On – Line Tracking of Manoeuvring and Ballistic Targets via Angle of Arrival and Doppler Measurements Taken by a Transmitter – Independent Receiver Network Using First Order Recurrent Linear Neural Networks.

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Abstract: – A totally passive multistatic radar or Transmitter-Independent Receiver Network (TIRN) [1], can be defined as a number of independent bistatic receivers [2], connected to a communication network, in order to detect and track targets in their coverage area using the signal(s) of non-cooperative transmitter(s). In this paper, an Angle of Arrival (AOA) method of transmitter and target detection is investigated. Linear systems of equations are extracted, and then solved by recurrent Artificial Neural Networks (ANN) for detection and tracking of moving and ballistic targets. These linear systems are often over determined by using a redundant number of receivers in order to achieve a minimal false alarm probability and increase the survivability of the TIRN. Finally it is shown that practical ANN designs are attractive and simple solutions for an AOA based TIRN for moving target tracking purposes, combining fast and robust convergence, ease of design and construction and – in case of adequate redundancy – adequate survivability

Key – words: Angle of Arrival, Doppler, Multistatic Radars, Passive Target tracking, On – Line Detection, Recurrent Neural Networks.

Abbreviations:

ANN:	Artificial Neural Network.
AOA:	Angle of Arrival.
CW:	Continuous Wave.
ESM:	Electronic Support Measures.
EW:	Early Warning.
IBR:	Independent Bistatic Receiver.
PRF:	Pulse Repetition Frequency.
PRI:	Pulse Repetition Interval.
RMSE:	Root Mean Square Error.
RS:	Range Sum.
TIRN:	Transmitter – Independent Receiver Network.
TDOA:	Time Difference Of Arrival.
UCAV:	Unmanned Combat Aerial Vehicle

1. Introduction.

A Transmitter – Independent Receiver Network is a number of Independent Bistatic Receivers (IBR) [2] connected to a communications network for target data information interchange in order to detect and track moving targets illuminated by non-cooperative transmitters [1]. The constraint of non-cooperative operation is essential for operational and survivability reasons. Enemy transmitters may be used as well as friendly ones.

Initially, the location of the non-cooperative transmitter must be identified and the signal it transmits must be analysed and have its parameters defined. Then the scattered signal on a possible target must be received and processed to uncover the target location and velocity. False targets or target ghosts must be identified and rejected. To accomplish that, several target location methods must be used. These are the Triangulation (or “angle of arrival” AOA), Time Difference of Arrival (TDOA) or other methods like the Range Sum method known for cooperative bistatic radars [3].

Triangulation, or Angle of arrival – AOA method is a simple technique of target location. This consists of the definition of at least three intersecting planar surfaces. Each surface contains one receiver and the target point, which is the common intersection point with the others. This method, although simple, needs receivers with monopulse antennas [4] and it is vulnerable to ghosting. This occurs when a receiver locks on a target and another receiver locks on another nearby target. Then under some circumstances a false target will appear in the intersection point of the planar surfaces created by the receivers and their corresponding targets. In order to minimize the possibility of this phenomenon the number of receivers must be increased or a second detection method like TDOA or Range Sum (RS) must be used.

On the other hand AOA is signal independent, and almost any signal may be used for target detection, provided that the receiver and the antenna used cover the band in which the signal is transmitted. Then, in case the received signal properties allow Doppler shift measurements (e.g. a CW signal) an ANN architecture similar to that used for position determination, can be used for velocity vector determination.

2. Formulation of the AOA Problem.

As referred in the introduction, a monopulse receiver can produce can produce two perpendicular surfaces each one containing the target and the receiver points. Two monopulse receivers at different locations can produce can produce in most cases four surfaces intersecting on the target. Special cases are:

- The target is located at equal or opposite azimuths or equal elevations from the receivers: Three surfaces are given.
- The target is on the straight line connecting the receivers: Only two surfaces are given.

From these special cases only the second can actually create a detection problem that will occur for a very short time for a moving target. If three or more receivers are used this problem is eliminated. In this paper a four-receiver model is used on a random constellation so none of the mentioned cases can occur. A for receiver model is required for velocity vector discrimination as it will be shown later.

