Analytical Capacity Evaluation of CDMA Networks with SIR-Based Power Control and Poisson Distributed Interferers

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Abstract: - In Signal to Interference ratio (SIR)-based power control CDMA networks, the received power level from different users is not constant. This power level is a function of the number of active users at a given base station and the amount of intercell interference. In this paper a new analytical approach for the capacity evaluation of SIR-based power control CDMA systems is presented taking into account spatial Poisson traffic. This study differs from others related to SIR-based power control systems in considering an unequal number of active users per cell at a given instant, leading to more realistic results. A Poisson distribution is employed to determine the number of active interfering users. Our results show that the statistics of the received power and the system capacity are closely related to the power control limitations.

Key-Words: - SIR-based power control. CDMA networks, Poisson distribution.

1 Introduction

Accurate capacity evaluation of cellular wireless networks based on CDMA is of great importance due to the selection of this access technique for third generation mobile radio systems, such as UMTS. In spite of the vast amount of literature dealing with CDMA networks, some theoretical aspects have not been fully addressed. In particular, many works concerning capacity evaluation of this kind of networks consider a strength-based power control [1-5]. However, second and third generation cellular radio standards employing CDMA (e.g. IS-95 and UMTS, respectively) define a power control which is a function of received Signal to Interference ratio (SIR). An exception is the work of Kim et all. [6]. In that paper, however, every cell is considered to be equally loaded, which is not realistic. Moreover, users are considered to connect to the nearest base station in that paper. A more realistic approach is to consider that cells are unequally loaded at a given instant and that users connect to a given base station on a minimum attenuation basis.

The goal of this contribution is to characterize the capacity and the statistics of the received power of a cellular CDMA network. We provide a novel closed-form expression for the maximum attainable capacity when the required power is unlimited. Many works dealing with CDMA capacity consider a uniform distribution of users. However a more realistic analysis should consider a unequal number of active users per cell at a given instant. As in [2-3] we consider that the active number of users per cell is Poisson distributed and users connect to the base

station with minimum attenuation. In particular, our approach is similar to [7-8], where the intracell interference is modeled as Poisson and the intercell interference as independent Gaussian. However, our work differs from [2-3], in that we consider a SIRbased power control, instead of the strength-based power control considered in those papers. In addition, we explicitly take into account the practical restriction of limited power, and its effect on received power statistics and capacity.

This paper is organised as follows. The system model is presented in section 2. The derivation of intercell interference statistics is undertaken in section 3 and the power statistics and maximum capacity is provided in section 4. Section 5 presents the calculation of the outage probability in terms of the required energy per bit to interference density ratio. Numerical results are shown in section 6. Finally, concluding remarks are provided in section 7.

2 System Model

A bidimensional layout of hexagonal cells is assumed, with a base station located at the centre of every cell, as shown in figure 1. We consider a power-controlled DS/CDMA system with frequency duplex division, that is, the uplink and downlink channels are allocated to different bands. In the following, we will focus our attention on the uplink channel.

The radio channel is affected by long- and shortterm fluctuations. Long-term fluctuations are due to shadowing and distance variations, while short-term fluctuations are due to multipath fading. In our system, we assume that power is measured on a time scale that takes into account both distance and shadowing effects but not multipath effects, which occurs on a much faster time scale. The channel attenuation between mobile located at x and base station k will thus be given by:

$$\Gamma_{x}[k] = d[x,k]^{\gamma} \cdot 10^{a(\xi_{x}/10)} \cdot 10^{b(\xi_{x,k}/10)}$$
(1)

where d[x,k] is the distance between location x and cell site k, a=b=1/2, ξ_x and $\xi_{x,k}$ are two independient realizations from a zero-mean Gaussian random variable with standard variation σ , and γ is the pathloss exponent. The term $10^{a(\xi_x/10)}$ is a log-normal term to deal with the propagation environment local to x (the near field) and the term $10^{b(\xi_{x,k}/10)}$ is a lognormal term to deal with the path between the near field and cell site k (the far field). It follows for a particular location x and two cell sites that $\Gamma_x[k_1]$ and $\Gamma_x[k_2]$ are correlated random variables since they both share the same near-field terms. However, for two separate locations x_1 and x_2 , $\xi_{x1,k}$ and $\xi_{x2,k}$ are assumed to be independient.

