
Pittsburgh, PA 15261, USA

Abstract: - Orthogonal Frequency Division Multiplexing (OFDM) has become a very popular method for high data rate wireless communications because of its advantages over single carrier modulation schemes on multi-path, frequency selective fading channels. However, inter-carrier interference due to Doppler frequency shifts, and multi-path fading severely degrades the performance of OFDM systems. Estimation of channel parameters is required at the receiver. In this paper, we present a channel modeling and estimation method based on time-frequency representation of the received signal. The Discrete Evolutionary Transform provides a time-frequency procedure to obtain a complete characterization of the multi-path, fading and frequency selective channel. Performance of the proposed method is tested on different levels of channel noise, Doppler frequency shifts, and jamming interference powers.

Key-Words: - Wireless communications, OFDM, Channel estimation, Evolutionary spectrum.

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is considered an effective technique for broadband wireless communications because of its great immunity to fast fading channels and inter-symbol interference (ISI). It has been adopted in several wireless standards such as digital audio broadcasting (DAB), digital video broadcasting (DVB-T), the wireless local area network (W-LAN) standard; IEEE 802.11a, and the metropolitan area network (W-MAN) standard; IEEE 802.16a [1, 2]. OFDM partitions the entire bandwidth into parallel subchannels by dividing the transmit data bitstream into parallel, low bit rate data streams to modulate the subcarriers of those subchannels. As such OFDM has a relatively longer symbol duration than single carrier systems (due to the lower bit rate of subchannels) which makes it very immune to fast channel fading and impulse noise. The independence among the subchannels simplifies the design of the equalizer. Because of all these advantages, OFDM is becoming a standard in digital audio / video broadcasting and wireless communications. However, inter-carrier interference (ICI) due to Doppler shifts, phase offset, local oscillator frequency shifts, and multi-path fading severely degrades the performance of multi-carrier communication systems [1, 3]. For fast-varying channels, especially in mobile systems, large fluctuations of the channel parameters are expected between consecutive transmit symbols. Estimation of the channel parameters is required to employ coherent receivers. Most of the channel estimation methods assume a linear time–invariant model for the channel, which is not valid for fast varying environments [4, 5]. A complete time-varying characterization of the channel can be obtained by employing time-frequency representation methods.

We present a time–varying channel modeling and estimation method based on the time–frequency representation of channel output. The Discrete Evolutionary Transform (DET) provides a time-frequency representation of the received signal by means of which the spreading function of the multi-path, fading and frequency-selective channel can be modeled and estimated.

[1] This work was supported by The Scientific and Technical Research Council of Turkey, TUBITAK-BAYG and by The Research Fund of The University of Istanbul, Project numbers: UDP-329/03062004 and BYP-324/18122003.

2 OFDM System Model

In an OFDM communication system, the available bandwidth B_d is divided into K subchannels. The input data is also divided into K-bit parallel bit streams. These bit streams are mapped into some complex constellation points: $X_{n,k}, \quad k = 0, 1, \cdots K-1$ where n is the time index and k is the frequency index. Blocks of data are modulated onto a set of subcarriers of corresponding subchannels with bandwidth $\Delta_f = B_d/K$. The modulation is efficiently implemented using a K-point Inverse Discrete Fourier Transform (IDFT). Then the data are passed through a Parallel/Serial (P/S) converter to form a serial data stream $x_{n,k}$. Before sending the $x_{n,k}$'s to the channel, last L_{CP} samples are inserted in front and called the Cyclic Prefix (CP). This is done to mitigate the effects of intersymbol interference (ISI) caused by the channel time spread [1, 2]. The length of the CP is taken at least equal to the length of the channel impulse response.As a result, the effects of the ISI are easily and completely eliminated. Furthermore, the receiver can implement demodulation of the OFDM by using fast signal processing algorithms such as FFT.