Let $r_i(x_i, y_i, z_i)$ and $r_T(x, y, z)$ be the locations of one of the receivers ($i \in \{1,2,3,4\}$) and the target respectively. Then the equations connecting the Cartesian coordinates with the spherical coordinates measured on each receiver by its monopulse antenna are given below:

$$\begin{aligned} x - x_i &= R_i \cos\phi_i \sin\vartheta_i \\ y - y_i &= R_i \sin\phi_i \sin\vartheta_i \\ z - z_i &= R_i \cos\vartheta_i \end{aligned} \tag{1}$$

Note that R_i is the range from the receiver to the target, and it can be eliminated. After some algebraic calculations system (1) is equivalent to:

$$\begin{aligned} &\begin{bmatrix} \cos\vartheta_i & 0 & -\cos\phi_i \sin\vartheta_i \\ 0 & \cos\vartheta_i & -\sin\phi_i \sin\vartheta_i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \\ &\begin{bmatrix} \cos\vartheta_i & 0 & -\cos\phi_i \sin\vartheta_i \\ 0 & \cos\vartheta_i & -\sin\phi_i \sin\vartheta_i \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \end{aligned} \tag{2}$$

This is an equation of a straight line connecting the receiver and the target, and it is actually the equation of the bore sight of the monopulse antenna expressed in Cartesian coordinates.

Similar expressions can be extracted for all the four receivers and after the combination of them, the system becomes:

$$\mathbf{Ax} = \mathbf{b} \tag{3}$$

With:

$$\mathbf{x} = \mathbf{r_T} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{4a}$$

$$\mathbf{A} = \begin{bmatrix} \cos\vartheta_1 & 0 & -\cos\phi_1 \sin\vartheta_1 \\ 0 & \cos\vartheta_1 & -\sin\phi_1 \sin\vartheta_1 \\ \cos\vartheta_2 & 0 & -\cos\phi_2 \sin\vartheta_2 \\ 0 & \cos\vartheta_2 & -\sin\phi_2 \sin\vartheta_2 \\ \cos\vartheta_3 & 0 & -\cos\phi_3 \sin\vartheta_3 \\ 0 & \cos\vartheta_3 & -\sin\phi_3 \sin\vartheta_3 \\ \cos\vartheta_4 & 0 & -\cos\phi_4 \sin\vartheta_4 \\ 0 & \cos\vartheta_4 & -\sin\phi_4 \sin\vartheta_4 \end{bmatrix} \tag{4b}$$

$$\mathbf{b}=\begin{bmatrix}\cos\vartheta_1 & 0 & -\cos\phi_1\sin\vartheta_1\\ 0 & \cos\vartheta_1 & -\sin\phi_1\sin\vartheta_1\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ z_1\end{bmatrix}+ \quad (4c)$$

$$+\begin{bmatrix}0&0&0\\ 0&0&0\\ \cos\vartheta_2 & 0 & -\cos\phi_2\sin\vartheta_2\\ 0 & \cos\vartheta_2 & -\sin\phi_2\sin\vartheta_2\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\end{bmatrix}\begin{bmatrix}x_2\\ y_2\\ z_2\end{bmatrix}+$$

$$+\begin{bmatrix}0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ \cos\vartheta_3 & 0 & -\cos\phi_3\sin\vartheta_3\\ 0 & \cos\vartheta_3 & -\sin\phi_3\sin\vartheta_3\\ 0&0&0\\ 0&0&0\end{bmatrix}\begin{bmatrix}x_3\\ y_3\\ z_3\end{bmatrix}+$$

$$+\begin{bmatrix}0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ \cos\vartheta_4 & 0 & -\cos\phi_4\sin\vartheta_4\\ 0 & \cos\vartheta_4 & -\sin\phi_4\sin\vartheta_4\end{bmatrix}\begin{bmatrix}x_4\\ y_4\\ z_4\end{bmatrix}$$

This is an overdetermined 8×3 system. It is known though that at least a solution exists in this case, at the target location point. Bearing in mind that an exact algebraic solution may not be possible due to angular measurement random errors a minimum RMSE solution will be given by a gradient descent method.

