Full-Wave Analysis of Indoor Communication Problems, based on a Parabolic Equation – Finite Element Technique

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Abstract: **-** This paper presents a new methodology to compute the electromagnetic field in practical cases of indoor communications. In particular, we consider the highly interesting case of the so-called interface modeling, i.e. when a plane wave propagates to the interior of a building. The proposed technique is based on a parabolic wave equation in three dimensions, while the cross section of the computational domain, as the wave propagates, is modeled by means of a finite element technique, terminated by highly absorbing PML boundaries. This formulation enables a robust computational modeling of obstacles of complex shape in a fully automated way. Unlike standard ray-tracing techniques, the proposed scheme makes it possible to solve a series of complex geometries and structures that would require vast computational resources with existing approaches.

Key-Words: **-** Wave propagation, indoor communications, ray-tracing, parabolic equation, finite element method.

1 Introduction

The introduction and rapid expansion of wireless local area networks (WLAN) and metropolitan area networks, over the last few years, has resulted in a considerable progress of wideband communications for both personal and corporate, or academic use. However, compared to conventional broadcasting, the characterization of the channel has become extremely complex. For example, both in urban or indoor communications, the multipath effect becomes so pronounced, that in many cases it causes not only fading and other forms of attenuation, but also serious degradation of the signal integrity, due to different propagation delays for each signal path, i.e. the so-called delay spread. Therefore, it is highly important to perform a thorough analysis of the propagation characteristics in the complex urban or indoor environment, in order to predict the received signal accurately and efficiently, especially in the case of broadband communications.

Several empirical models for the modeling of propagation in either urban or indoor environment exist and are usually based on a combination of simplified calculations and extensive measurements [1], [2]. However, these models give only a rough and in many cases unreliable estimate of the received signal strength, especially in the cases where the exact geometric configuration of buildings and the construction materials are not considered. Therefore, more accurate field prediction techniques have been sought, which has led to a wide class of ray-tracing techniques [3]-[7]. Such techniques are definitely the state of the art in the analysis of propagation in urban or indoor channels and are extensively applied. However, they are still based on simplified approaches, i.e. simple reflection, refraction and diffraction models, while the number of rays that has to be considered is, sometimes, excessive and requires vast computational resources. The quest for more accurate and widely applicable full-wave models is, thus, a matter of further investigation.

In this paper, we present a quite unusual electromagnetic formulation, based on a three dimensional Parabolic Equation (PE) approach [8]-[13]. The method presented is used to calculate the electromagnetic field in the interior of a building, taking into account wall structures, doors or other obstacles. The use of the parabolic equation model enables a radical reduction of unknowns, compared to a direct 3D treatment of Maxwell or Helmholtz equations, which would have been computationally prohibitive. The use of the parabolic equation enables the treatment of the 3D problem as a sequence of successive 2D problems, while the different mechanisms of field propagation, such as direct wave, reflection, refraction or diffraction are automatically taken into account, without the necessity of considering separate rays or propagation paths. Of particular significance is the use of the Finite Element Method (FEM) to model the wave propagation in the planes normal to the propagation direction, which enables accurate and fully automated treatment of complex objects, compared to more conventional models, based on finite differences or the Fourier Transform. The proposed technique is successfully applied to characteristic indoor wave propagation problems.

2 The Parabolic Equation Model

2.1 The Paraxial Approximation

The basic concept that facilitates the application of a full wave model in realistic propagation problems, where the computational domain spans several wavelengths in all directions, is the assumption that the wave propagates only within a narrow cone centered along a main direction of propagation. This is fully justified in microwave radiolinks, due to high antenna gains and narrow beamwidths. At first glance, this assumption may sound restrictive, but a general propagation problem, involving many paths can be easily decomposed to a reduced set of basic directions. Under the aforementioned hypothesis, the electric field can be expressed in terms of a slowly varying envelope and a phase variation along the main direction, which is assumed to be toward the *x*-axis, i.e.

$$\mathbf{E}(x,y,z) = \tilde{\mathbf{E}}(x,y,z) e^{-jk_0 x} \quad (1)$$

The wave envelope $\tilde{\mathbf{E}}(x,y,z)$ incorporates all slower field variations, while (1) holds only for relatively small angles of propagation. By introducing (1) to the Helmholtz equation, a similar equation is obtained for the slowly varying envelope, involving both first and second order derivatives, with respect to the direction of propagation. By applying the paraxial approximation, the second order variations are considered negligible and the parabolic equation

$$\begin{aligned} &\frac{\partial \tilde{\mathbf{E}}(x,y,z)}{\partial x} + j\frac{k_0(n^2-1)}{2}\tilde{\mathbf{E}}(x,y,z) = \\ &-j\frac{1}{2k_0}\nabla_t^2 \tilde{\mathbf{E}}(x,y,z) \end{aligned} \quad (2)$$

is obtained.

