Tracking of Maneuvering and Ballistic Targets with a Transmitter – Independent Receiver Network using Time Difference Of Arrival and a Second Order Recurrent Artificial Neural Network.

NIKOS J. FARSARIS (*), Dr. THOMAS D. XENOS (*), Prof. PETER P. STAVROULAKIS (**)

(*) Telecommunications Department Electrical and Computer Engineering Faculty Aristotle University of Thessaloniki. 54006 Thessaloniki GREECE

(**) Electronics and Computer Engineering Department Technical University of Crete 73100 Chania Crete GREECE

Abstract: – In this paper the time difference of arrival (TDOA) method is evaluated as a location and tracking method of maneuvering targets via measurements taken by totally passive multistatic radars (or transmitter – independent receiver networks – TIRN). For real time target detection a second order recurrent artificial neural network (ANN) is used. In order to examine the capabilities and limitations of such a system, simulation of several target trajectories are made proving that a system implementing ANN and TDOA can be constructed and exploited as a tracking radar with the obvious advantages of a passive system (no transmitted power) and relatively low gain antennas (of small dimensions thus easily portable or mobile).

Key – Words: – Multistatic Radar, TDOA, Neural Networks.

Abbreviations:

- ANN: Artificial Neural Network.
- ESM: Electronic Support Measures.
- EW: Early Warning.
- PRF: Pulse Repetition Frequency:
- TIRN: Transmitter Independent Receiver Network.

TDOA: Time Difference Of Arrival.

1. Introduction.

A Transmitter – Independent Receiver Network [1] consists of a random constellation of fixed, portable or mobile radar signal receivers similar to those used as ESM receivers. Although some distinctive optimized constellation geometries have been studied [2] the random constellation gives the best tactical flexibility especially in mobile (vehicle or ship) platforms are used.

The detection method of a target used here consists of multiple TDOA measurements of a radar signal reflected on a target like aircraft, missile or ship. The radar may be either cooperative or non cooperative, the general case being the second. In that case the bistatic radar ranging equations [3] of the bistatic radars formed by the – possibly enemy operated – radar, the target and each one of the TIRN receivers constitute a system of nonlinear equations that connect the target coordinates, the coordinates of the receivers and the TDOA measurements.

The above system then leads to an unconstrained optimization problem. Using a second order recurrent neural network can solve this problem with reasonable approximation. Then the reliability and stability of the solution is tested using simulated maneuvering and ballistic targets.

2. Formulation of the TDOA problem.

The scenario used here consists of a radar transmitter that transmits a signal intended (and possibly optimized) for range measurement accuracy, a target and a number of receivers exchanging information over a communication network. The signal mentioned here can be of a low PRF form possibly involving pulse compression techniques. This type of signal is typical of land or ship based EW radars. The ranging equations then are extracted as follows. Considering r_t , r_i , t_i , t_i , c, being the transmitter – target range, the receiver – target range, the transmitter – target signal travel time, the receiver – target signal travel time and the signal propagation velocity (the speed of light for microwaves) respectively the following equations are extracted for *n* receivers:

$$r_t + r_i = ct_t + ct_i$$

$$i = 1, 2, \dots, n$$
(1)

Then by subtracting them for eliminating the radar transmitter related terms the system transforms to the following:

$$r_i - r_j - c(t_i - t_j) = 0,$$

 $i, j = 1, 2, ..., n \quad i \neq j$
(2)

This yields $n^2 - n$ equations, n-1 of them being independent, with the target coordinates being 3 independent variables. So the minimum number of receivers needed is 4. In order to extract 3 independent equations one receiver might be used as reference receiver to the others. Then for the 4receiver model if the first receiver is used as reference the equation system (2) gives:

$$\sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} + \sqrt{(x-x_j)^2 + (y-y_j)^2 + (z-z_j)^2} + (3) -c(t_1-t_j) = 0, \ j = 2,3,4$$

In the above systems x, y, z are the target coordinates, $x_i, y_i, z_i, i = 1,2,3,4$ are the receiver coordinates and $t_i - t_j = \Delta t_{ij}$ is the measured TDOA for two receivers.