For a channel of bandwidth B_d and K subchannels, symbol duration is $T = \frac{1}{\Delta_f} = \frac{1}{B_d/K} = \frac{K}{B_d}$. However, the actual block duration is $T_f = \frac{K+L_{CP}}{B_d}$. For a system with $B_d = 800kHz.$, $K = 512$ and $L_{CP} = 64$, $T_f = \frac{512+64}{800kHz} = 720\mu s.$

At the receiver, the CP part is eliminated. Demodulation is performed by a K-point DFT operation on $x_{n,k}$ to get $R_{n,k}$. If the CP is long enough, the interference between two OFDM blocks is eliminated and the subchannels can be viewed as independent of each other, i.e., $R_{n,k} = H_{n,k}\ X_{n,k} + N_{n,k}$, where $H_{n,k}$ are the samples of channel frequency response at $n\Delta_f$ of the n^{th} block, and $N_{n,k}$ is the Fourier transform of the additive Gaussian white (AGWN) channel noise with zero mean and σ^2 variance. A simple equalizer is sufficient for each subchannel at the receiver, i.e., $\hat{X}_{n,k} = R_{n,k}/H_{n,k}$. Then the decision is made upon $\hat{X}_{n,k}$. The channel estimation problem is to obtain the channel parameters $H_{n,k}$.

2.1 Channel Model

In wireless communications, the multi-path, fading channel with Doppler frequency shifts is generally modeled as a linear time-varying system with the following impulse response [6, 7]:

$$h(t, \tau) = \sum_{i=0}^{L-1} \gamma_i(t)\delta(\tau - \tau_i) \tag{1}$$

where $\gamma_i(t)$ are independent Gaussian processes with zero mean, σ_i^2 variance, and normalized overall power, τ_i are delay profiles describing the channel dispersion with τ_{max} as the maximum delay and L is the total number of paths. The variance σ_i^2 is a measure of the average signal power received at path i, characterized by the relative attenuation of that path, α_i. In the dicrete–time, the channel can be modeled by

$$h(m, \ell) = \sum_{i=0}^{L-1} \alpha_i\ e^{j\psi_i m}\ \delta(\ell - N_i) \tag{2}$$

where ψ_i represents the Doppler frequency, α_i is the relative attenuation, and N_i is the time delay caused by path i. The Doppler frequency shift ψ_i, on the carrier frequency ω_c, is caused by an object with radial velocity v and can be approximated by

$$\psi_i \cong \frac{v}{c}\ \omega_c, \tag{3}$$

where c is the speed of light in the transmission medium [8]. In wireless mobile communication systems, with high carrier frequencies, Doppler shifts become significant and have to be taken into consideration. The channel parameters cannot be easily estimated from the impulse response, however the estimation problem can be solved in the time-frequency plane by means of the so called spreading function. The generalized transfer function of this linear time–varying channel is obtained by taking the discrete Fourier transform (DFT) with respect to ℓ, i.e.,

$$H(m, \omega_k) = \sum_{i=0}^{L-1} \alpha_i\ e^{j\psi_i m}\ e^{-j\omega_k N_i} \tag{4}$$

where $\omega_k = \frac{2\pi}{K}k, k = 0, 1, \cdots, K-1$. Furthermore, the spreading function of the channel is obtained by calculating the DFT of $h(m, \ell)$ with respect to m,

$$S(\Omega_s, \ell) = \sum_{i=0}^{L-1} \alpha_i \delta(\Omega_s - \psi_i)\delta(\ell - N_i) \tag{5}$$

which displays peaks located at the time-frequency positions determined by the delays and the corresponding Doppler frequencies, and with α_i as their amplitudes

[8]. If we extract this information from the received signal, we should then be able to figure out the transmitted data symbol.

3 Channel Modeling and Estimation for Wireless OFDM Systems

Assume we are given bit stream b_n converted into N-bit parallel blocks, and then mapped onto some transmit symbols $X_{n,k}$ drawn from an arbitrary constellation points where $n \in \mathcal{Z}$ is the time index, $\mathcal{Z}$ is the set of integers, and $k = 0, 1, \cdots, K-1$, denotes the frequency or subcarrier index. We then insert some pilot symbols, $p_{n,k} \in \{-1, 1\}$ at some pilot positions (n', k'), known to the receiver: $(n', k') \in \mathcal{P} = \{(n', k')|n' \in \mathcal{Z}, k' = iS + (n'\text{mod}(S)), i \in [0, P-1]\}$ where P is the number of pilots, and the integer $S = K/P$ is the distance between adjacent pilots in an OFDM symbol [6].

