Macrodiversity in Wireless Sensor Networks Based on the Generalized Approach to Signal Processing

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Abstract: - The effect of correlated lognormal shadowing on the average probability of error performance of narrow-band wireless sensor network systems with micro- and macrodiversity reception in a Rician fading channel is investigated by considering a constant correlation model for the shadowed signals at the fusion center or sensor sink of wireless sensor network with radio channel. The performance degradation due to correlated shadowing is illustrated by considering both coherent and differentially coherent binary phase-shift keying schemes. Numerical results presented show that when the sensors in a wireless sensor network system are very closely spaced, the effect of correlated shadowing on system performance cannot be ignored. The obtained results allow us to define the lower and upper bounds on wireless sensor network system performance.

Key-Words: - Wireless sensor network, correlated lognormal shadowing, macrodiversity, generalized detector, Rician fading channel.

1 Introduction

With the increasing demand for wireless sensor network systems and services, a problem to increase wireless sensor network system capacity has a great interest. This problem may be solved under the use of the generalized approach to signal processing in noise [1]–[5]. Propagation measurements in environment have shown that the received signal envelope has a Rician distribution [6]. In addition to the instantaneous Rician fading, the received signal in these wireless sensor network systems also suffers lognormal shadowing that is caused by obstructions between the sensor nodes and the sensor sink (radio port) [7].

Diversity reception is a well-known means to mitigate the effects of fading and shadowing. It exploits the random nature of the radio propagation by combining, or selecting from two or more independent (or at least highly uncorrelated) fading signals paths, resulting in improved system performance. Microdiversity reduces the effect of instantaneous short-term fading at the radio port and can result from explicit or implicit diversity [8].

In the presence of long-term fading (shadowing), macrodiversity, which involves the use of a group of geographically distributed radio ports to serve sensor sink, has been shown to significantly improve wireless sensor network system performance. The most commonly used macrodiversity combining scheme is based on the simple selection of the radio port with the largest local mean signal power [9], although other macrodiversity combining methods have been proposed to further improve the diversity gain and the system capacity [10].

The use of composite microdiversity has drawn considerable attention recently since this simultaneously combats both short-term fading and long-term shadowing [11]–[14]. In most of these and other existing literature on the effect of macrodiversity on the average probability of error, the assumption of uncorrelated shadowing between different radio ports is usually made. This requires large spatial separations between the radio ports. However, correlated shadowing may exist in some practical wireless sensor network systems as a result of closely spaced sensor nodes as they are likely to be shadowed by the same obstacles [13], [14].

In this paper, we investigate the effect of correlated lognormal shadowing on the average probability of error of wireless sensor network system with micro- and macrodiversity reception in a correlated lognormal-shadowed Rician fading channel based on the generalized approach to signal processing in the presence of noise [1]–[5]. We present a semianalytical expression for the probability distribution function at the generalized detector output of the sensor sink of a selection-based macrodiversity wireless sensor network system. The performance degradation due to correlated shadowing is investigated for a special uniform correlation model. We define the lower and upper bounds on wireless sensor network system performance.

2 System Model

We consider a wireless sensor network system in which N radio ports are used to serve the sensor sink. Each radio port has a L-branch microdiversity and operates over a lognormal-shadowed Rician fading channel. Let $x_{i1}, x_{i2}, ..., x_{iL}$ be the statistically independent envelopes of the faded signals incoming at the input of the generalized detector of the sensor sink. These signals are received on the L microdiversity branches at the i-th radio port of the sensor sink. Each input signal can be presented in the form

$$x_{ij} = \begin{cases} n_{ij} \to H_0 \\ s_{ij} + n_{ij} \to H_1 \end{cases}, \quad j = 1, ..., L \quad , \tag{1}$$

where s_{ij} is the transmitted information signal and n_{ij} is the additive Gaussian noise.