For the computational implementation of (2), a discretization is performed along the *x*-axis, using central differences. Hence, the first order derivative is approximated via

$$\frac{\partial \tilde{\mathbf{E}}(x,y,z)}{\partial x} = \frac{\tilde{\mathbf{E}}^{i+1}(x,y,z) - \tilde{\mathbf{E}}^{i}(x,y,z)}{\Delta x} \quad (3)$$

where Δx is the propagation step. Its values can be of the order of the wavelength or more, since (2) involves slower field variations along the propagation path. Due to the central differencing implied in (3), the values of the other two terms in (2) have to be considered also in the middle between the propagation planes *i* and *i*+1. However, the resulting scheme is marginally stable, hence we implement the Crank-Nicholson approach, in which the value of any quantity, $\mathbf{F}$, within the interval $(i, i+1)$ is approximated by

$$\mathbf{F}^{i+1}(x,y,z) = a\mathbf{F}^{i+1}(x,y,z) + (1+a)\mathbf{F}^{i}(x,y,z)\,. \quad (4)$$

It can be proven that if $a \geq 1/2$ this method is unconditionally stable (regardless of the propagation step). For the best accuracy, we usually choose values slightly greater than 0.5. Under the assumptions presented, the parabolic equation can be written, in discretized form along the propagation path as

$$\begin{aligned} &\left\{1 + j\Delta x\frac{k_0(n^2-1)}{2}a + j\frac{\Delta x}{2k_0}a\nabla_t^2\right\}\tilde{\mathbf{E}}^{i+1} \\ &= \left\{1 - j\Delta x\frac{k_0(n^2-1)}{2}(1-a) + j\frac{\Delta x}{2k_0}(1-a)\nabla_t^2\right\}\tilde{\mathbf{E}}^{i} \end{aligned} \quad (5)$$

2.2 Wide Angle Correction

The paraxial approximation has the drawback of being correct only for small angles around the propagation direction. However, a significant correction that extends the use of the parabolic equation for angles up to 30 degrees is readily available, if we do not consider the second order variations of the envelope negligible. Hence, the Helmholtz equation takes the form

$$\begin{aligned} &\frac{\partial}{\partial x}\left[\frac{\partial \tilde{\mathbf{E}}(x,y,z)}{\partial x} - 2jk_0\tilde{\mathbf{E}}(x,y,z)\right] \\ &= -\nabla_t^2\tilde{\mathbf{E}}(x,y,z) - k_0^2(n^2-1)\tilde{\mathbf{E}}(x,y,z) \end{aligned} \quad (6)$$

To dispense with the second order derivative we simply use an operator notation and solve (6) in terms of the first order derivative, i.e.

$$\frac{\partial}{\partial x} = \frac{-\nabla_t^2 - k_0^2(n^2-1)}{-2jk_0 + \partial/\partial x} \tag{7}$$

which can be thought of as a recursive relation. When applied again in (6) we get the wide angle parabolic equation

$$\left\{-2jk_0 + \frac{\nabla_t^2 + k_0(n^2-1)}{2jk_0}\right\}\frac{\partial}{\partial x}\tilde{\mathbf{E}}(x,y,z) = -\left\{\nabla_t^2 + k_0(n^2-1)\right\}\tilde{\mathbf{E}}(x,y,z) \tag{8}$$

The latter one is discretized in exactly the same way, hence (3) is introduced for the propagation step, while the Crank-Nicholson scheme is, similarly, applied. It is interesting and surprising to note that (8) results in exactly the same form (5) provided that the Crank-Nicholson parameter is replaced by the modified wide-angle parameter

$$a_{WA} = a + \frac{1}{2jk_0\Delta x} \tag{9}$$

Therefore, the very simple replacement (9) to (5) is sufficient to account for propagation angles up to 30 degrees with respect to the main axis, without any further modification. This wide angle approach is essential to the application to realistic outdoor to indoor propagation problems. Although the transmitter may be at a distance long enough to ensure propagation along a main axis, deviations from this main propagation route are naturally expected in the complex indoor environment and have to be taken into account.

