

# MC-CDMA Systems with Uplink and Downlink Receivers over Fading Channels

MINH HUNG LE<sup>1</sup> and NIKOS E. MASTORAKIS<sup>2</sup>

<sup>1</sup>School of Engineering and Industrial Design

University of Western Sydney

Second Avenue, Kingswood, 2747, N.S.W.

AUSTRALIA

<sup>2</sup>Department of Computer Science

Military Institutions of University Education, Hellenic Naval Academy

Terma Hatzikyriakou, 18539, Piraeus

GREECE

[http://www.geocities.com/minhle\\_uws](http://www.geocities.com/minhle_uws)

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*Abstract:* - Bit error rates of uplink and downlink Multicarrier Code Division Multiple Access (MC-CDMA) system using coherent Maximal-Ratio Combining (MRC) and Equal Gain Combining (EGC) receivers are estimated over Rayleigh fading channels. The analysis assumes that different subcarriers experience independent fading channels, but not necessary identically distributed. The analysis is based on Gaussian approximation of the multiple access interference. Generalized bit error rate (BER) expressions for both uplink and downlink with MRC and EGC receivers were derived. The analytical results are supported with computer simulation results. The effect of fading parameters, number of users, and number of subcarriers were presented. The BER performance of the EGC receiver in the uplink is highly influenced by the fading parameter compared with the MRC receiver. The EGC receiver achieves advantages compared to the MRC receiver in the downlink, the MRC receiver obtains small improvement compared to the EGC in the uplink.

*Key-Words:* - Multicarrier Code Division Multiple Access, Maximal Ratio Combining, Equal Gain Combining, Uplink MC-CDMA Receiver, Downlink MC-CDMA Receiver.

## 1 Introduction

Multicarrier code division multiple access has attracted significant attention as a downlink communication method for fourth generation mobile communication systems. MC-CDMA achieves high quality and high capacity communication in a multipath fading environment because frequency domain spreading signals generate frequency diversity without any error correcting code. The application of direct-sequence (DS) spread spectrum (SS) code-division multiple-access (CDMA) to cellular communications systems has been analysed. DS-CDMA system has been considered to be a candidate to support multimedia services in mobile communications because it has capabilities to cope with asynchronous nature of multimedia data traffic, to provide higher capacity over conventional wireless access schemes and to cope with a hostile multipath fading environment. However, the computational complexity of Rake receiver in a CDMA system increases exponentially as the data transmission rates and bandwidths increase. In addition, multipath propagation causes interchip-interference (ICI) in the DS-CDMA system and severe intersymbol-

interference (ISI) in high data rate systems if the channel delay spread exceeds the symbol duration [1].

Multicarrier code division multiple access has been proposed to offer high data rate services in fading channels since 1993. Basically, MC-CDMA is a combination of CDMA with orthogonal frequency division multiplexing (OFDM) signalling. It is considered to be more suitable to solve the problems associated with the DS-CDMA schemes. In an MC-CDMA system, a single data symbol is transmitted over multiple narrowband subcarriers. The narrowband signals on subcarriers experience frequency non-selective fading that eliminates the need for additional equalizer at the receiver side. Orthogonality of transmitted subcarrier signals is a crucial factor in maintaining the spectral efficiency at maximum level. Recent advances in computation power make the implementation of multicarrier systems feasible using, e.g., fast Fourier transform (FFT). Since the MC-CDMA technique possesses the advantages of both OFDM and CDMA, it has the properties desirable for future high data-rate wireless multimedia services such as insensitivity to frequency-selective channel with a simple one-tap

frequency domain equalizer, frequency diversity, and capability of handling diverse multimedia traffic.

The performance of MC-CDMA in both uplink and downlink of a mobile communication system based on different approximation techniques for Rayleigh fading channels were investigated. Numerous papers have been dedicated to the bit error rate analysis of MC-CDMA. In this paper, we present manifestation for BER of MC-CDMA with Rayleigh fading channel using numerous users, various types of receivers both for uplink and downlink [2].

## 2 Multicarrier CDMA Systems

MC-CDMA system model is examined. A single cell in MC-CDMA system obtains  $M$  simultaneous users, each user has  $N$  subcarriers. Propagation channels of all users are assumed to be uncorrelated.