The instantaneous signal-to-noise ratio (SNR) at the output of the generalized detector of the i-th radio port (the sensor sink) according to the generalized approach to signal processing in the presence of noise [1]–[5] is given by

$$q_b = \frac{E_b}{4\sigma^4} \sum_{k=1}^{L} z_{ik} \quad , \tag{2}$$

where E_b is the transmitted signal energy per information bit, σ^2 is the variance of the additive white Gaussian noise, and

$$z_{ik} = 2s^*_{ik} x_{ik} - x^2_{ik} + n_{2ik} = \begin{cases} n^2_{2ik} - n^2_{1ik} \to H_0 \\ s^2_{ik} + n^2_{2ik} - n^2_{1ik} \to H_1 \end{cases} \tag{3}$$

is the sufficient statistic at the output of the generalized detector of the i-th radio port of the sensor sink, where s^*_{ik} is the reference information signal, $n^2_{2ik} - n^2_{1ik}$ is the background noise forming at the output of the generalized detector, n_{1ik} is the noise forming at the output of the preliminary filter of input linear tract of the generalized detector, n_{2ik} is the noise forming at the output of the additional filter of input linear tract of the generalized detector. The noise n_{1ik} and n_{2ik} are uncorrelated between each other and have the same statistical characteristics as the noise n_{ik} at the input of the generalized detector [2].

The probability distribution function (pdf) of the instantaneous SNR, conditioned on the local-mean SNR per microdiversity branch

$$q_{0i} = M[z_{ij}] \frac{E_b}{4\sigma^4} = Q_i \frac{E_b}{4\sigma^4} \quad , \tag{4}$$

where $M[.]$ is the mean, is given by

$$f_{q_b}(q_b \mid q_{0i}) = \frac{K+1}{q_{0i}} e^{-\left[\frac{(K+1)q_b}{q_{0i}} + KL\right]} \left[\frac{(K+1)q_b}{KL_{q_{0i}}}\right]^{\frac{L-1}{2}} \times J_{L-1}\left(2\sqrt{\frac{KL(K+1)q_b}{q_{0i}}}\right) \quad , \tag{5}$$

where $J_{L-1}(x)$ is the Bessel function of the $L-1$ order of an imaginary argument, K is Rice factor defined as the ratio power in the specular and scattered components [6]. Due to the presence of shadowing, the local mean power of the received signal at the i-th radio port Q_i is lognormally distributed. Therefore, in decibels, the local mean power

$$\mathcal{Q}_i = 10 \log Q_i \tag{6}$$

is a normally distributed random variable, with standard deviation typically between 6–12 dB. For a local-mean-based macrodiversity scheme in which each sensor is connected to the radio port with the largest local-mean power, the output of the generalized detector with microdiversity combiner is

$$\mathcal{Q} = \max\{\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_N\} \quad . \tag{7}$$

It is noted that the local radio port that yields the largest local-mean power $\mathcal{Q}$ also yields the largest local -mean SNR, i.e.,

$$Q_0 = 10^{0.1\mathcal{Q}} \frac{E_b}{4\sigma^4} \quad . \tag{8}$$

The overall average probability of error with micro- and macrodiversity reception is then given by

$$\overline{P}_{er} = \int_{-\infty}^{\infty} \overline{P}_{er}(Q_0 \mid \mathcal{Q}) f_{\mathcal{Q}}(q) dq \quad , \tag{9}$$

where $f_{\mathcal{Q}}(q)$ is the pdf of $\mathcal{Q}$ and $\overline{P}_{er}(Q_0 \mid \mathcal{Q})$ is the conditional average probability of error for a given local-mean SNR at the selected radio port. Closed-form expressions for $\overline{P}_{er}(Q_0 \mid \mathcal{Q})$ in a slow-fading Rice channel are available for both coherent and noncoherent (differentially coherent) M-ary phase-shift keying (PSK) signals [15], [16], in the presence of microdiversity combining techniques [17].