3. Solution of the TDOA problem.

The problem of TDOA belongs to a class of nonlinear problems, the equations presented in (3) being second order surfaces (in that case hyperboloids with foci at the receivers). Their common intersection point then is on the target. Systems like (3) is difficult to solve analytically.

Extended form of each equation is presented here:

$$\Delta t_{ij} = t_i - t_j \quad and \\ 4 \Big[(x_i - x_j)^2 - (c\Delta t_{ij})^2 \Big] x^2 + \\ + 4 \Big[(y_i - y_j)^2 - (c\Delta t_{ij})^2 \Big] y^2 + \\ + 4 \Big[(z_i - z_j)^2 - (c\Delta t_{ij})^2 \Big] z^2 + \\ + 8 (x_i - x_j) (y_i - y_j) xy + \\ + 8 (x_i - x_j) (z_i - z_j) xz + \\ + 8 (y_i - y_j) (z_i - z_j) yz + \\ + [4 (x_i - x_j) (x_j^2 + y_j^2 + z_j^2 - x_i^2 - y_i^2 - z_i^2) + \\ + 4c^2 (\Delta t_{ij})^2 (x_i + x_j)] x + \\ + [4 (y_i - y_j) (x_j^2 + y_j^2 + z_j^2 - x_i^2 - y_i^2 - z_i^2) + \\ + 4c^2 (\Delta t_{ij})^2 (y_i + y_j)] y + \\ + [4 (z_i - z_j) (x_j^2 + y_j^2 + z_j^2 - x_i^2 - y_i^2 - z_i^2) + \\ + 4c^2 (\Delta t_{ij})^2 (z_i + z_j)] z + \\ + (x_i^2 + y_i^2 + z_i^2 + x_j^2 + y_j^2 + z_j^2 - c^2 (\Delta t_{ij})^2)^2 + \\ - 4 (x_i^2 + y_i^2 + z_i^2) (x_j^2 + y_j^2 + z_j^2) = 0 \\ \text{This belongs to the equation class of:}$$

$$\mathbf{x}^{\mathrm{T}} \cdot \mathbf{A}_{i,j} \cdot \mathbf{x} + \mathbf{b}_{i,j} \cdot \mathbf{x} + c = 0$$
(5)

On the other hand a more practical solution is given if the problem of the solution of system (3) is transformed to an optimization problem. The equation system (3) may be written as:

$$F_{i}(x, y, z) = 0 \quad i = 1, 2, 3 \quad j = 2, 3, 4 \quad i = j - 1$$

$$F_{i}(x, y, z) = \sqrt{(x - x_{1})^{2} + (y - y_{1})^{2} + (z - z_{1})^{2}} + (7)$$

$$-\sqrt{(x - x_{j})^{2} + (y - y_{j})^{2} + (z - z_{j})^{2}} - c(\Delta t_{1j})$$

If G is a large positive real number this has the equivalent

$$E(x, y, z) = GF_1^2(x, y, z) + +GF_2^2(x, y, z) + GF_3^2(x, y, z) = 0$$
(8)

The last expression E(x, y, z) is zero at the target and positive elsewhere, so it can be used as an energy function with at least a global minimum (equal to zero) at the target position. So the problem already set leads to the unconstrained minimization of E(x, y, z) with the additional information that the minimum is actually zero.