### 2.1 Transmitter Model

Transmitter model of MC-CDMA system is similar to that of OFDM system used, e.g., in the Digital Audio Broadcasting (DAB) system. The main difference is that a MC-CDMA system transmits the same symbol in parallel through several subcarriers whereas an OFDM system transmits different symbols at different subcarriers. Figure 1 illustrates the block diagram of the transmitter in MC-CDMA system.

Every user has a pseudo random signature sequence  $c_m$  given by:

$$c_m = \left\{ c_m[n] : \begin{array}{l} n = 0, 1, \dots, N-1 \\ m = 0, 1, \dots, M-1 \end{array} \right\} \quad (1)$$

where the elements  $c_m[n]$  are modelled as independent and identically distributed (i.i.d.) random variables (r.v.s) with equal probability,  $Prob(c_m[n]=+1) = Prob(c_m[n]=-1) = 1/2$ . The same signature sequence chip is used to modulate each of the  $N$  carriers of the  $m$ th user. The maximum number of users in the system is  $M$ .

The input data symbols of the  $m$ th user generate a stream of equal probability data symbols  $d_m$  with  $(E[d_m[l]^2] = 1)$  given by

$$d_m = \{d_m[l] : l = -\infty, \dots, -1, 0, 1, \dots, \infty\} \quad (2)$$

with frequency domain spreading, each data bit is multiplied by  $N$  chips of the signature sequence [3].

Transmitted signal  $G_m(t)$  corresponding to the  $l$ th data bit of the  $m$ th user is defined by:

$$G_m(t) = \sqrt{\frac{2P_m}{N}} \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} c_m[n] d_m[l] \cdot \cos\left(2\pi\left(f_c + n\frac{Q}{T_b}\right)t\right) \Pi_{T_b}(t - lT_b) \quad (3)$$

where  $P_m$  is the power of data bit.  $\Pi_{T_b}(t)$  is the rectangular pulse defined in the  $[0, T_b]$ .

### 2.2 Receiver Model

Receiver model with  $M$  active users in the MC-CDMA system, the receiver signal can be written as:

$$h(t) = \sum_{m=0}^{M-1} \sqrt{\frac{2P_m}{N}} \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} \phi_{m,n} c_m[n] d_m[l] \cdot \cos\left(2\pi\left(f_c + n\frac{Q}{T_b}\right)t + \psi_{m,n}\right) \Pi_{T_b}(t - lT_b - \rho_m) + \sigma(t) \quad (4)$$

where  $n(t)$  is the additive white Gaussian noise (AWGN) with double-sided power spectral density of  $N_o/2$ . The mean power of the  $n$ th subcarrier signal is

defined as  $P_{m,n} = \frac{P}{N} E[\phi_{m,n}^2]$ . Assuming that the

mean powers of all subcarriers are equal, the mean total power of the  $m$ th user signal is  $P_m = NP_{m,n}$ .

The operation of an equalizer is necessary to upgrade the performance of the system, e.g. multiplying each subcarrier by a factor  $S_{m,n}$ . Hence only one tap frequency domain equalizers are considered here. The block diagram of the receiver in MC-CDMA system is depicted in Fig. 2. With coherent demodulation the decision variable  $w_{j,l}$  of the  $l$ th data bit of the  $j$ th user is given by:

$$w_{j,l} = \frac{1}{T_b} \int_{lT}^{(l+1)T} h(t) \sum_{n=0}^{N-1} S_{j,n} c_j[n] \cdot \cos\left(2\pi\left(f_c + n\frac{Q}{T_b}\right)t + \hat{\psi}_{j,n}\right) dt \quad (5)$$

where  $\hat{\psi}_{j,n}$  denotes the receiver's estimation of the phase at the  $n$ th sub carrier of the desired signal [3].

In straightforward manner, it is assumed that different subcarriers experience i.i.d. fading channels although the assumption is not necessary for the analysis. Walsh-Hadamard codes are employed as the signature sequences in this paper. Assuming that the users are time synchronous, after demodulation and combining subcarrier signals, the decision variable is obtained as:

$$w_{j,l} = G + K + \sigma \quad (6)$$

The decision variable consists of three components, the first term corresponds to the desired signal, the second term corresponds to the multiple access interference from other users and the last term corresponds to the noise. Assuming perfect phase synchronization of users,  $\hat{\psi}_{j,n} = \psi_{j,n}$ . The desired signal component

$$G = \frac{1}{2} \sqrt{\frac{2P_j}{N}} \sum_{n=0}^{N-1} d_j[l] \phi_{j,n} S_{j,n} \quad (7)$$