3 Correlated Shadowing

Consider an effect of correlated shadowing based on techniques discussed in [18]. In evaluating the average probability of error of wireless communication system with micro- and macrodiversity reception, it is usually assumed, for analytical convenience, that the macrodiversity branches are uncorrelated. However, measurements in several environments have shown that, for example, in mobile communication a lognormal shadow fading on the paths between a mobile user and the surrounding base stations are usually correlated [19], [20]. We may apply this example for wireless sensor network system assuming that a lognormal shadow fading on the paths between a sensor sink and sensors are correlated.

The degree of correlation depends not only on the separation between sensors but also on the sensor field (geographical location, surrounding terrain, medium, and so on), the angle of arrival of the received signals, and other factors. In order to study the effects of correlated lognormal shadowing on the macrodiversity system, we consider the general case of normal variables with correlation coefficients under the use of the generalized approach to signal processing in the presence of noise [1]–[5],

$$\rho_{ij} = M\Big[\frac{(Q_i - \mu_i)(Q_j - \mu_j)}{4\sigma_i^2\sigma_j^2}\Big] , \quad i \neq j . \quad (10)$$

The cumulative distribution function (cdf) of Q is given by

$$\mathscr{F}_Q(\mathbf{t} \mid N) = \Pr\,(Q_1 < q; Q_2 < q; \cdots; Q_N < q;)$$
$$= \frac{1}{\sqrt{(2\pi)^N \det(\mathbf{R})}} \int_{-\infty}^{t_1}\int_{-\infty}^{t_2} \cdots \int_{-\infty}^{t_N} \exp(-0.5\mathbf{X}^T\mathbf{R}^{-1}\mathbf{X})d\mathbf{X} \quad (11)$$

and

$$f_Q(q) = \frac{d\mathscr{F}_Q(\mathbf{t} \mid N)}{dq} , \quad (12)$$

where

$$t_i = \frac{q - \mu_i}{2\sigma^2}, \quad i = 1,2,\ldots,N , \quad (13)$$

$$\mathbf{t} = \{t_1, t_2, \ldots, t_N\} , \quad (14)$$

$$\mathbf{X} = \{x_1, x_2, \ldots, x_N\} , \quad (15)$$

and $\mathbf{R}$ is the $N \times N$ correlation matrix. In general, it is difficult to evaluate (11) analytically. The special case of dual-branch macrodiversity $(N = 2)$ has been discussed in [13]. In [14], we can find the Monte Carlo technique to perform the performance analysis for arbitrary *N*; however, this method is quite time consuming.

To simplify the evaluation of (11), we use the procedure discussed in [18]. We assume that there exists a constant correlation between the mean signal powers at any two sensor nodes, i.e., $\rho_{ij} = \rho$ for $i \neq j$. It can be shown that with this model, the correlation between local-mean powers of the received signals at the sensor sink may be the same even when the sensor nodes are at unequal distances from the sensor sink since the correlation depends on the standard deviation of the shadowing process rather than the area mean, which is distance dependent [19]. For the constant correlation model, it is straightforward to show that using results in [21] the cdf of Q is given by

$$\mathscr{F}_Q(q \mid N) = \int_{-\infty}^{\infty}\prod_{i=1}^{N} \mathscr{F}\Big[\frac{q - (\mu_i + 2\sqrt{\rho}\sigma_i^2 y)}{2\sqrt{1-\rho}\;\sigma_i^2}\Big] f(y)dy , \quad (16)$$

where $\mathscr{F}(\cdot)$ and $f(y)$ are the standard normal cdf and pdf, respectively, for $\rho \geq 0$. From (16), we obtain the pdf of Q as

$$f_Q(q \mid N) = \int_{-\infty}^{\infty}\sum_{i=1}^{N} \frac{1}{\sqrt{8\pi(1-\rho)}\sigma_i^2} \exp\Big\{-\frac{[q - (\mu_i + 2\sqrt{\rho}\sigma_i^2 y)]^2}{8(1-\rho)\sigma_i^4}\Big\}$$
$$\times \prod_{\substack{j=1 \\ j \neq i}}^{N} \mathscr{F}\Big[\frac{q - (\mu_j + 2\sqrt{\rho}\sigma_j^2 y)}{2\sqrt{1-\rho}\;\sigma_j^2}\Big] f(y)dy . \quad (17)$$