Every base station transmits a pilot signal. Mobiles connect to the base station whose pilot signal is received with the highest average power level. As time averaging of these signals removes the rapid fluctuations due to multipath fading, these fluctuations are not taken into consideration in base station selection. Hence, a user will connect to the base station towards which the average attenuation is minimum.

System performance is analyzed at the central base station, denoted by BS0. We consider a system with N interfering users per cell. Given that a mobile may not be able to measure the pilot signals of all the base stations in the system, in our system we will consider that the selection is limited to the Nc nearest base stations. Therefore, the area of the system will be divided into two regions, as shown in figure 1: region R0, which contains the points having BS0 among the Nc nearest base stations; and region R1, which contains the points not having BS0 among the Nc nearest base stations. The size and the shape of these regions are a function of Nc. The cases for Nc=4 and Nc=9 are shown in figure 1.

In the teletraffic theory of wired networks, the celebrated model is that of the Poisson process. It is still used to model the arrival and departures of the calls in the network. The Poisson process is a reasonable model for the spatial distribution traffic in wireless network. [2]. In [3], the use of the Poisson distribution for a CDMA system with a finite and

large number of users is justified. Furthermore, if two disjoint regions are considered, the Poisson distribution for the number of users in both regions are statically independient.

Many previous studies assumed strength-based power control, which maintains received power at a desired level regardless of changes in the number of active users and in the amount of the total other cell interference. We consider a SIR-based power control where received power is a function of the number of active home users and total other cell interference.

3 Intercell interference statistics

In this section we address the problem of intercell interference characterization in a SIR-based power control CDMA system with a Poisson distribution of users under the assumption of perfect power control.

We denote as *E* the location space of the mobile radio network. Now let us suppose that the users in the system form a spatial Poisson point pattern on *E* with mean measure m(dx). Let us mark every position $x \in E$ by the normalized intercell interference it creates at cell site *k*. Thus we set:

$$\phi_{x}[k] = \begin{cases} 0 & C_{x} = k \\ \frac{\Gamma_{x}[C_{x}]}{\Gamma_{x}[k]} & C_{x} \neq k \end{cases}$$
(2)

where C_x is the cell site selected by the user (minimum attenuation) and Γx is the path-loss variable. Then, the intercell interference at cell site k is:

$$I_{out} = \sum_{x \in E} \phi_x[k] \cdot S_x \tag{3}$$

where S_x is the received power from an user in position x (it is not constant in a SIR-based power control).

In [2], a CDMA network with spatial Poisson traffic and strength-based power control (constant power) is studied. Following a similar approach and assuming that the received power and the distance are mutually independent in a power-controlled system as in [6], we can write:

$$\kappa_i = \int_E E[\phi_x[k]^i] \cdot E[S_x^{\ i}] \cdot m(dx)$$
(4)

where κi denote the ith cumulant of intercell interference, which will be given by:

$$\begin{split} \kappa_{i} &= \int_{E} e^{i^{2}\beta^{2}b^{2}\sigma^{2}} \sum_{\substack{j \in \zeta(x) \\ j \neq qk}} \left(\frac{d[x, j]}{d[x, k]} \right)^{i\gamma} \times \\ &\times \left(\int_{-\infty}^{\infty} \frac{e^{-y^{2}/2}}{\sqrt{2\pi}} \cdot D_{i}(y, j, k, x) \cdot \prod_{\substack{l \in \zeta(x), l \neq j \\ i=1, 2, \dots}} Q \left(y + \frac{M_{j} - M_{l}}{b\sigma} + i\beta b\sigma \right) dy \right) \times \\ &\times E[S^{i}] \cdot m(dx) \end{split}$$
(5)

where $\beta = \ln 10/10$, $M_l = 10\gamma \cdot \log 10d[x, l]$, Q(x) is the complementary cumulative distribution function of a standard (zero mean, unit variance) Gaussian process, and

$$D_{i}(y, j, k, x) = \begin{cases} Q \left(y + \frac{M_{j} - M_{k}}{b\sigma} + 2i\beta\sigma \right), & \text{if } k \in \zeta(x) \\ 1 & \text{otherwise} \end{cases}$$
(6)

being $\zeta(x)$ the set of the points included in region R0.