3. AOA problem solution using an on – line method.

An on – line method of linear equation solving has been described thoroughly in [5] with a method that can be easily adapted for the triangulation method. In order to solve (3), an energy function for an estimated $\mathbf{x_e}$ must be defined which in this case is:

$$E(x)=0.5(\mathbf{Ax_e}-\mathbf{b})^T(\mathbf{Ax_e}-\mathbf{b}) \qquad (5)$$

The differential equation system below (where t is in time units) describes gradient descent approximation for the minimization of the energy function $E(x)$ for an initial value $\mathbf{x}_e(0)$ (initial value problem).

$$\frac{d\mathbf{x_e}}{dt}=-\mu\nabla E(\mathbf{x_e})$$
$$\nabla E(\mathbf{x_e})=\mathbf{A}^{\mathrm{T}}(\mathbf{Ax_e}-\mathbf{b})$$
$$\mathbf{x}_e(0)=\mathbf{x}^{(0)}=0 \qquad (6)$$

The above system in its analytical form is equivalent to:

$$\frac{dx_j}{dt}=-\sum_{p=1}^{n}\mu_{jp}\left(\sum_{i=1}^{m}a_{ip}\left(\sum_{k=1}^{n}a_{ik}x_k-b_i\right)\right), \qquad (7)$$
$$x_j(0)=x_j^{(0)}\ j=1,\ 2,\ \ldots,\quad n$$

Choice of μ_{jp} must ensure the stability of the differential equation and an appropriate convergence speed to the desired solution. It has been proven that the system (6) or (7) is stable and has a solution that converges to a vector $\mathbf{x}$ as t tends to the infinite as it is:

$$\frac{dE}{dt}=\sum_{j=1}^{n}\frac{\partial E(\mathbf{x})}{\partial x_j}\frac{dx_j}{dt}= \qquad (8)$$
$$-(\nabla\mathbf{E(x)})^{\mathbf{T}}\mathbf{M}\cdot\nabla\mathbf{E(x)}\le 0$$

The above is always true if $\mathbf{M}$ with elements μ_{jp} is a (predefined) positive definite matrix. Further analysis of (7) gives:

$$e_i(\mathbf{x})=\sum_{k=1}^{n}a_{ik}x_k-b_i,\ i=1,2,\ldots,m \qquad (9a)$$

$$\frac{\partial E(\mathbf{x})}{\partial x_p}=\sum_{i=1}^{m}a_{ip}e_i(\mathbf{x}), \qquad (9b)$$
$$p=1,2,\ldots,n,\ x_j(0)=x_j^{(0)}$$

$$\frac{dx_j}{dt}=-\sum_{p=1}^{n}\mu_{ip}\frac{\partial E(\mathbf{x})}{\partial x_p},\quad j=1,2,\ldots,n \qquad (9c)$$

The recurrent ANN, shown in fig.A.1, (Appendix A) consists of integrators (as many as the dimension of the problem) and weighted input adders. The weights α_{ij} and μ_{ij} are the elements of the matrices $\mathbf{A}$ and $\mathbf{M}$, Second array elements they are constant. That makes this network easy to construct. In this particular case, in equations (6) to (9), $m = 8$ and $n = 3$. They are small values that make the construction even easier.

There are three layers in that ANN connected in feed – forward mode. First layer named "sensor layer" because it senses the actual variables x_i and computes errors $e_i(x)$ as they defined in (9a). Error signals $e_i(x)$ are inputs to the second " association " layer, which gives the gradient components of the system. Here the weights are approximately equal to weights of the first layer as (6) denotes. The third, "response layer" is consisted of response elements, which define the convergence rate.

In this case μ_{ij} (third layer elements) are considered constant for simplicity reasons. In order to simplify the network, μ_{ij} may be considered elements of a positively defined diagonal matrix, thus making the elimination of the adders in the third layer possible. This is the case simulated here in order to prove that even a simplified design can give the expected results.

4. ANN Simulation results and error analysis.

Test system was simulated using Matlab's SIMULINK ®. This consists of four receivers at:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.414 \end{bmatrix} Km \quad \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 20.544 \\ 0.743 \\ 0.525 \end{bmatrix} Km$$
$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} -6.614 \\ -17.218 \\ 0.545 \end{bmatrix} Km \quad \begin{bmatrix} x_4 \\ y_4 \\ z_4 \end{bmatrix} = \begin{bmatrix} -10.487 \\ 20.443 \\ 0.822 \end{bmatrix} Km \tag{10}$$

This set of receivers is placed on a rough land surface as receiver altitudes denote. Coordinate axes $x'x$ and $y'y$ denote position from West (negative) to East and from South to North respectively while axis $z'z$ denotes altitude (height) placement.

The target trajectories simulated here are chosen between extremely manoeuvrable airborne targets and tactical ballistic missiles.