The multiple access interference term is described as

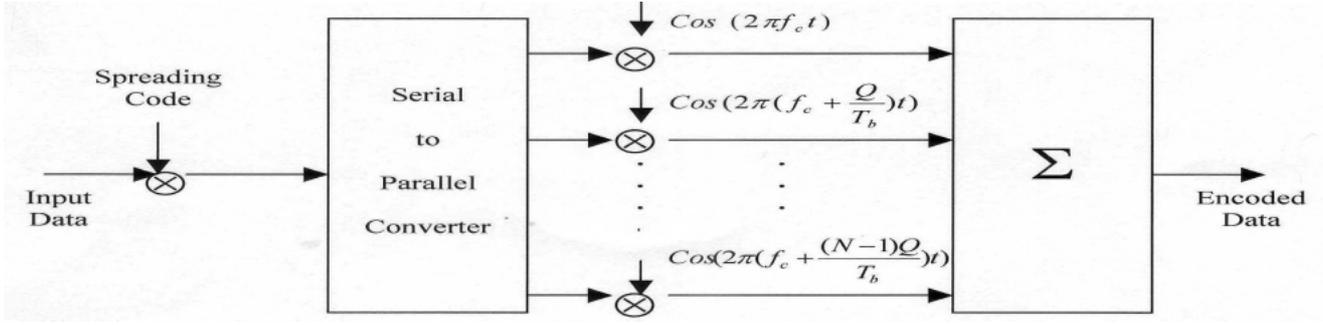


Fig. 1 Transmitter in MC-CDMA system

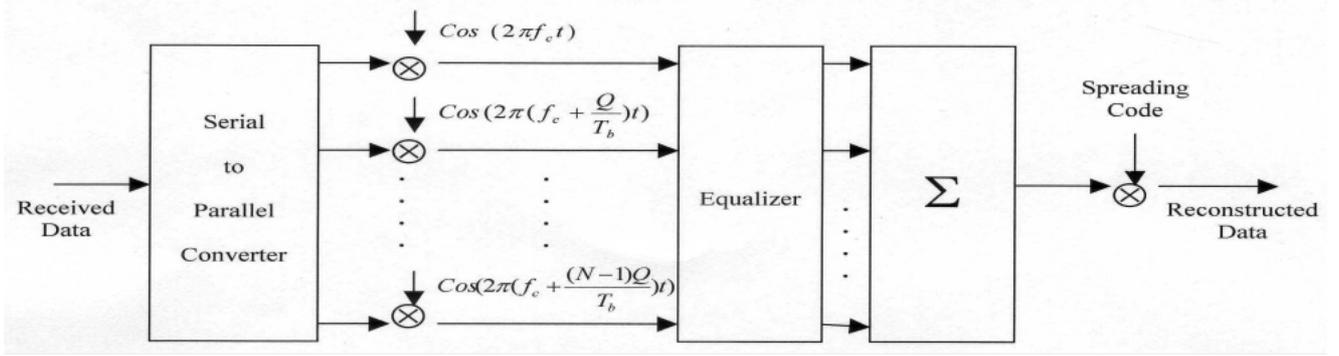


Fig. 2 Receiver in MC-CDMA system

$$K = \frac{1}{2} \sum_{m=0}^{M-1} \sqrt{\frac{2P_m}{N}} d_m [l] \sum_{n=0}^{N-1} \phi_{m,n} S_{j,n} c_m [n] c_j [n] \cdot \cos(\psi_{j,n} - \psi_{m,n}) \quad (8)$$

Similarly, the noise term can be written as

$$\sigma = \frac{1}{T_b} \sum_{n=0}^{N-1} c_j [n] S_{j,n} \int_{T_b}^{(l+1)T_b} \sigma(t) \cdot \cos\left(2\pi\left(f_c + n\frac{Q}{T_b}\right)t + \psi_{j,n}\right) dt \quad (9)$$

Two combining methods of maximal-ratio combining and equal gain combining are presented in this paper.