Therefore, the ith cumulant κi can be simply rewritten as the product of $E[S_i]$, N (Poisson distribution mean) and h_i , which is obtained by the numerical evaluation of the above integral. Then, the mean and the variance of intercell interference are:

$$E[I_{out}] = E[S] \cdot h_1 \cdot N \tag{7}$$

$$Var[I_{out}] = E[S^2] \cdot h_2 \cdot N \tag{8}$$

A Gaussian approximation to intercell interference is employed motivated by the central limit theorem [3].

4 Calculation of $E[S^n]$

With perfect power control and assuming internal interference Poisson distribution with mean N, the following expression holds for the ratio of bit energy to interference density (normalized by the background noise power) for the desired user:

$$\Gamma = \frac{G \cdot S}{k \cdot S + I_{out} + 1} \tag{9}$$

where G is the processing gain, k is the number of interfering active users in reference cell and I_{out} is the intercell interference.

For a Poisson distribution of users, the probability of *n* active users is defined as:

$$p(n) = e^{-N} \cdot \frac{N^n}{n!} \tag{10}$$

Following this, the probability of k internal interferers on the desired user is [3]:

 $\pi_k = \text{Prob}\{k \text{ internal interferers } | \text{ one desired user}\}= \Pr{ob}\{k+1 \text{ users}\}$

Pr ob{at least one user connected to reference cell}

$$= \frac{p(k+1)}{1-p(0)} =$$

$$= e^{-N} \cdot \frac{N^{k+1}}{(k+1)!} \cdot \frac{1}{1-e^{-N}}$$
(11)
From (9) we can solve for S obtaining:

From (9) we can solve for S obtaining:

$$S = \frac{1 + I_{out}}{\frac{G}{\Gamma} - k}$$
(12)

The maximum number of active users is limited because, if the required power turns negative, the power control becomes unfeasible and system enters an outage state. Hence, the maximum number k_{max} is estimated holding the following inequality:

$$\frac{G}{\Gamma} - k > 0 \tag{13}$$

Thus

$$k_{\max} = \frac{G}{\Gamma} - 1 \tag{14}$$

We consider two cases for the evaluation of the power statistics:

Case 1: No limit of power

The n_{th} order moment of power is equal to:

$$E[S^{n}] = \sum_{i=0}^{k_{\max}} \pi_{i} \cdot E\left[(S \mid i)^{n}\right]$$
(15)

where E[(S|i)] is the required power S when *i* intracell interferers are active. Therefore the n_{th} order moment of *S* is given by:

$$E[S^{n}] = \sum_{i=0}^{k_{\max}} \pi_{i} \cdot E\left[\left(\frac{1+I_{out}}{\frac{G}{\Gamma}-i}\right)^{n}\right]$$
(16)

Then the mean of *S* will be:

$$E[S] = \sum_{i=0}^{k_{\max}} \pi_i \cdot E[(S \mid i)] = \sum_{i=0}^{k_{\max}} \pi_i \cdot \frac{1 + E[I_{out}]}{\frac{G}{\Gamma} - i}$$
(17)

Equating (7) and (17), we obtain:

$$E[S] = \frac{\sum_{i=0}^{k_{\max}} \frac{\pi_i}{G}}{1 - h_1 \cdot N \cdot \sum_{i=0}^{k_{\max}} \frac{\pi_i}{G}}$$
(18)

The second order moment of S is:

$$E[S^{2}] = \sum_{i=0}^{k_{\max}} \pi_{i} E\left[(S \mid i)^{2}\right] = \sum_{i=0}^{k_{\max}} \pi_{i} E\left[\left(\frac{1+I_{out}}{\frac{G}{\Gamma}-i}\right)^{2}\right] \quad (19)$$