One reasonable solution method is to initially estimate the target location in a coarse manner (as a random point in a large sector) and then use a gradient descent method in order to minimize the energy function. An algorithm or an ANN may achieve this. The second is preferred for real time solution. This is done [4] by the definition of positive real numbers e, g and a symmetric positive definite matrix **T** such as:

$$e\frac{d^2\mathbf{x}}{dt^2} = -g\mathbf{T}\frac{d\mathbf{x}}{dt} - \nabla E(\mathbf{x}) , \ \mathbf{x} = \begin{bmatrix} x & y & z \end{bmatrix}^T \quad (9)$$

With initial conditions:

$$\mathbf{x}(0) = \mathbf{x}^{(0)}, \ \frac{d\mathbf{x}}{dt}(0) = \dot{\mathbf{x}}^{(0)}$$
(10)

For practical implementation the above derivative in (10) can be either estimated or taken as zero at the beginning. The target then is at the convergence point of $\lim_{t\to\infty} \mathbf{x}(t)$ and there, the gradient in (9) is zero. At this (theoretically infinite but practically very little time) the derivative in (10) is the approximate target velocity for a moving target.

The term $\nabla E(\mathbf{x})$ is extracted by (8) as follows:

$$\nabla E(\mathbf{x}) = 2 \begin{bmatrix} GF_1(\mathbf{x})\nabla F_1(\mathbf{x}) + \\ +GF_2(\mathbf{x})\nabla F_2(\mathbf{x}) + \\ +GF_3(\mathbf{x})\nabla F_3(\mathbf{x}) \end{bmatrix}$$
(11)

In the above the gradient terms are:

$$\nabla F_i(\mathbf{x}) = \frac{\partial F_i(\mathbf{x})}{\partial x} \hat{\mathbf{x}} + \frac{\partial F_i(\mathbf{x})}{\partial y} \hat{\mathbf{y}} + \frac{\partial F_i(\mathbf{x})}{\partial z} \hat{\mathbf{z}} , \quad i = 1, 2, 3$$
(12)

The gradient $\nabla E(\mathbf{x})$ of course may be expressed directly as:

$$\nabla E(\mathbf{x}) = \frac{\partial E(\mathbf{x})}{\partial x} \hat{\mathbf{x}} + \frac{\partial E(\mathbf{x})}{\partial y} \hat{\mathbf{y}} + \frac{\partial E(\mathbf{x})}{\partial z} \hat{\mathbf{z}} , \quad i = 1, 2, 3$$
(13)

The last equations (11,12,13), lead to circuit implementations for an approximate extraction of the gradient.

4. ANN System Design and Simulation.

The design used for target detection and tracking consists of an ANN a TIRN TDOA Generator and a TDOA Gradient Generator circuit with scopes for visualization of data. It is shown at figure 1 at the Appendix 1. The ramp and sine functions shown represent movement and maneuvering of the target. For simulation of ballistic targets a Ballistic Target Generator replaces this component.

The ANN described by equations (9) - (13) is according to [4] presented at fig.2. All forward gains are equal to 1/e and recurrent gains are equal to g. Matrix **T** elements are also represented by gains. Outputs of the network are the correction of target coordinates and inputs are approximations of the three components of the energy function gradient:

$$DEx \cong -\frac{\partial E}{\partial x},$$

$$DEy \cong -\frac{\partial E}{\partial y},$$

$$DEz \cong -\frac{\partial E}{\partial z}$$
(14)

The TDOA Generator is the TIRN itself. Its four receivers receive the reflected signal from the target and by comparing the reception times produce the TDOA components of $\Delta t_{1,j}$ of equations (7). For simulation reasons an equivalent range comparison system is presented at figure 3.

The same equivalent system is the basis of the TDOA Gradient Generator shown at figure 4, where the approximate gradient components (14) of the energy function (8) are composed in a straightforward way described in (13) Components of the gradient is approximated by:

$$DEx = \frac{-E(x + \Delta x) + E(x - \Delta x)}{2\Delta x}$$
$$DEy = \frac{-E(y + \Delta y) + E(y - \Delta y)}{2\Delta y}$$
$$DEz = \frac{-E(z + \Delta z) + E(z - \Delta z)}{2\Delta z}$$
(15)

The blocks of extracting the energy function E(x, y, z) are given in figure 5.