The n^{th} OFDM symbol $s_n(m)$ is obtained by taking the inverse discrete Fourier transform (IDFT) and then adding a cyclic prefix of length L_{CP} (where L_{CP} is chosen such that $L \leq L_{CP} + 1$, and L is the time-support of the channel impulse response.)

$$s_n(m) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{n,k} e^{j\omega_k m} \tag{6}$$

$m = -L_{CP}, -L_{CP}+1, \cdots, 0, \cdots, K-1$ where again $\omega_k = \frac{2\pi}{K}k$, and each OFDM symbol has $N = K + L_{CP}$ length. The overall transmit symbol is then given by $s(m) = \sum_n s_n(m - nN)$. The channel output suffers from multi-path propagation, fading and Doppler frequency shifts introduced by the nature of the wireless channel:

$$\begin{aligned} y_n(m) &= \sum_{\ell=0}^{L-1} h(m, \ell)\, s_n(m-\ell) \\ &= \sum_{i=0}^{L-1} \alpha_i\, e^{j\psi_i m}\, s_n(m - N_i) \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{n,k} \sum_{i=0}^{L-1} \alpha_i\, e^{j\psi_i m}\, e^{j\omega_k (m-N_i)} \end{aligned}$$

The transmit signal is also corrupted by Additive White Gaussian Noise $\eta(m)$ over the channel. The received signal for the n^{th} frame can then be written as $r_n(m) = y_n(m) + \eta_n(m)$. The receiver discards the Cyclic Prefix and demodulates the signal using a K-point DFT as

$$\begin{aligned} R_{n,k} &= \frac{1}{\sqrt{K}} \sum_{m=0}^{K-1} [y_n(m) + \eta_n(m)]\ e^{-j\omega_k m} \\ &= \frac{1}{K} \sum_{s=0}^{K-1} X_{n,s} \sum_{i=0}^{L-1} \alpha_i\, e^{-j\omega_s N_i} \\ &\times \sum_{m=0}^{K-1} e^{j\psi_i m}\, e^{j(\omega_s - \omega_k)m} + Z_{n,k} \end{aligned} \tag{7}$$

If the Doppler effects in all the channel paths are negligible, $\psi_i = 0,\ \forall i$, then the channel is almost time–invariant within one OFDM symbol. In that case, the last summation in the above equation gives $K\,\delta(s-k)$, and

$$\begin{aligned} R_{n,k} &= X_{n,k} \sum_{i=0}^{L-1} \alpha_i\, e^{-j\omega_k N_i} + Z_{n,k} \\ &= X_{n,k}\, H_{n,k} + Z_{n,k} \end{aligned} \tag{8}$$

where the channel frequency response $H_{n,k}$ is the discrete Fourier transform of $h(nN, \ell)$, and $Z_{n,k}$ is the Fourier transform of the noise, $\eta(nN + m)$. By estimating the channel frequency response coefficients $H_{n,k}$, data symbols, $X_{n,k}$, can be recovered according to equation (8). However, if there are large Doppler frequency shifts in the channel, then the time–invariance assumption above is no longer valid. Here we consider time–varying channel modeling and estimation and approach the problem from a time–frequency point of view [7, 8]. In the following we briefly explain the Discrete Evolutionary Transform as a tool for the time–frequency representation of non–stationary signals.