For the special case of uncorrelated lognormal shadowing $(\rho = 0)$, we obtain the result that is analogous to [11]. The integral in (17) can be easily evaluated numerically using Hermite integration, i.e.,

$$f_Q(q \mid N) \cong \frac{1}{\sqrt{\pi}}\sum_{k=1}^{n} w_k g(y_k) \quad (18)$$

with

$$g(y) = \sum_{i=1}^{N} \frac{1}{\sqrt{8\pi(1-\rho)}\sigma_i^2} \exp\Big\{-\frac{[q - (\mu_i + 2\sqrt{\rho}\sigma_i^2 y)]^2}{8(1-\rho)\sigma_i^4}\Big\}$$
$$\times \prod_{\substack{j=1 \\ j \neq i}}^{N} \mathscr{F}\Big[\frac{q - (\mu_j + 2\sqrt{\rho}\sigma_j^2 y)}{2\sqrt{1-\rho}\;\sigma_j^2}\Big] f(y)dy , \quad (19)$$

where w_k and y_k are the weight factors and roots of the *n*-th order Hermite polynomial, which are tabulated in [22] for different values of *n*. We note that in (18), a value of $n = 20$ yields sufficiently accurate results.

4 Numerical Results

In this section, we present the average bit-error rate (BER) obtained by numerically evaluating the derived expressions for the wireless sensor network system employing micro- and macrodiversity. We assume that the sensor sink is located at the point which is equidistant to all sensor nodes, so that $\mu_i = \mu$ for $i = 1,2,..., N$. Then, since the common area mean μ of the lognormal shadowing is a scaling factor, it is assumed to be 0 dB, without loss of generality. In addition, all the radio ports are assumed to experience equally severe shadowing, i.e., $\sigma_i^2 = \sigma^2$ for $i = 1,2,...,$ N.

To illustrate the effect of both micro- and macrodiversity on the average probability of error in a correlated lognormal shadowing environment, the numerical results are presented for binary phase-shifted keying signals (BPSK) in a Rician fading channel with Rice factor $K = 5$ dB. Analogous forms of expressions for the conditional average probability of error, for a given local-mean SNR q_0, in a Rician fading channel with L-branch microdiversity that are available elsewhere in the literature are used to obtain the numerical results. For coherent BPSK, we use [15, Eq. (28)], while for differentially coherent BPSK (DPSK) with post detection combining, we use [15, Eq. (54)] and (18) in (9). The resulting integral is then evaluated numerically. On all the graphs, we plot the average probability of error versus the average SNR defined by (8).

Figure 1 shows the effect of correlated lognormal shadowing on wireless sensor network system performance for BPSK modulation schemes, with $N = 3$, $L = 2$, and $\sigma^2 = 9$ dB. It is observed that as the correlation coefficient ρ increases, the wireless sensor network system performance deteriorates. For example, at an average BER of 10^{-4} , as much 7.2 dB margin is required for the wireless sensor network system with correlated shadowing $(\rho = 0.6)$ to provide the same performance as that for the wireless sensor network system with uncorrelated shadowing $(\rho = 0)$ As the correlation coefficient tends to unity, the performance obtained coincides with that with no macrodiversity. As expected, the performance of DPSK is about 1.5 dB worse than that for coherent BPSK detection. The BER performance for wireless sensor network system employing the Neyman–Pearson detector (NP) for coherent BPSK detection (no macrodiveristy) is shown for comparison. The presented performances show a great superiority of employment of the generalized detector (GD) in wireless sensor networks.

In Figure 2, the average probability of error is plotted with different orders of macrodiversity N under a light shadowing condition $\sigma^2 = 6$ dB with $\rho = 0$ and $L = 2$ for coherent BPSK signals. It is obvious that the use of macrodiversity reception enhances wireless sensor network system performance. At an average BER of 10^{-4} , for a five-branch macrodiversity system ($N = 5$), the reduction in the required average SNR is about 8 dB compared with the case of no macrodiversity ($N = 1$). For comparison, Fig. 2 presents the BER performance under the use of the NP detector ($N = 1$) to underline a superiority of employment of the generalized detector over NP detector.