Test system was simulated using Matlab's Simulink [®]. This consists of four receivers at:

$$\begin{bmatrix} x_{1} \\ y_{1} \\ z_{1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.414 \end{bmatrix} km \begin{bmatrix} x_{2} \\ y_{2} \\ z_{2} \end{bmatrix} = \begin{bmatrix} 20.544 \\ 0.743 \\ 0.525 \end{bmatrix} km$$

$$\begin{bmatrix} x_{3} \\ y_{3} \\ z_{3} \end{bmatrix} = \begin{bmatrix} -6.614 \\ -17.218 \\ 0.545 \end{bmatrix} km \begin{bmatrix} x_{4} \\ y_{4} \\ z_{4} \end{bmatrix} = \begin{bmatrix} -10.487 \\ 20.443 \\ 0.822 \end{bmatrix} km$$
(16)

This set of receivers is placed on a rough land surface as receiver altitudes denote. Coordinate axes x'x and y'y denote position from West (negative) to East and from South to North respectively while axis z'z denotes altitude (height) placement. Other parameters according to equations (8) and (9) are chosen to be:

$$e = 2, g = 10, G = 10^{12},$$

$$T = \frac{1}{10} \begin{bmatrix} 8 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 8 \end{bmatrix}$$
(17)

Finally the simulation step is chosen in a way to simulate a low PRF radar signal, similar to the signal produced (for example) by an SRE – M2a air surveillance radar [5].

The target trajectories simulated here are chosen between extremely maneuverable airborne targets and tactical ballistic missiles.

- Target 1: A tri-sonic aircraft approaching from a 300 km West 200 km South horizontal velocity of 1km/sec (0.6 km/sec towards East and 0.4 km/sec towards North) while performing vertical maneuvers between 6 and 10 km altitudes with angular frequency of 0.05 rad/sec. The initial target position guess is at 140km South – West at an altitude of 5 km
- Target 2: A bi-sonic aircraft approaching from the West at 600 m/sec performing an elliptic spiral roll with major axis of 6 km and minor axis of 2 km at 6 km altitude with a radial velocity of 0.05pi rad/sec This target is maneuvering, developing a maximum acceleration estimated at 148.33

m/sec² (about 15 g). Initial target guess is at 150 km West 20 km South.

• Ballistic Target: A tactical ballistic missile launched from 50 km East 300 km South at a take – off angle of 400 mils and a velocity of 3200 km/sec towards the North. Initial launch position guess is at 30 km East 200 km South.

5. Simulation Results.

Simulation results are shown to figures 6 to 11 at the Appendix 2. All targets are shown in Range Azimuth and Elevation vs. time. Next the difference of the real Range Azimuth and Elevation and the indicated by the TIRN is shown. Simulation times are 1000 seconds for maneuvering and the entire time of flight (~250 seconds) for ballistic targets.

For target 1 (figures 6 and 8) target detection occurs at about 250 km for approaching target while tracking is maintained up to 220 km for retreating target after that, the range and elevation indicated values are lower than real. It is interesting to observe that azimuth tracking is easier to achieve and it is maintained after range tracking loss, but the errors are more significant at close ranges, still the indicated value differs less than one degree from real target azimuth.

For target 2 and (figures 8, 9) the situation is different. Target detection occurs closer; at a range about 120 km for an approaching target but tracking is maintained for retreating target at distances near 300 km. Difference figures indicate a lag at azimuth tracking that becomes significant at close ranges. This leads to a conclusion that effort must be made to improve convergence rate of the ANN in order to achieve better performance, or simply use a complementary azimuth tracking method, monopulse $\Sigma - \Delta$ (sum – angular or phase difference) antenna patterns being one suggestion.

Finally for the ballistic target (figures 10 and 11) adequate tracking is achieved at 220 km for an approaching and maintained up to 270 km for a retreating missile. After that the TIRN indicates range and elevation values lower than the real ones.

Several simulations have been executed for random directions, all giving similar results.

6. Conclusions.

Bearing in mind that the objective is to design a totally passive multistatic detection and tracking radar a TIRN using TDOA method and an ANN is an attractive solution. It must be noted that no special antenna parameters is used in the analysis above, that means that the only antenna limitation is to produce a gain adequate for signal detection as described in [3].