The maximal-ratio combining is based on correcting the phase shift and weighting the received signal with the channel fading parameter. The assigned equalization coefficients are

$$S_{m,n} = \hat{J}_{m,n}^* \quad (10)$$

where  $\hat{J}_{m,n}$  is the estimated value of  $J_{m,n}$ , and  $(\cdot)^*$  denotes the complex conjugation. MRC is the optimum diversity combining technique with respect to the bit error rate. The uncorrelated diversity branches and no interference are assumed [4].

The equal gain combining is also known as phase equalization, and it is based on correcting only the phase shift introduced by the channel. Each subcarrier is correcting its phase by selecting:

$$S_{m,n} = \frac{\hat{J}_{m,n}^*}{\left| \hat{J}_{m,n} \right|} \quad (11)$$

### 3 Uplink and Downlink MC-CDMA Receivers over Fading Channel

#### 3.1 Uplink MC-CDMA Receivers

In uplink MC-CDMA receivers, the signal to interference plus noise ratio is abbreviated as SNR, need to be defined first. For simplicity, the noise components are approximated as zero-mean Gaussian distributed random variables. Similarly, it is assumed that the spreading code chips and input data bits are random and independent. The multiple access interference (MAI) components are approximated to be zero mean Gaussian distributed r.v.s. This approximation is based on the Central Limit Theorem (CLT), which is considered fairly accurate at least for large number of users ( $M > 10$ ). Generally speaking the  $j$ th user conditional SNR (Conditioned on  $\vec{\phi}$ ) can be expressed as:

$$SNR = \frac{E\left[\left(w_j \vec{\phi}_j\right)^2\right]}{W\left(w_j \vec{\phi}_j\right)} \quad (12)$$

where  $\vec{\phi}_j = [\phi_{j,0}, \phi_{j,1}, \dots, \phi_{j,N-1}]$ , the fading amplitudes vector of the  $j$ th user.  $W$  is variance. Assume independent users and independent subcarriers,

$$E\left[\left(w_j \left| \vec{\phi}_j \right|^2\right)\right] = \frac{1}{4} \frac{2P_j}{N} \left( \sum_{n=0}^{N-1} \phi_{m,n} S_{j,n} \right)^2 \quad (13)$$

and

$$W\left(w_j \left| \vec{\phi}_j \right|\right) = W\left(K \left| \vec{\phi}_j \right|\right) + W\left(\sigma \left| \vec{\phi}_j \right|\right) \quad (14)$$

where  $\Delta\psi_{j,n} = \psi_{j,n} - \psi_{m,n}$ ,  $\psi_{j,n}$  and  $\psi_{m,n}$  and are i.i.d r.v.'s uniformly distributed in the range of  $[0, 2\pi]$ . The probability density function (pdf) of  $\Delta\psi_{j,n}$  can be obtained with  $E[\cos(\Delta\psi_{j,n})] = 0$ ,  $E[\cos^2(\Delta\psi_{j,n})] = \frac{1}{2}$ . Such phase distortion will randomize the phase and consequently cancel the effect of the orthogonality of the spreading codes at the receiver [4].

The conditional variance of the MAI can be expressed as

$$W\left(K \left| \vec{\phi}_j \right|\right) = \frac{1}{8} \sum_{m=0}^{M-1} \frac{2P_m}{N} E[\phi_{m,n}] \sum_{n=0}^{N-1} S_{j,n}^2 \quad (15)$$

and similarly the conditional variance of the noise components is:

$$W\left(\sigma \left| \vec{\phi}_j \right|\right) = \frac{N_0}{4T} \sum_{n=0}^{N-1} S_{j,n}^2 \quad (16)$$

Substituting (16), (15) and (13) into (12), we obtain the following expression for the conditional SNR (Conditioned on  $\vec{\phi}$ ):

$$SNR = \frac{\frac{1}{4} \frac{2P_j}{N} \left( \sum_{n=0}^{N-1} \phi_{j,n} S_{j,n} \right)^2}{\frac{N_0}{4T} \sum_{n=0}^{N-1} S_{j,n}^2 + \frac{1}{8} \sum_{m=0}^{M-1} \frac{2P_m}{N} E[\phi_{m,n}] \sum_{n=0}^{N-1} S_{j,n}^2} \quad (17)$$

For uplink MC-CDMA receiver with MRC, the assigned equalization coefficient is the complex conjugate of

$$S_{j,n} = \phi_{j,n}^* \quad (18)$$

Hence, from (17), the SNR for uplink MC-CDMA receiver with MRC can be written

$$SNR = \frac{2\delta_{0j} \sum_{n=0}^{N-1} \phi_{j,n}^2}{N + \sum_{m=0}^{M-1} \delta_{0m}} \quad (19)$$

where  $\delta_{0m} = \frac{P_m T}{N_0} = \frac{E_b}{N_0}$  is the bit energy to noise ratio of the  $m$ th user and  $E[\phi_{m,n}] = 1$ .