- Target 1: A tri-sonic aircraft approaching from a 300 Km West 200 Km South with a horizontal velocity of 1 Km/sec (0.6 Km/sec towards East and 0.4 Km/sec towards North) while performing vertical manoeuvres between 6 and 10 km altitudes with angular frequency of 0.05 rad/sec.
- Target 2: A bi-sonic aircraft approaching from the West at 600 m/sec performing an elliptic spiral roll with major axis of 6 Km and minor axis of 2 Km at 6 Km altitude with a radial velocity of 0.05π rad/sec This target is manoeuvring, developing a maximum acceleration estimated at 148.33 m/sec^2 (about 15 g). This will be the case of some UCAV being under development today.
- Ballistic Target: A tactical ballistic missile launched from 50 Km East 300 Km South at a take – off angle of 400 mils and a velocity of 3.2 Km/sec towards the North.

Simulation results are shown to figures A.1 to A.6 at the Appendix A. All targets are shown in Range Azimuth and Elevation vs. time. Next the difference of the real Range Azimuth and Elevation and the indicated by the TIRN is shown. Simulation times are 1000 seconds for manoeuvring, and the entire time of flight (~250 seconds) for the ballistic target. Initial guess is zeroed for every target. Although the initial guess is totally wrong the network converges fast to the actual target location and tracking continues at all ranges. As shown in fig.A.1 to A.6 only marginal errors are generated; for example azimuth and elevation errors do not exceed 0.1 degree and range errors are confined to 10 meters for airborne and 30 meters for ballistic targets. After a closer look range error is proportional to the derivative of the range under measure (radial velocity) and that is the case of the first order systems. This gives a way of elimination of that error if applications of the TIRN are more demanding. The range error is given by:

$$D_R = -k_R \dot{R} \tag{11}$$

For this test model after a brief analysis (R in Km $\dot{R}$ in km/sec)

$$k_R \cong 0.0083 \text{ sec} \tag{12}$$

This is close to the sample time of the simulation (or the PRI of the transmitter if it is a Low PRF radar). Similar expressions can be made for the other coordinates (spherical or Cartesian), depending only on the sample time, the geometry and the convergence rate. (In this model convergence rate is high enough to avoid errors).

5. Velocity Vector Synthesis.

In a bistatic system the Doppler frequency shift is given by the equation:

$$f_{Di} = -f_T \frac{\dot{r}_T + \dot{r}_i}{c} \qquad (13)$$

In this equation $\dot{r}_T$ and $\dot{r}_i$ is the radial velocity of the target at the target-transmitter and target-receiver directions, while f_T, f_{Di}, c is the transmitted signal (carrier) frequency, the Doppler shift frequency and the light velocity in the atmosphere. This is equivalent to:

$$v_T + v_i = \dot{r}_T + \dot{r}_i = -c\frac{f_{Di}}{f_T} \qquad (14)$$

If the TIRN is synchronized with a stable local oscillator, an error in Doppler shift measurement can be tolerated since it will be the same error f_{de} at all the receiver measurements:

$$v_e + v_T + v_i = \dot{r}_T + \dot{r}_i = -c\frac{f_{Di}}{f_T} - c\frac{f_{de}}{f_T} \qquad (15)$$

In this case a systematic error in velocity is:

$$v_e = -c\frac{f_{de}}{f_T} \qquad (16)$$

The measured Doppler shift on any receiver is then f_{dmi} and the following equations apply:

$$v_{te} = v_T + v_e \qquad (18a)$$

$$f_{dmi} = f_{de} + f_{Di} \qquad (18b)$$

$$v_{te} + v_i = -c\frac{f_{dmi}}{f_T} = \delta_i \qquad (18c)$$

The above analysis shows that it is possible to measure the velocity projection on a target-receiver direction approximated by an unknown constant v_{te}, identical for all the receivers of a TIRN. Since the velocity vector can be analyzed in three dimensions then there are four unknown quantities that must be identified. That is why a TIRN must be created by at least four receivers in order to exploit its full capabilities.

For formulating and solving the velocity vector synthesis problem it is better to analyze the velocity using a target-centered Cartesian coordinate system [1], [6], since the velocity is a vector bound on target. If $\vec{v}_i$ is the vector velocity projection on a target-receiver direction and $\vec{v}(u,w,s)$ the actual velocity of the target:

$$\vec{v}\cdot\vec{v}_i = \vec{v}_i^{\,2} = v_i^2 = \dot{r}_i^2 \qquad (19)$$

Combining (18c) and (19) and after some algebraic calculations [6], [7] a linear system is generated:

$$\begin{bmatrix} \cos\phi_1\sin\vartheta_1 & \sin\phi_1\sin\vartheta_1 & \cos\vartheta_1 & 1 \\ \cos\phi_2\sin\vartheta_2 & \sin\phi_2\sin\vartheta_2 & \cos\vartheta_2 & 1 \\ \cos\phi_3\sin\vartheta_3 & \sin\phi_3\sin\vartheta_3 & \cos\vartheta_3 & 1 \\ \cos\phi_4\sin\vartheta_4 & \sin\phi_4\sin\vartheta_4 & \cos\vartheta_4 & 1 \end{bmatrix} \cdot \begin{bmatrix} u \\ w \\ s \\ v_{te} \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} \quad (20)$$