3.1 The Discrete Evolutionary Transform

A non-stationary signal, $x(n), 0 \leq n \leq N-1$, may be represented in terms of a time-varying kernel $X(n, \omega_k)$ or its corresponding bi-frequency kernel $X(\Omega_s, \omega_k)$. The time–frequency discrete evolutionary representation of $x(n)$ is given by [9],

$$x(n) = \sum_{k=0}^{K-1} X(n, \omega_k) e^{j\omega_k n}, \tag{9}$$

where $\omega_k = 2\pi k/K$, K is the number of frequency samples, and $X(n, \omega_k)$ is the evolutionary kernel.

The discrete evolutionary transformation (DET) is obtained by expressing the kernel $X(n, \omega_k)$ in terms of the signal. This is done by using conventional signal representations [9]. Thus, for the representation in (9) the DET that provides the evolutionary kernel $X(n, \omega_k)$, $0 \leq k \leq K-1$, is given by

$$X(n, \omega_k) = \sum_{\ell=0}^{N-1} x(\ell) \mathrm{w}_k(n, \ell) e^{-j\omega_k \ell}, \qquad (10)$$

where $\mathrm{w}_k(n, \ell)$ is, in general, a time and frequency dependent window. The DET can be seen as a generalization of the short-time Fourier transform, where the windows are constant. The windows $\mathrm{w}_k(n, \ell)$ can be obtained from either the Gabor representation that uses non-orthogonal bases, or the Malvar wavelet representation that uses orthogonal bases [9]. Details of how the windows can be obtained for the Gabor and Malvar representations are given in [9]. However, for the representation of multipath wireless channel outputs, we need to consider signal-dependent windows that are adapted to the Doppler frequencies of the channel.

3.2 Channel Estimation using DET

We will now consider the computation of the spreading function by means of the evolutionary transformation of the received signal. The output of the channel $y_n(m)$ for the n^{th} OFDM symbol can be written as,

$$y_n(m) = \frac{1}{\sqrt{K}} \sum_{i=0}^{L-1} \sum_{k=0}^{K-1} \alpha_i \, e^{j\psi_i m} \, e^{j\omega_k (m-N_i)} X_{n,k}$$

Now calculating the discrete evolutionary representation of $y_n(m)$:

$$\begin{aligned} y_n(m) &= \sum_{k=0}^{K-1} Y_n(m, \omega_k) e^{j\omega_k m} \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} H_n(m, \omega_k) X_{n,k} e^{j\omega_k m} \quad (11) \end{aligned}$$

By comparing the above two representations of $y_n(m)$, we get the corresponding evolutionary kernel as

$$Y_n(m, \omega_k) = \frac{1}{\sqrt{K}} \sum_{i=0}^{L-1} \alpha_i \, e^{j\psi_i m} \, e^{-j\omega_k N_i} X_{n,k} \quad (12)$$

Finally, the channel frequency response for the n^{th} OFDM symbol can be obtained by

$$H_n(m, \omega_k) = \frac{\sqrt{K} \, Y_n(m, \omega_k)}{X_{n,k}} \qquad (13)$$

The evolutionary kernel $Y_n(m, \omega_k)$ can be calculated directly form $y_n(m)$ [9] and channel parameters α_ℓ, ψ_ℓ, and N_ℓ can be obtained form the spreading function $S(\Omega_s, \ell)$. However, (13) indicates that to estimate the channel frequency response, we need the input data symbols $X_{n,k}$ at pilot positions. Two possible solutions can be implemented:

1. One complete OFDM symbol, after every C symbols can be sent as pilot so that $X_{n,k} = p_{n,k} \in \{-1, 1\}, \forall k, n = rC, r \in \mathcal{Z}$. In this case, the spreading function can be calculated by these pilot values, and can be used until the next pilot OFDM symbol.

2. Other pilot symbol patterns [2, 5, 6] can be used and data symbols $X_{n,k}$ can be detected using any of the pilot aided channel estimation and filtering methods [2, 5] by $\hat{X}_{n,k} = R_{n,k}/\hat{H}_{n,k}$. Then the detected data can be used for the estimation of the spreading function via DET.

Using either of these approaches, the DFT of $H(m, \omega_k)$ with respect to m, and the inverse DFT with respect to ω_k, gives us the spreading function $S(\Omega_s, \ell)$ from which all the parameters of the channel will be obtained and the transmitted data symbol will be detected.