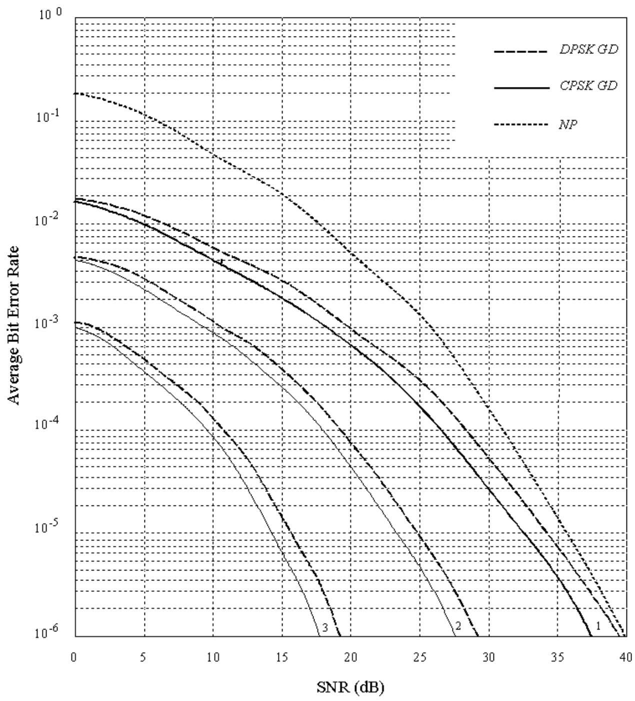

Figure 1. Average BER performance under correlated lognormal shadowing for coherent (CPSK) and differentially coherent (DPSK) BPSK modulation schemes: $K = 5$ dB; $L = 2$; $N = 3$; $\sigma^2 = 9$ dB; $1 - \rho = 1.0$ (no macrodiversity); $2 - \rho = 0.6$; $3 - \rho = 0$.

Figure 3 shows the wireless sensor network system performance for the similar conditions as in Fig. 2 but with correlated shadowing $(\rho = 0.5)$. From Figs. 2 and 3, we notice that the significance of the macrodiversity gain decreases as the order of macrodiversity N increases, especially for $N > 3$. For example, at an average BER of 10^{-4} **,** the diversity gain is 3.5 dB for $N = 3$ while the gain is 4.0 dB for $N = 4$ (only 0.5 dB difference), for correlated shadowing with $\sigma^2 = 6$ dB and $\rho = 0.5$. The diversity gain here is defined as

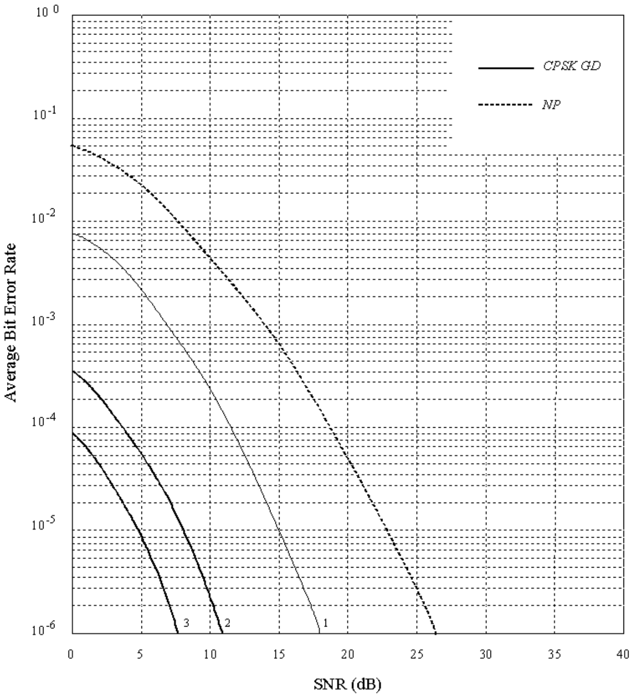

Figure 2. Average BER performance under a light shadowing condition ($\sigma^2 = 6$ dB) for coherent BPSK detection with $K = 5$ dB, $L = 2$, $N = 3$, $\rho = 0$, and different orders of macrodiversity: $1 - N = 1$, $2 - N = 5$, and $3 - N = 10$.