E,

Taking into account that :

$$E[(S \mid i)^{2}] = Var[S \mid i] + E^{2}[S \mid i]$$
We get: (20)

$$E[S^{2}] = \sum_{i=0}^{k_{\text{max}}} \pi_{i} \cdot \left[Var[S \mid i] + E^{2}[S \mid i] \right] =$$

$$= \sum_{i=0}^{k_{\text{max}}} \pi_{i} \cdot \left[\frac{Var[I_{out}]}{\left(\frac{G}{\Gamma} - i\right)^{2}} + \left(\frac{1 + E[I_{out}]}{\frac{G}{\Gamma} - i}\right)^{2} \right]$$
(21)
Einelly, considering that:

Finally, considering that:

 $Var[S] = E[S^2] - E^2[S]$

Equating (7) and (8), we obtain: $Var[S] = E[S^{2}] - E^{2}[S] =$

$$=\sum_{i=0}^{k_{\max}} \pi_i \cdot \left[\frac{E[S^2] \cdot h_2 \cdot N}{\left(\frac{G}{\Gamma} - i\right)^2} + \left(\frac{1 + E[S] \cdot h_1 \cdot N}{\frac{G}{\Gamma} - i}\right)^2 \right] - E^2[S]$$

(23)

And manipulating this expression and (22), we get:

$$Var[S] = \frac{h_2 \cdot N \cdot \sum_{i=0}^{k_{\max}} \frac{\pi_i \cdot E^2[S]}{(\frac{G}{\Gamma} - i)^2} + \sum_{i=0}^{k_{\max}} \pi_i \cdot \left(\frac{1 + E[S] \cdot h_1 \cdot N}{(\frac{G}{\Gamma} - i)}\right) - E^2[S]}{1 - h_2 \cdot N \cdot \sum_{i=0}^{k_{\max}} \frac{\pi_i}{(\frac{G}{\Gamma} - i)^2}}$$

(24)

The required values for E[S] and Var[S] must be positive for the power control to be feasible. This condition is necessary to avoid that the system enters an outage state. From (18) the condition E[S] > 0 leads to the following inequality:

$$1 - h_1 \cdot N \cdot \sum_{i=0}^{k_{\text{max}}} \frac{\pi_i}{\frac{G}{\Gamma} - i} > 0$$
(25)

And therefore the number of users per cell (mean of Poisson distribution) must verify:

$$N < \frac{1}{h_1} \cdot \frac{1}{\sum_{i=0}^{k_{\text{max}}} \frac{\pi_i}{\frac{G}{\Gamma} - i}}$$
(26)

On the other hand from (24), the condition Var[S] > 0implies that:

$$1 - h_2 \cdot N \cdot \sum_{i=0}^{k_{\text{max}}} \frac{\pi_i}{\left(\frac{G}{\Gamma} - i\right)^2} > 0$$
(27)

where

(22)

$$N < \frac{1}{h_2} \cdot \frac{1}{\sum_{i=0}^{k_{\text{max}}} \frac{\pi_i}{\left(\frac{G}{\Gamma} - i\right)^2}}$$
(28)

It is clear from (26) and (28) that we have two conditions for the maximum number of users, thus:

$$N \max = Min \left(\frac{1}{h_1} \cdot \frac{1}{\sum_{i=0}^{k_{\max}} \frac{\pi_i}{G_{-i}}}, \frac{1}{h_2} \cdot \frac{1}{\sum_{i=0}^{k_{\max}} \frac{\pi_i}{(\frac{G}{\Gamma} - i)^2}} \right)$$
(29)

We have verified that for values of σ lower than around 9 dB, the mean condition is the most restrictive in order to estimate N_{max}. Otherwise, the variance condition determines this value.

Case 2: Power limit S_{max}

The transmitted power of mobiles is limited in real systems. In our study the system is modeled from the viewpoint of the base station to make the problem mathematically tractable as in [6]. Therefore the system is in an outage state when the required power exceeds a given value S_{max}. If the required power turns negative, the power control also becomes unfeasible (outage state). Then, from (9), the maximum level of intercell interference is equal to:

$$I_{\max,k} = S_{\max} \cdot \left(\frac{G}{\Gamma} - k\right) - 1 \tag{30}$$

As a result, the statistical moments of *S* are:

$$E[S^{n}] = \sum_{i=0}^{k_{\max}} \pi_{i} E[(S \mid i)^{n}] =$$

$$= \sum_{i=0}^{k_{\max}} \pi_{i} \int_{0}^{S_{\max}} (S \mid i)^{n} \cdot f_{s}(S) dS =$$

$$= \sum_{i=0}^{k_{\max}} \pi_{i} \int_{0}^{I_{\max}} \left(\frac{1+x}{\frac{G}{\Gamma}-i}\right)^{n} \cdot f_{I_{out}}(x) \cdot dx$$
(31)

In the above expression $f_{Iout}(x)$ is the Gaussian PDF of intercell interference with mean and variance given by (7) and (8).