The time-frequency evolutionary kernel of the channel output is obtained by replacing $y_n(m)$ in equation (10), or

$$\begin{aligned} Y_n(m, \omega_k) &= \sum_{\ell=0}^{K-1} y_n(\ell) \mathrm{w}_k(m, \ell) e^{-j\omega_k \ell} \\ &= \frac{1}{\sqrt{K}} \sum_{s=0}^{K-1} X_{n,s} \sum_{i=0}^{L-1} \alpha_i e^{-j\omega_s N_i} \\ &\times \sum_{\ell=0}^{N-1} \mathrm{w}_k(m, \ell) e^{j(\psi_i + \omega_s - \omega_k)\ell} \quad (14) \end{aligned}$$

We consider windows of the form $\mathrm{w}_p(m, \ell) = e^{j\psi_p (m-\ell)}$ presented in [7] that depends on the Doppler frequency ψ_p. This window will give us the correct representation of $Y_n(m, \omega_k)$ only when $\psi_p = \psi_i$, in fact, using the window $\mathrm{w}_i(m, \ell) = e^{j\psi_i (m-\ell)}$, above representation of $Y_n(m, \omega_k)$ becomes,

$$Y_n(m, \omega_k) = \sqrt{K} \sum_{i=0}^{L-1} \alpha_i e^{j(\psi_i m - \omega_k N_i)} \, X_{n,k}$$

which is the expected result multiplied by K. We consider windows $\mathrm{w}_p(m, \ell) = e^{j\omega_p (\ell - m)}$ where $0 \leq \omega_p \leq$

π. When ω_p coincides with one of the Doppler frequencies, the spreading function displays a large peak at the time-frequency position (N_i, ψ_i), corresponding to delay and Doppler frequency of that transmission path, with magnitude proportional to attenuation α_i. When ω_p does not coincide with any of the Doppler frequencies, the spreading function displays a random sequence of peaks spread over all possible delays. Then it is possible to determine a threshold that permits us to obtain the most significant peaks of the spreading function corresponding to possible delays and Doppler frequencies. Finally, estimated channel frequency response can be used to detect the data symbols.

4 Simulation Results

In the experiments, the wireless channel is simulated randomly, i.e, the number of paths, $1 \leq L \leq 5$, the delays, $0 \leq N_i \leq L_{CP} - 1$ and the doppler frequency shift $0 \leq \psi_i \leq \psi_{\max}$, $i = 0, 1, \cdots, L-1$ of each path are picked randomly. Input data is BPSK coded and modulated onto $K = 128$ sub-carriers, 12 % of which is assigned to the pilot symbols. The OFDM symbol duration is chosen to be $T = 200\mu$s, and $T_{CP} = 50\mu$s. Frequency spacing between the sub-carriers is $F = 5k$Hz. First, the Signal-to-Noise Ratio (SNR) of the channel noise is changed between 0 and 15dB, for fixed values of the maximum doppler $\psi_{\max}$ on each path, and the bit error rate (BER) is calculated by four different approaches: 1) No Channel Estimation, 2) Pilot Symbol Assisted (PSA) Channel Equalization 3) Proposed Approach, and 4) Known Channel parameters. The spreading function, hence all the parameters of the channel are estimated by the proposed method and shown in Fig. 1. Figures 2 and 3 show the BER versus SNR for normalized Doppler frequency $\psi_{\max} = 0.0001\pi\text{rad}(50\text{Hz})$ and $\psi_{\max} = 0.001\pi\text{rad}(500\text{Hz})$ respectively. Notice that our proposed mothod improves the performance of PSA channel estimation even with low SNR values. Finally, the SNR is fixed to 15dB while the normalized Doppler frequency is changed from 500Hz to 5kHz, and BER is shown in 4 for each of the above methods.