the reduction in the average SNR compared with the case of no macrodiversity $(N = 1)$. In addition, the diversity gain obtained by increasing N may be countered by the loss in BER performance due to the corresponding increase in the correlation.

5 Conclusions

We have derived semianalytical expressions for the average probability of error for a wireless sensor network system based on the generalized approach to signal processing in the presence of noise with micro- and macrodiversity in a Rician fading channel and compared the BER performances under the use of the generalized detector and NP detector. The analysis incorporated the effect of correlated lognormal shadowing on the wireless sensor network system performance. Since the general result for an arbitrary correlation model is computationally difficult, we have analyzed a simple constant correlation model, with which the correlation coefficient between any two-macrodiversity branches is a constant ρ. The results showed that macrodiversity could significantly mitigate the effect of shadowing on the wireless sensor network system performance, particularly in heavy shadowing environments. The results obtained also indicate that when the sensor nodes are closely located in a sensor field, the effect of correlation on the wireless sensor network system with macrodiversity cannot be ignored. If the correlation is very strong, i.e., $\rho \to 1$, the performance coincides with that with no macrodiversity. The combination of micro- and macrodiversity is more effective in providing performance improvement than either microdiversity or macrodiversity even under a correlated shadowing environment.

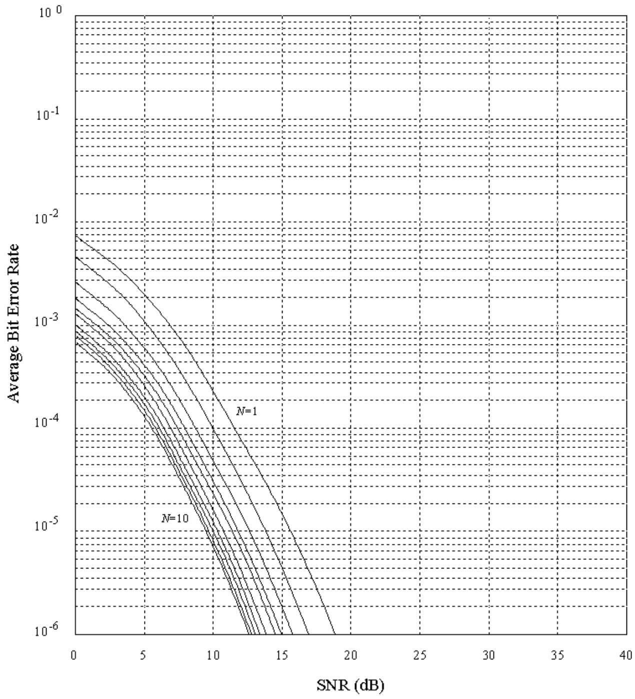

Figure 3. Average BER performance with different orders of macrodiversity $N = 1,2,\ldots10$ for coherent BPSK detection under a light shadowing condition ($\sigma^2 = 6$ dB) with $K = 5$ dB, $L = 2$, and $\rho = 0.5$.

Most of the results available in the literature on the effects of macrodiversity have assumed uncorrelated $(\rho = 0)$ shadowing. This provides a lower bound on wireless sensor network system performance in the presence of correlated shadowing. The results presented in this paper for the constant correlation model provides an upper bound on wireless sensor network system performance when the value of the correlation coefficient is chosen to correspond to

$$\rho = \max\{\rho_{ij}, i \neq j, i, j = 1,2,\ldots, N\}.$$

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