In this equation the angular coordinates are translated from the target-centered coordinate system to the correspondents receiver-centered parallel to that, since the corresponding coordinates are π radians supplementary.

Expressing this in matrix-vector form we get:

$$\mathbf{G}\cdot\tilde{\mathbf{v}} = \boldsymbol{\delta} \qquad (21)$$

The above is a linear fully determined system the solution method is similar to the detection problem. Considering v_{te} in (20) or (21) as a total systematic error and applying a similar ANN solution the ANN used has a similar architecture the only differences being that matrix $\mathbf{G}$ is a 4×4 matrix, and the solution vector $\tilde{\mathbf{v}}$ differs to the actual velocity vector $\mathbf{v}$ by its fourth term only, which is the systematic error mentioned.

6. Simulation of Velocity Synthesis method on a TIRN.

The target simulated is similar to the first target in the AOA problem; a tri-sonic aircraft yawing with 10g at an altitude of 6 to 10 Km with a maximum vertical speed of 300 m/sec.

The system description and simulation result alongside with systematic error is seen in Appendix B (figures B.1 to B.5). Figure B.1 describes the test model. The only difference in this model to the AOA one is the “accelerator” ramp function which changes the convergence rate once the target is due to be tracked. In figure B.3 a comparison of the measured velocity vector components (u,w,s) with the real ones is shown – fourth graph being the total systematic error taken as 500 m/sec – and figure B.4 shows the difference of the measured to the actual value of each component. Finally figure B.5 shows the target location in Cartesian coordinates.

The ANN is capable of tracking the velocity of this target with an accuracy depending only on the distance of the target and the convergence rate. In fact (comparing Fig B.3 to B.4) errors never exceed 1 m/sec near the TIRN area, and error in s (z-

component) is only dependent on the convergence speed.
The dependence on the target distance is also easily explained: At long distance the Doppler shift is nearly the same for all the receivers.

7. Conclusions.

Accuracy of measurements can actually be optimised depending only on the geometry and the antennas used if only location is needed. This is a signal independent method as regards target location accuracy. Only angular data is processed.

On the other hand velocity vector synthesis depend on the signal received in the way that Doppler measurements must be taken (range derivatives). This is easier for example if CW illuminates the target. The conclusion here is that the radar ambiguity function as described for monostatic radars [8] does not apply since CW here is the case that gives the best detection and velocity determination of a target.

In the above paragraphs it is shown that a TIRN with monopulse antennas is capable of target tracking if a relatively simple ANN processes measurements taken. The fact that accurate solutions can be given to the detection and velocity tracing of demanding targets makes it an attractive alternative to the active tracking radars especially in a battlefield saturated by electromagnetic emissions. It is also an attractive solution for civil use where accurate tracking of aerial targets is needed (e.g. near airfield control) with a cost only a fraction of that of conventional radar and more, environment friendly since a single radar transmitter may be used with more than one TIRN in a wide area.

References:

[1] Nikos J. Farsaris and Prof. Peter P. Stavroulakis: "Target Detection via Measurements Taken by a Transmitter Independent Receiver Network." AGARD CPP-582, April 1996.
[2] N. Mavrokoukoulakis, A. Pollatos, J. Forrest, J. Schoenenberger, and H. Griffiths: "Bistatic Radar Studies." AGARD Contract AVP /82/571 September 1984.
[3] Nicholas Willis: "Bistatic Radars", Artech House 1991.
[4] Philip L. Bogler: Radar Principles With Applications To Tracking Systems, Chap 2. Wiley – Interscience 1990. pp. 9-16.
[5] A. Cichocki, R. Unbehauen: Neural Networks for Optimization and Signal Processing, John Wiley & Sons Ltd 1993. pp. 102-104.
[6] Nikos J. Farsaris, Prof. Peter P. Stavroulakis, and Prof. Stamatis S. Kouris: Neural Network Designs For TIRN Applications. COMCON VI Kerkyra Greece 1997.
[7] Michalis Damianakis: Detection of Moving Targets using TIRN and Artificial Neural Networks (Diploma thesis in Greek Language supervised by Prof. Stavroulakis and consulted by Dipl. Eng. N. J. Farsaris) Technical Univ. of Crete, Sep.2001.
[8] Nadav Levanon: "Radar Principles." Wiley and sons 1991 chap.5 "The Ambiguity Function".