5 Conclusions

In this work, we present a complete characterization of the multi-path, fading OFDM channels with Doppler frequency shifts using a time-frequency approach. The Discrete Evolutionary Transform allows us to obtain a representation of the time-dependent channel transfer function from the noisy channel output. At the same time, using the estimated channel parameters, a better detection of the input data can be achieved. Examples show that, our method has a considerably better BER performance than PSA channel estimation. Alternative to the DET method proposed here, other time-frequency analysis techniques or wavelet transform can be used to characterize the time-varying communication channel.

References:

[1] Petropulu, A., Zhang, R., and Lin, R., "Blind OFDM Channel Estimation Through Simple Linear Precoding," *IEEE Trans. on Wireless Com.*, Vol. 3, No. 2, pp. 647–655, Mar. 2004.

[2] Stuber, G.L., Barry, J.R., McLoughlin, S.W., Li, Y.G., Ingram, M.A., Pratt, T.G., "Broadband MIMO-OFDM wireless communications," *IEEE Proceedings*, Vol. 92, No. 2, pp. 271–294, Feb. 2004.

[3] Cai, X., Giannakis, G.B., "Error probability minimizing pilots for OFDM with M-PSK modulation over Rayleigh-fading channels," *IEEE Trans. on Vehicular Tech.*, Vol. 53, No. 1, pp. 146–155, Jan. 2004.

[4] Simeone, O., Bar-Ness, Y., and Spagnolini, U., "Pilot-Based Channel Estimation for OFDM Systems by Tracking the Delay-Subspace," *IEEE Trans. on Wireless Com.*, Vol. 3 , No. 1, pp. 315–325, Jan. 2004.

[5] Kang, S.G., Ha, Y.M., Joo, E.K., "A comparative investigation on channel estimation algortihms for OFDM in mobile communications," *IEEE Trans. on Broadcasting*, Vol. 49, No. 2, pp. 142–149, June 2003.

[6] Schafhuber, D., Matz, G., Hlawatsch, F., "Adaptive Wiener filters for time–varying channel estimation in wireless OFDM systems," *IEEE ICASSP'03*, Vol. 4 , pp. IV - 688–691, Hong Kong, Apr. 6-10, 2003.

[7] Chaparro, L.F., and Alshehri, A.A., "Channel Modeling for Spread Spectrum via Evolutionary Transform," *IEEE ICASSP'04*, Vol. II, pp. 609–612, Montreal, Quebec, Canada, May 17–21, 2004.

[8] Hahm, M.D., Mitrovski, Z.I., and Titlebaum, E.L., "Deconvolution in the Presence of Doppler with Application to Specular Multipath Parameter Estimation," *IEEE Trans. on Signal Proc.*, Vol. 45, No. 9, pp. 2203–2219, Sep. 1997.

[9] Suleesathira, R., Akan, A. and Chaparro, L. F., "Discrete Evolutionary Transform for Time-Frequency Signal Analysis," *J. Franklin Institute*, pp. 347-364, Vol. 337, No. 4, Jul. 2000.

[10] Garcia, M.J.F.-G.; Zazo, S.; Paez-Borrallo, J.M.; "Pilot patterns for channel estimation in OFDM," *Electronics Letters*, Vol. 36, No. 12, pp. 1049–1050, June 2000.

[11] Sayeed, A.M., Aazhang, B., "Joint Multipath-Doppler Diversity in Mobile Wireless Communications," *IEEE Trans. on Com.*, Vol. 47, No. 1, pp. 123 - 132, Jan. 1999.

[12] Bello, P.A., "Characterization of Randomly Time-Variant Linear Channels," *IEEE Trans. on Com. Systems*, Vol. CS.11, pp. 360–393, 1963.