As the simulation of many targets has investigated it, the tracking sensitivity depends on the convergence rate and the distances between the receivers. An empirical rule is that tracking may be achieved at maximum ranges 5 to 15 times greater than the maximum distance between the TIRN receivers for random constellations. Random constellations offer more tactical flexibility in rough or island environments. Since rural these environments is the most probable areas of operation for a TIRN no effort has been made to optimize detection by optimizing the geometry (in contrast to [2]). The only limitation is to avoid placing three TIRN receivers to a straight line as noted in [1].

Care must be taken to cure the limitations observed during the simulation. The first is the tracking loss in long ranges and the other one is the azimuth deviation in close ranges. The first problem occurs because the TDOA Δt_{ij} variations (eq. 4, 7) with range become small at long target ranges relative to TIRN receivers distance. Since range limitation is not exceeding that of the antenna mentioned this is not a significant problem. Else, conventional non - cooperative bistatic location methods analyzed in [1], [3] may be used. The second problem is only occurring when tracking of an extremely maneuvering target is taking place. Improving convergence rate by using variable e, g, G (eq. 8, 9) can solve this problem. Actually, only G can be replaced with a dynamic equivalent:

$$G_{dynamic} = G_{static} \frac{(Maximum Range)}{(Indicated Range) + 1}$$
(18)

This settles down the problem adequately, the azimuth tracking error being reduced to less than one degree again. Another way to avoid this problem is using complementary azimuth tracking method (monopulse tracking for example).

It must be also noted that this is a second order model. That means that steady-state acceleration errors will be always present, but if acceleration limits are clear (e.g. $\pm 20g$) for extremely maneuvering targets this may be reasonably minimized. Higher order models may solve this problem completely but they suffer some stability problems making them rather impractical. Finally an interesting observation is that for ballistic targets tracking is maintained for adequate time to extract by rearward extrapolation the launch site of the missiles. This is important if the TIRN is used (among others) in counter – artillery role.

In that case the acceleration error may be diminished during the extrapolation process using robust models of polynomial fitting with estimated acceleration components for aerodynamic forces and gravity respectively.

Anyway it must be obvious by now that the advantages of totally passive location and tracking system is more important than the problems noted here and in practice most of the problems occurred are rather easy to solve. Note that the system described is one tracking radar, difficult if not practically impossible to detect and jam, and it may use totally non – cooperative transmitter(s) as signal sources, giving its operator a great tactical advance.

References.

- [1]. Nikos J. Farsaris and Prof. Peter P. Stavroulakis: "Target Detection via Measurements Taken by a Transmitter Independent Receiver Network" AGARD CPP-582, April 1996
- [2]. Yin Chengyou, Xu Shanjia Wang Dongjin: "Location Accuracy of Multistatic Radars Based on Ranging Information. Department of Electronic Engineering and Information Science, University of Science and Technology China". 230027.
- [3]. Nicholas Willis: "Bistatic Radars", Artech House 1991.
- [4]. A. Cichocki, R. Unbehauen: "Neural Networks for Optimization and Signal Processing", John Wiley & Sons Ltd 1993. pp102-104.
- [5]. NAMSA NAMFI, "Technical Manual for Air Surveillance Radar" SRE-M2a TME 2702 vol.1 March 1984.

Appendix 1. Target detection and tracking system models



Figure 1: Target Detection and Tracking System.



Figure 2: Optimizer ANN: parameters described at section 4.



Figure 3: TDOA Generator (TIRN Equivalent)



Figure 4: Gradient Generator Circuit.



Figure 5:Energy Function Generator "Stetrag" in fig. 4



Figure 7: Differences between Real and Detected Range Azimuth and Elevation vs. Time for Target 1.



Figure 9: Differences between Real and Detected Range Azimuth and Elevation vs. Time for Target 2.



Figure 11: Differences between Real and Detected Range Azimuth and Elevation vs. Time for Ballistic Target