Appendix A.

Simulation results as described in section 4.

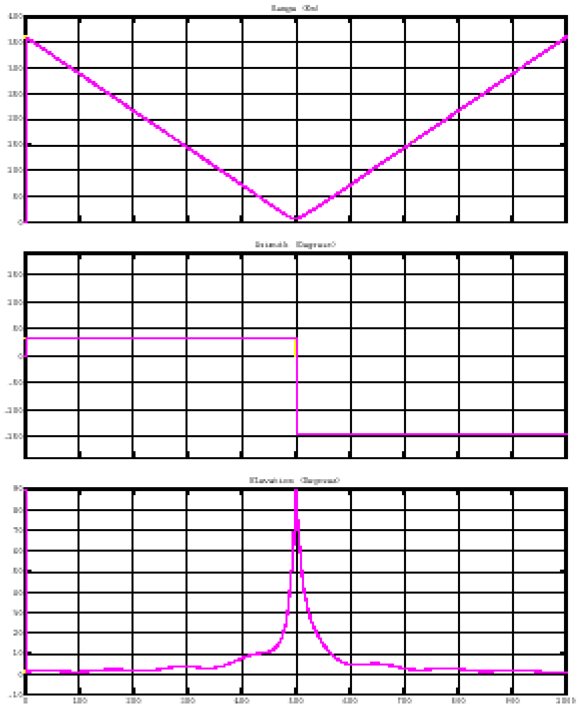
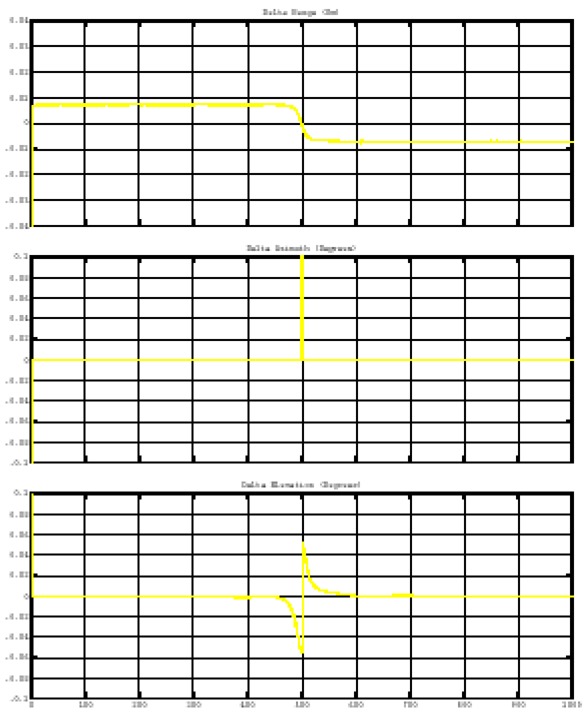


Fig A1: Simulation results for real and tracked position of a Target.1 (Ragne AimuthElevation)



FigA.2: Absolute difference values for Range, Azimuth and Elevation (Target 1).

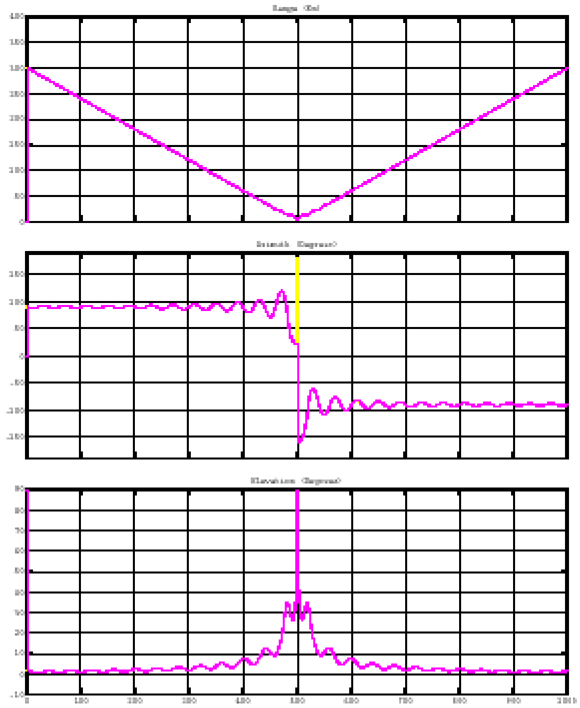


Fig A.3: Simulation results for real and tracked position of a Target.2 (Range, Azimuth, Elevation)