For uplink MC-CDMA receiver with EGC, the weights are set all to unity, i.e.

$$S_{j,n} = 1 \quad (20)$$

which is much simpler than MRC as it does not require channel estimation. Using (17), the SNR for EGC for uplink MC-CDMA receiver can be written as [5]:

$$SNR = \frac{2\delta_{0j} \left( \sum_{n=0}^{N-1} \phi_{j,n} \right)}{N^2 + N \sum_{m=0}^{M-1} \delta_{0m}} \quad (21)$$

### 3.2 Downlink MC-CDMA Receivers

In downlink MC-CDMA receivers, the different users arrive at one particular receiver through the same channel. Thus, we assume perfect phase correction for the interference. Contrary to the uplink, the phase distortion is not present and the orthogonality of the subcarrier codes is not destroyed by the channel. Since Walsh codes are employed as the signature sequence for all users, the MAI term can be re-written as:

$$K = \frac{1}{2} \sum_{m=0}^{M-1} \sqrt{\frac{2P_m}{N}} d_m[l] \cdot \left[ \sum_{n=0}^{\frac{N-1}{2}} \phi_{m,n} S_{j,n} - \sum_{n=0}^{\frac{N-1}{2}} \phi_{m,n} S_{j,n} \right] \quad (22)$$

where, the power of all uses is the same,  $P_m = P_j = P$ .

The examination of downlink MC-CDMA receiver with MRC is similar to that of the uplink and using (22) and (18), the conditional variance of the MAI is written as:

$$W\left(K \left| \vec{\phi}_j \right|\right) = \frac{1}{4} (M-1) \frac{2P}{N} \sum_{n=0}^{N-1} \phi_{j,n}^2 \quad (23)$$

and the corresponding SNR is expressed as:

$$SNR = \frac{2\delta_{0j} \sum_{n=0}^{N-1} \phi_{j,n}^2}{N + 2(M-1)\delta_{0j}} \quad (24)$$

The investigation of downlink MC-CDMA receiver with EGC is similar to that of the uplink and using (22) and (20), the conditional variance of the MAI is written as [5]:

$$W\left(K \left| \vec{\phi}_j \right|\right) = \frac{1}{4} (M-1) 2P \left[ 1 - \left( \frac{\Gamma\left(i + \frac{1}{2}\right)}{\Gamma(i)} \frac{1}{\sqrt{i}} \right)^2 \right] \quad (25)$$

and the corresponding SNR is expressed as:

$$SNR = \frac{2\delta_{0j} \left( \sum_{n=0}^{N-1} \phi_{j,n} \right)^2}{N^2 + 2N(M-1) \left[ 1 - \left( \frac{\Gamma\left(i + \frac{1}{2}\right)}{\Gamma(i)} \frac{1}{\sqrt{i}} \right)^2 \right]} \delta_{0j} \quad (26)$$

### 3.3 Uplink and Downlink MC-CDMA Receivers

The conditional probability of bit error for coherent BPSK is:

$$P(e|\delta) = \frac{1}{2\sqrt{\pi}} \Gamma\left(\frac{1}{2}, \frac{\delta}{2}\right) \quad (27)$$

The Laplace Transform methods are applied to find the pdf of  $SNR = \delta$ ,  $f(\delta)$ , the average BER can be found by,

$$P_{ave} = \int_0^{\infty} P(e|\delta) f(\delta) d\delta \quad (28)$$

The pdf of sum of i.i.d. Gamma distributed Random variable is  $G = \sum_{n=0}^{N-1} \phi_{j,n}^2$ . Since the multiuser systems are examined in this paper, the average BER of the system is provided by:

$$BER = \frac{1}{M} \sum_{m=0}^{M-1} P_{ave}(m) \quad (29)$$

## 4 Numerical Results

The BER performance of MC-CDMA receivers are defined in the closed form solutions. The explanations are generalized for both up and down links using two different equalization techniques, namely EGC and MRC. To calculate the BER, it is assumed that the mean power of each interference term is equal to the mean power of the desired signal. It is assumed that the users in uplink are synchronous within the cyclic prefix. Flat fading channel at each subcarrier is used and i.i.d. fading among different subcarriers is assumed in the analysis and simulations.