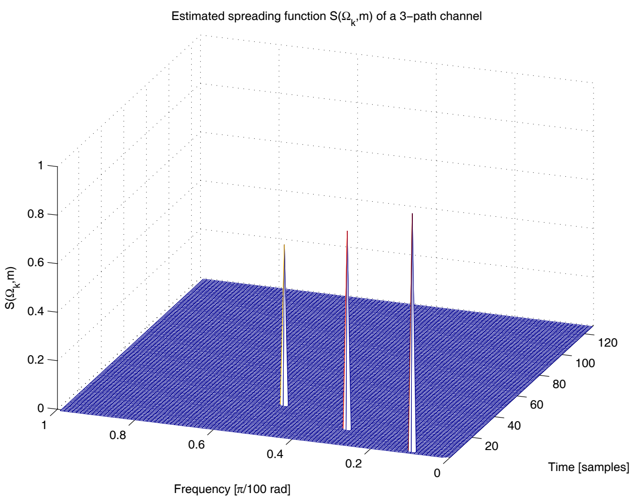

Fig. 1. Estimated spreading function for one symbol.

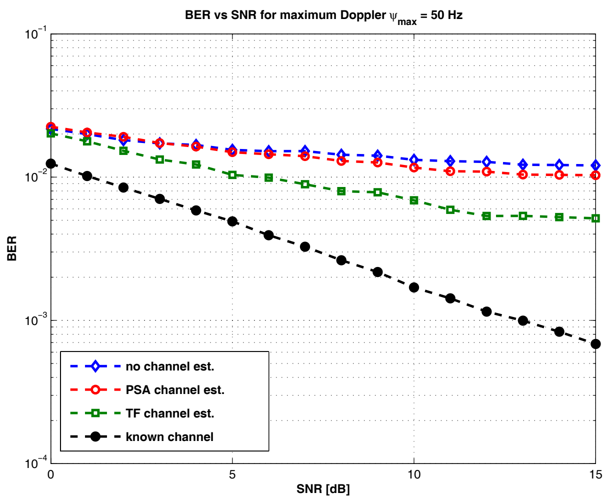

Fig. 2. BER versus SNR at $\psi_{\text{max}} = 50Hz$.

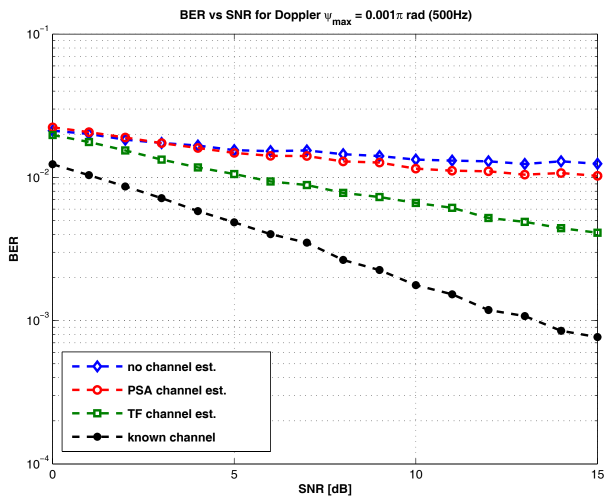

Fig. 3. BER versus SNR at $\psi_{\text{max}} = 500Hz$.

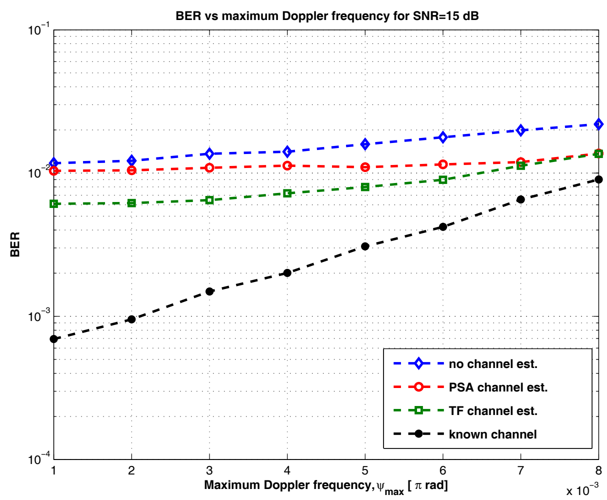

Fig. 4. BER versus ψ_{max} at SNR=15 dB.