Since S and Iout are closely related with each other, they are calculated recursively according to the following steps:

- 1. Set $E[I_{out}]$ and $Var[I_{out}]$ at zeros
- 2. Calculate E[S] and E[S²] from (17) and (21), respectively.
- 3. Calculate $E[I_{out}]$ and $Var[I_{out}]$ from (7) and (8), respectively.
- 4. Repeat steps 2 and 3 until the errors of $E[I_{out}]$ and $Var[I_{out}]$ are within a given bound.

5 Probability of outage

Once the mean and variance of both the intercell interference and power have been found, the system capacity is evaluated in terms of the probability of outage. The probability of outage can be defined as:

Case 1: No limit of power

$$P_{out} = \Pr(S < 0) = 1 - \Pr(k < k \max) =$$

= $1 - \sum_{i=0}^{k_{\max}} \pi_i$ (32)

Although the transmitted power is not limited, the mean number of users per cell *N* is limited by (29).

Case 2: Power limit S_{max}

In this case the probability of outage can be calculated:

$$P_{out} = 1 - \Pr(0 < S < S_{\max}) =$$

$$= 1 - \Pr(I_{out} < I_{\max,k} | k < k_{\max}) =$$

$$= 1 - \sum_{i=0}^{k_{\max}} \pi_i \Pr(I_{out} < I_{\max,i}) =$$

$$\frac{k_{\max}}{1 - \sum_{i=0}^{k_{\max}} I_{\max,i} - E[I_{\max}]}$$

$$=1-\sum_{i=0}^{N_{\text{max}}}\pi_{i}\cdot(1-Q(\frac{I_{\text{max},i}-E[I_{out}]}{\sqrt{Var[I_{out}]}}))$$
(33)

6 Numerical results

This section presents curves showing the results obtained for the mean and variance of S and the probability of outage. A standard variation σ of 8 dB, a path loss exponent of 4, N_c nearest base stations of 4, a processing gain of 256 and a required energy per bit to interference density ratio of 3 dB are considered for all the following figures. For these values of the parameters h₁ and h₂ are 0.5619 and 0.2267 respectively. Also, we consider for all the figures a comparison between the no power limit case and the power limited case for a limit of S_{max} equal to 0 and 10 dB.

Figure 2 provides a comparison of the mean of S. In the case of no power limit, the maximum number of users per cell N (mean of Poisson distribution) is equal to 81. Our results show that the curves are similar until the number of 81 users is reached. For a higher number of users, the power control is not feasible in the case of no power limit. For limited power control the figure shows that E[S] reaches very high values which become nearly constant with the average numbers of users.

Figure 3 shows the variance of S. It is clear from the figure that the variance has a similar behaviour to the one obtained for the mean of S.

Figure 4 present the outage probability. The value of S_{max} affects outage significantly, but the effect is much more pronounced for a number of users near 81 due to the impact of the number of users on the mean and variance of S. For example, for 80 users, an increase in probability of outage of around two orders of magnutide can be seen for S_{max} of 0 and 10 dB.

7 Conclusions

A new analytical approach for the capacity evaluation of SIR-based power control CDMA systems is shown. In this study a Poisson distribution is employed to determine the number of active interferer users in the system cells. Our results show the statistics of the received power and the system capacity which are closely related to the limitations of terminal power. In the case of no power limit, the maximum number of users per cell N (mean of Poisson distribution) is limited by the feasibility of the power control.

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Figure 1. Cellular system



Figure 2. Mean of S for no power limit and Smax equal to 0 and 10 dB.



Figure 3. Variance of S for no power limit and Smax equal to 0 and 10 dB.



Figure 4. Outage probability for no power limit and Smax equal to 0 and 10 dB.