The analytical outcomes are presented to examine the BER performance of MC-CDMA receivers in Rayleigh fading channel with multiple users. The effect of the number of users, fading parameter  $i$ , the number of subcarriers and equalization method were investigated. The number of subcarriers is equal to the length of the signature sequence. The analytical results are carried with computer simulation results for Rayleigh fading as a special case. The simulation system utilizes Walsh-Hadamard codes as the signature sequence, which results in zero cross correlation, providing the signal sequences are bit synchronized without channel fading.

The uplink MC-CDMA system with MRC receiver for  $N=8$  subcarriers and  $i=1$  for Rayleigh channel is illustrated in Figure 3. The uplink

MC-CDMA system with EGC receiver for  $N=8$  subcarriers and  $i=1$  for Rayleigh channel is demonstrated in Figure 4. Figures 3 and 4 show the analytical results and computer simulation for the MRC and EGC receivers in a Rayleigh fading channel ( $i=1$ ) with different number of active users. The number of subcarriers used in the simulation system is 8, and this means the case of 8 active users corresponds to the fully loaded case. From Figures 3 and 4, it should be noted that MRC uplink method provides better performance than EGC uplink technique. The effect of the number of active users is presented to show the influence of the MAI on the BER performance. It is obvious that the uplink EGC receiver provides higher BER than the uplink MRC receiver.

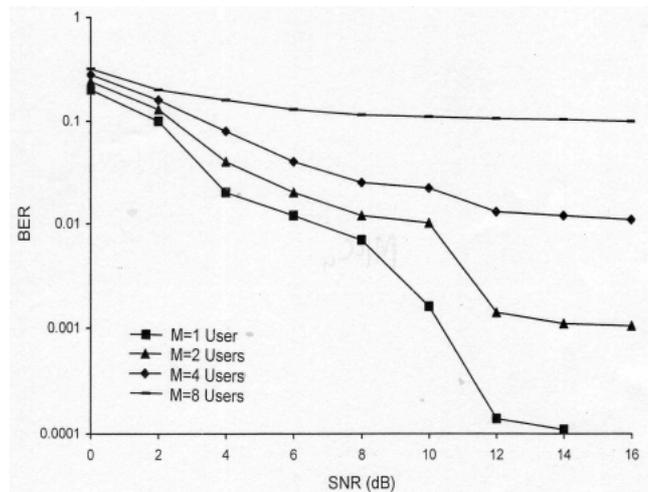


Fig. 3 Uplink MC-CDMA system with MRC receiver for  $N=8$  subcarriers and  $i=1$  for Rayleigh channel.

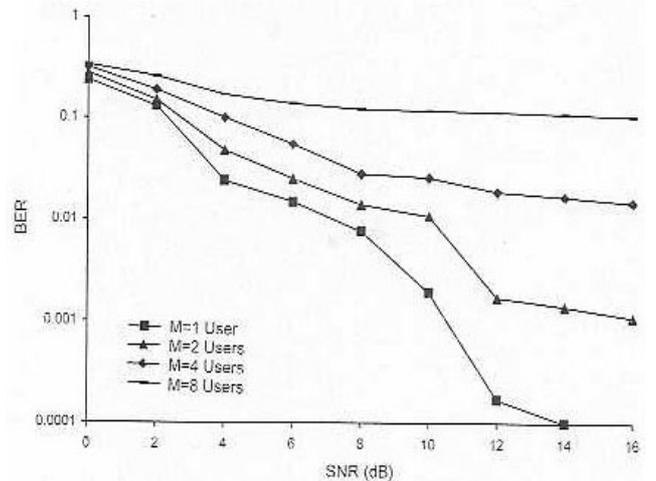


Fig. 4 Uplink MC-CDMA system with EGC receiver for  $N=8$  subcarriers and  $i=1$  for Rayleigh channel.