2.3 Mesh Termination with PML

One of the fundamental requirements of the computational technique is the application of an appropriate mesh termination method to truncate the infinite domain. In the case of a parabolic equation based marching scheme, the importance of a highly accurate absorbing boundary condition becomes much more evident, due to possible accumulation of reflection errors. Thus, it seems that the most appropriate choice is the implementation of the perfectly matched layer (PML) and especially its anisotropic rendition [14], which is more appropriate for finite element approaches.

In particular, the anisotropic PML is defined by its permeability and permittivity tensors

$$\overline{\overline{\mu}} = \mu_0\overline{\overline{\Lambda}},\ \overline{\overline{\varepsilon}} = \varepsilon_0\overline{\overline{\Lambda}} \tag{10}$$

where the matrix $\overline{\overline{\Lambda}}$ has been determined to provide zero reflection for incident waves of arbitrary angles of incidence. It can be shown that for a PML region normal to the *y*- and *z*-axis, (Fig. 1) it is given by

$$\overline{\overline{\Lambda}}_y = \begin{bmatrix} s_y & 0 & 0 \\ 0 & 1/s_y & 0 \\ 0 & 0 & s_y \end{bmatrix} \text{ and } \overline{\overline{\Lambda}}_z = \begin{bmatrix} s_z & 0 & 0 \\ 0 & s_z & 0 \\ 0 & 0 & 1/s_z \end{bmatrix} \tag{11}$$

respectively, whereas for a corner region,

$$\overline{\overline{\Lambda}} = \overline{\overline{\Lambda}}_y\overline{\overline{\Lambda}}_z = \begin{bmatrix} s_y s_z & 0 & 0 \\ 0 & s_z/s_y & 0 \\ 0 & 0 & s_y/s_z \end{bmatrix} \tag{12}$$

The PML parameters can be, virtually, arbitrary, however they should have a properly chosen imaginary part to provide the necessary absorption. In most cases, the imaginary part has a gradual profile from zero to the maximum value, i.e.

$$s_y = 1 - j\tan\delta\left(\frac{y}{d}\right)^2 \tag{13}$$

where *d* is the PML width, *y* the distance from where it starts and normal to its boundary and $\tan\delta$ a sufficiently large loss tangent. In finite difference codes this is very important, since a material discontinuity would cause severe dispersion effects. In finite element formulations, though, we can chose a constant PML profile. It has been numerically verified that this choice does not cause any considerable dispersion error, whereas is contributes to enhanced absorption properties.

The inclusion of the PML scheme to the parabolic equation is easily performed if we lay out the modified Helmholtz equation including the PML,

$$\nabla \times \overline{\overline{\Lambda}}^{-1} \nabla \times \mathbf{E} - k^2 \overline{\overline{\Lambda}} \mathbf{E} = 0 \tag{14}$$

and apply simple calculus to the resulting equation. The latter one is decomposed in three components, from which, only those transverse to the direction of propagation are kept to the formulation. From (14) we easily derive the scalar equations for each component and, finally, by applying the paraxial approximation, the resulting equation, in terms of the transverse component is

$$\begin{aligned} & s_y s_z \frac{\partial \tilde{\mathbf{E}}_t}{\partial x} \\ & = c_1 s_y s_z \tilde{\mathbf{E}}_t + c_2 \left\{ s_z \frac{\partial}{\partial y} \frac{1}{s_y} \frac{\partial \tilde{\mathbf{E}}_t}{\partial y} + s_y \frac{\partial}{\partial z} \frac{1}{s_z} \frac{\partial \tilde{\mathbf{E}}_t}{\partial z} \right\} \end{aligned} \tag{15}$$

where $c_1 = -j(k^2 - k_0^2)/2$ and $c_2 = -j/2k_0$. Thus, (15) holds for the entire domain, if we simply use the appropriate values for s_y, s_z according to Table I. The generic parameter, *s*, in Table I is given by either (13) or a constant with a sufficiently large imaginary part.