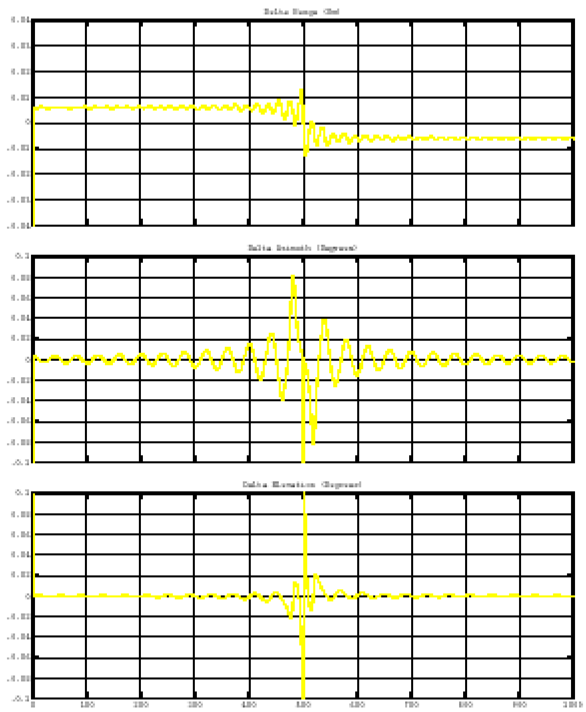


Fig.A4: Absolute difference values for Range, Azimuth and Elevation (Target 2).

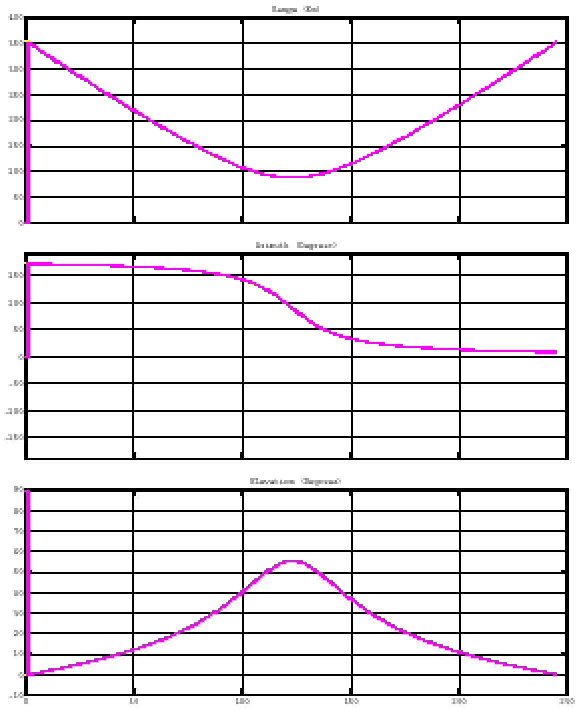


Fig A.5 Simulation results for real and tracked position of a Ballistic Target (Range, Azimuth, Elevation)

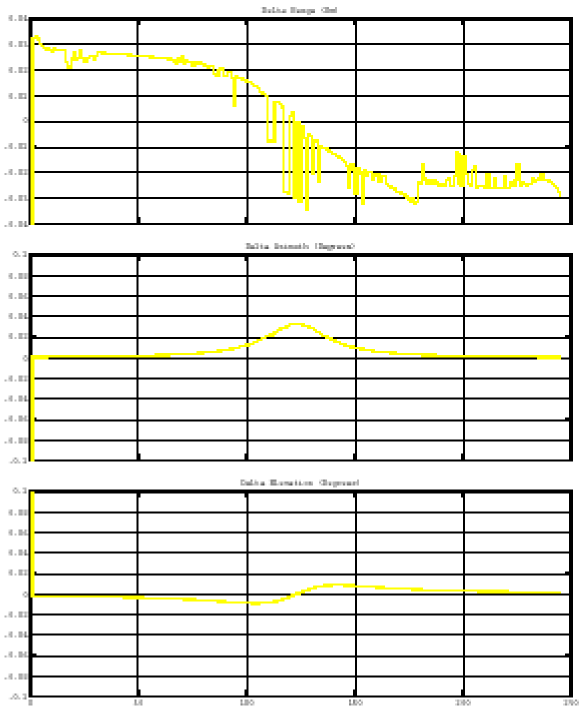


Fig.A6: Absolute difference values for Range, Azimuth and Elevation (Ballistic Target).