The downlink MC-CDMA system with MRC receiver for  $N=8$  subcarriers and  $i=1$  for Rayleigh channel is exemplified in Figure 5. The downlink MC-CDMA system with EGC receiver for  $N=8$

subcarriers and  $i=1$  for Rayleigh channel is shown in Figure 6. Figures 5 and 6 present the BER performance comparison of computer simulation and analytical results of MRC and EGC receivers for downlink in Rayleigh fading channel. The effect of the number of active users is obvious. From Figures 5 and 6, it should be noted that EGC downlink method offers better performance than MRC downlink technique. The MRC with its complexity will not be affected since the accuracy of the channel estimation would result into some degradation in the performance.

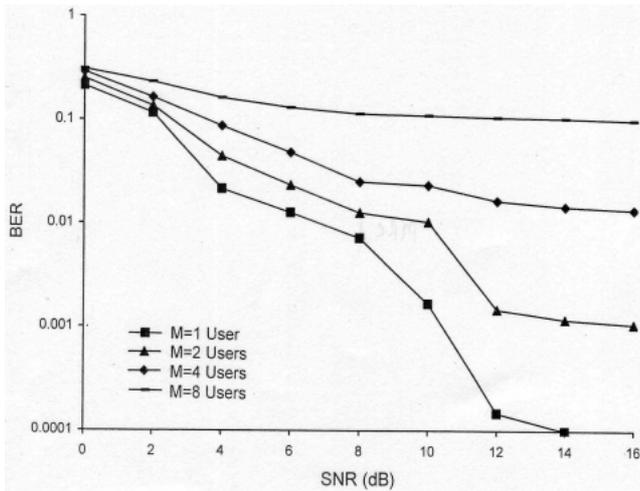


Fig. 5 Downlink MC-CDMA system with MRC receiver for  $N=8$  subcarriers and  $i=1$  for Rayleigh channel.

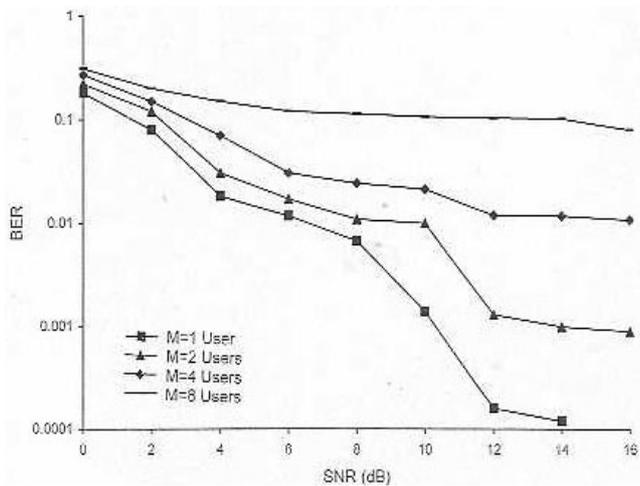


Fig. 6 Downlink MC-CDMA system with EGC receiver for  $N=8$  subcarriers and  $i=1$  for Rayleigh channel.

From Figures 3 and 5, the BER of MRC receiver in downlink is higher than uplink, the MRC receiver for uplink outperforms the MRC receiver for downlink. Comparing Figures 4 and 6, the BER of EGC receiver in downlink is lower than uplink, the EGC receiver for downlink outperforms the EGC receiver for uplink.

## 5 Conclusion

Performance analysis of BER for MC-CDMA receivers with multiple active users over Rayleigh fading channels was presented in this paper. The analysis was applied to evaluate the performance of both EGC and MRC receivers in uplink and downlink. A closed and general formula was derived for the BER that can be applied for EGC and MRC equalizers for uplink and downlink MC-CDMA receivers. The BER expression is valid for real fading parameters that make it applicable for other types of fading channel including Rician channel with proper mapping between fading parameter and Rice factor. The approach in this paper has the advantage of simplicity, generality, closed form expressions, and computational efficiency. Numerical results were presented based on the expressions derived and carried out for Rayleigh fading channel. The uplink MC-CDMA receiver with MRC obtains advantage compared with the uplink MC-CDMA receiver with EGC. However, the downlink MC-CDMA receiver with EGC has advantage compared with the downlink MC-CDMA receiver with MRC. Otherwise, the downlink MC-CDMA receiver with EGC outperforms the uplink MC-CDMA receiver with EGC and the uplink MC-CDMA receiver with MRC enhances the downlink MC-CDMA receiver with MRC.

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