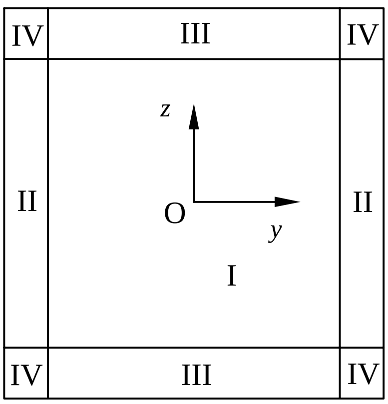

Fig 1. Perfectly matched layer regions

Table I. Values of s_x and s_y

Region	s_y	s_z
I	1	1
II	s	1
III	1	s
IV	s	s

2.4 Finite Element Discretization

Having discretized the parabolic equation along the paraxial direction, we can now focus on the discretization of the problem on the transverse plane, which will result in the calculation of the field values at any plane $x = x_0 + \Delta x$, provided that the field on the previous plane x_0 is known. In our case we have considered a scalar version of (15), by keeping only the vertical component, since in most cases the polarization is vertical. To account for possible polarization coupling in very complex obstacles, it is easy to incorporate both components in the finite element approach.

By applying a standard Galerkin procedure to (15), along with the Crank-Nicholson scheme (5) and the wide angle correction (9) we get the final system of equations

$$\begin{aligned} & \left[\left(\frac{1}{\Delta x} - c_1 a_{WA} \right) \mathbf{T} + c_2 a_{WA} \mathbf{S} \right] \mathbf{E}^{i+1} = \\ & = \left\{ \left[\frac{1}{\Delta x} + c_1 (1 - a_{WA}) \right] \mathbf{T} - c_2 (1 - a_{WA}) \mathbf{S} \right\} \mathbf{E}^{i} \end{aligned} \tag{16}$$

where $\mathbf{E}^i$ is the column vector the wave envelope values at the nodes of the finite element mesh on the *i*-th plane, $\mathbf{S}$, $\mathbf{T}$ are the stiffness and mass FEM matrices and the constants c_1, c_2 and a_{WA} have been defined above. Hence, at each step, the values of the previous plane act as an excitation and an FEM problem is solved to provide the solution at the next step.

In our case, the Finite Element Method is a perfect choice when it comes to obstacles of complex shape, due to its simplicity and versatility in dealing with changes in the geometry of the cross section, as the scheme marches towards the direction of propagation. The use of finite differences would be particularly difficult in dealing with grids that change as the method evolves towards the propagation path. In our case, we have used an automatic mesh generation, based on the Delaunay algorithm, which is performed each time the geometry of the cross section changes. Then, a simple interpolation is done to project the values of the previous plane to the new mesh and, finally, (16) is solved to provide the field values at the next step. Of course, when a change in the geometry of the cross section is encountered, the FEM matrices also change and have to be recomputed for the new mesh.

3 Computational Results

3.1 Performance evaluation of the PML

Before considering the application of the proposed algorithm in the presence of obstacles, we present the problem of a gaussian beam that propagates at a distance of several wavelengths. This is a simple test but is, actually, the worst case, since it involves propagation at grazing incidence which may degrade the PML performance. Moreover, this model has an analytical solution and can serve as a validation for our algorithm. In Fig. 2 we present a comparison between the analytical solution and the results obtained by a PML termination. Evidently, its performance is considered excellent even in this case, where almost the entire transmitted power has passed through the computational window. The situation is expected to be much better in cases of wide angle reflection or diffraction from obstacles, since those waves will not hit the PML at a grazing angle.