Appendix B

Simulation results for target velocity as described in section 5.

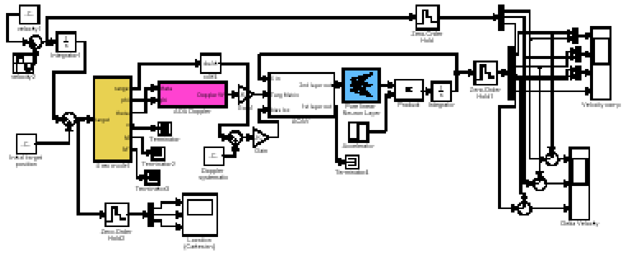


Fig.B1 Systems Architecture

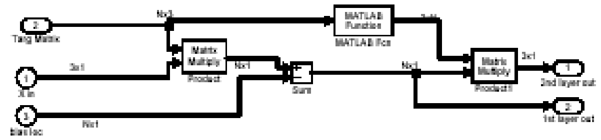


Fig B2 Sensor and Association Layer of ANN above.

Fig. B2 Velocity (Cartesian coordinates) And Systematic Velocity (Doppler) Error of Target.

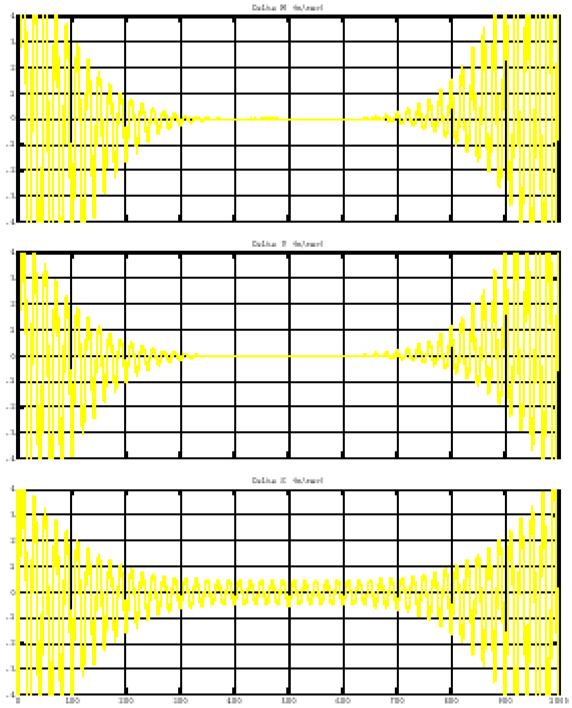


Fig. B3 Difference between real and tracked velocity.

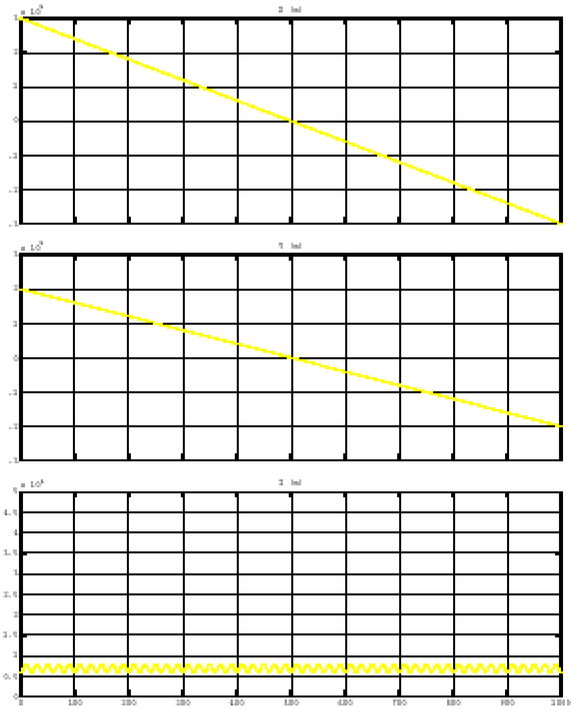


Fig B.5 Target Location (Cartesian)