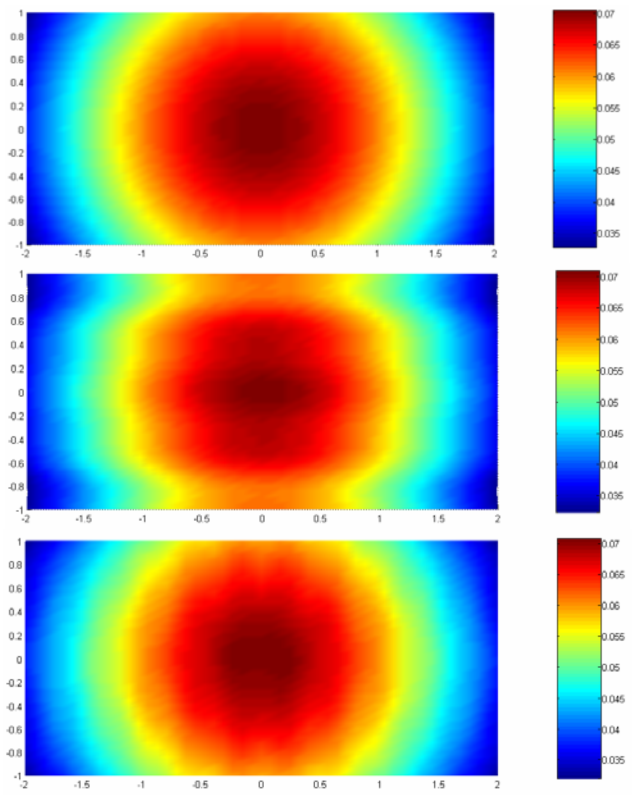

Fig. 2. Propagation of a gaussian beam at a distance of 100λ, within a 33λ×17λ window: (a) Analytical solution, (b) gradual PML profile, (c) constant PML profile

3.2 Three-Dimensional Room Model

The fully 3D model problem that has been chosen to demonstrate the method's potential is a structure of a single room of realistic dimensions (2×2.5×3m). However all the essential attributes of the method are demonstrated in this model, since the walls and their thickness are correctly represented and any more complex building configuration could be constructed using the single room as a fundamental block. The frequency of operation is that of a typical WLAN, i.e. 2.54 GHz. A plane wave from an external directive antenna is assumed to enter the room. Fig. 3 shows a characteristic cross section of the room, discretized via finite elements, where it is obvious how the plane crosses the walls. Thus, the importance of using FEM is clear, since any structure of this kind is discretized automatically. In Fig. 4 and 5, two characteristic graphs of the field distribution inside and behind the room are shown.

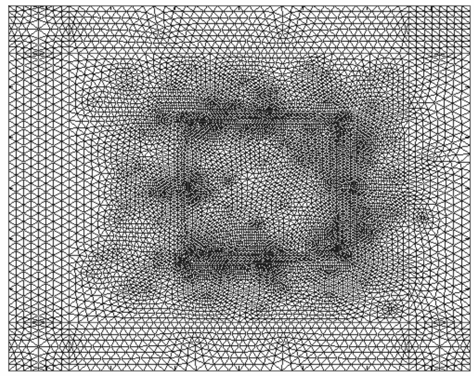

Fig. 3. A characteristic cross section of a room and its walls, perpendicular to the propagation path and its FEM mesh

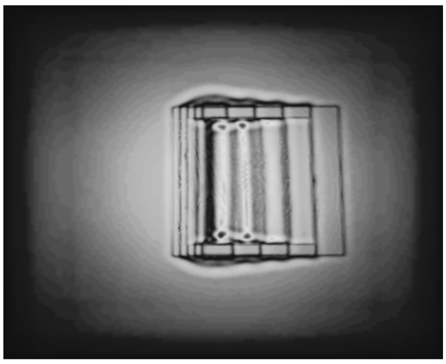

Fig. 4 Field distribution inside the room

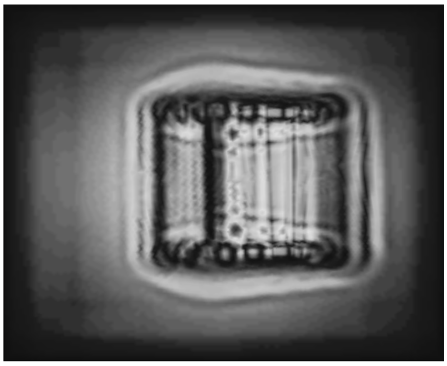

Fig. 5 Field distribution behind the room

4 Conclusions

We have successfully applied a 3D parabolic equation formulation to the analysis of model wave propagation problems in indoor communication environments. Through the use of an FEM approach for the transverse problem, the complexity and variation of the propagation channel has been dealt with easily and automatically, while the PML boundary condition has provided an excellent means of mesh truncation. The presented methodology serves as a fundamental tool for characteristic models of wave propagation along a main direction but has also the advantage of taking into account all mechanisms of propagation like reflection, diffraction or scattering, without considering separate rays as the conventional ray-tracing approximation does. Moreover, it is open to more general extensions, dealing with several basic propagation axes and is expected to provide an accurate means of indoor channel characterization and optimization.

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