Reference Generation for Harmonics Cancellation

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Abstract: - This paper presents two recursive schemes for current reference generation for shunt active filters under unknown fundamental frequency. The schemes are based on the liner Kalman filter that needs the knowledge of the fundamental frequency. In practice, the fundamental frequency of the power system grid can vary. If it differs from the fundamental frequency considered in the mathematical model used in the Kalman filter the estimates provided by the filter will not be accurate. This paper proposes the use of on-line identification of the fundamental frequency of the power system. The identified frequency is used to update de Kalman filter model.

Key-Words: - Reference generation, active filters, harmonics, Kalman filter, identification.

1 Introduction

The increasing use of nonlinear loads has implied in harmonic injection on power systems. Some drawbacks provided by these types of loads, according [1], are voltage distortion, increased losses and heating, and misoperation of protective equipment. The current harmonics can also increase losses in rotating machines. They are responsible for oscillatory torque that causes mechanical stress, leads to malfunctions in sensitive loads and can create interference with communication equipments [2]. Therefore, harmonics on power system have become an important issue for the electric utility companies.

Passive LC filters and capacitors have been used to eliminate line current harmonics and to increase the power factor. However, if the amplitude and frequency of the distortion power vary randomly, those conventional approaches become ineffective [2]. They also introduce resonances in the power system, tend to be bulky and the design can be complex [3].

An alternative to the passive filters is the shunt active filter that permits to compensate the harmonics and asymmetries of the mains currents caused by nonlinear loads. For harmonic cancellation, the shunt active filter injects ac three-phase currents in the system to cancel the harmonic content.

The harmonic measurement and reference generation are important for the design and control of active filters. Moreover, the technique used to obtain the reference currents will have a decisive influence on their efficiency and performance [3] [4].

Several authors have been addressed this subject using different techniques for obtaining the reference signal for the active filter. In [3] it is described the use of band-stop and band-pass filters. In [5] and [6] low-pass filters are used to generate the current references. These techniques are of simple implementation but, according [3], in practice, they suffer from residual current phase shift and magnitude.

Other methods of current reference generation are available. For instance, instantaneous active and reactive current component method [7], p-q theory method [8] among others. According [4] these techniques suffers from the influence of the distortion of the main voltage waveform. For digital implementations there is a delay due to the processing of the signal.

Another classic approach to obtain the current frequency spectra is the discrete Fourier transform (DFT) or the fast Fourier transform (FFT) algorithms. However, the application of the DFT or FFT relies on some basic assumptions, summarized in [9]. If one fails to fulfill the basic assumptions of DFT or FFT algorithms, they will lead to incorrect results.

In practice, the nonlinear loads operate at continuously variable power levels that result in a dynamic behavior of the harmonics in the system. Other practical aspects are the measurement noise and grid frequency variations. In strong grids, the frequency variations are usually small, but larger frequency variations occur in autonomous grids [10].

Under these practical issues, some authors proposed the use of the Kalman filter to estimate and track the harmonics of the grid [4], [9], [11] and [12]. In these formulations it is assumed that the frequency of the grid is known and is constant.

This paper proposes two on-line schemes, based on Kalman filter, for generation of the active filter current references. If the grid frequency differs from the assumed frequency in the mathematical model, the filter will provide erroneous tracking of the harmonics. In order to account for possible fundamental frequency changes, the first method makes use of a recursive least squares algorithm (RLS) that is responsible for the on-line identification of the fundamental frequency of the grid. Then, the identified frequency is used to update the mathematical model of the harmonics needed in the Kalman filter. The second method uses a nonlinear identifier proposed in [13] to update the Kalman filter model.

This paper is organized as follow: in section 2 it is presented the mathematical model for the harmonics and the parametrization used in one of the fundamental frequency identification methods (RLS) that are described in section 3. In section 4 the Kalman filter algorithm is summarized. Simulation results are given in section 5.

2 Mathematical Modelling

This section describes how the harmonics mathematical model is obtained. This model is necessary for the implementation of the Kalman filter. It also describes de parametrization used in the RLS method for identification of the fundamental frequency.

2.1 Harmonics Model

Consider a signal with amplitude $A(t_k)$, angular frequency ω and phase θ:

$$s(t_k) = A(t_k)\cos(\omega t_k + \theta) \tag{1}$$

Let $x_{1_k} = A(t_k)\cos(\omega t_k + \theta)$ and $x_{2_k} = A(t_k)\sin(\omega t_k + \theta)$. At t_{k+1}, which is $t_k + \Delta t$ the signal may be expressed [9] as:

$$\begin{aligned} s(t_{k+1}) &= A(t_{k+1})\cos(\omega t_k + \omega\Delta t + \theta) = x_{1_{k+1}} \\ &= x_{1_k}\cos(\omega\Delta t) - x_{2_k}\sin(\omega\Delta t) \end{aligned}$$

We also have

$$\begin{aligned} x_{2_{k+1}} &= A(t_{k+1})\sin(\omega t_k + \omega\Delta t + \theta) \\ &= x_{1_k}\sin(\omega\Delta t) + x_{2_k}\cos(\omega\Delta t) \end{aligned}$$

Therefore, the signal state variable representation becomes:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{k+1} = \begin{bmatrix} \cos(\omega\Delta t) & -\sin(\omega\Delta t) \\ \sin(\omega\Delta t) & \cos(\omega\Delta t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k + \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}_k \tag{2}$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k + \nu_k \tag{3}$$

where $\begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix}_k^T$ allow the system to random walk and υ_k represents the measurement noise.

If the signal includes n frequencies, that is

$$s(t_k) = \sum_{i=1}^{n} A_i(t_k)\cos(i\omega t_k + \theta_i) \tag{4}$$

the state variable representation becomes:

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n-1} \\ x_{2n} \end{bmatrix}_{k+1} = \begin{bmatrix} M_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & M_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n-1} \\ x_{2n} \end{bmatrix}_k + \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_{2n-1} \\ \gamma_{2n} \end{bmatrix}_k \tag{5}$$

$$y_k = \begin{bmatrix} 1 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n-1} \\ x_{2n} \end{bmatrix}_k + \nu_k \tag{6}$$

where

$$M_i = \begin{bmatrix} \cos(i\omega\Delta t) & -\sin(i\omega\Delta t) \\ \sin(i\omega\Delta t) & \cos(i\omega\Delta t) \end{bmatrix} \tag{7}$$

2.2 Parametrization for Fundamental Frequency Identification using RLS

Consider the following fundamental signal representation:

$$s_1(t) = A_1\cos(\omega_1 t + \theta_1) \tag{8}$$

where A_1, ω_1 and θ_1 are the magnitude, angular frequency and phase, respectively.

Let $x_1(t) = A_1\cos(\omega_1 t + \theta_1)$ and $x_2(t) = \dot{x}_1(t) = -\omega_1 A_1\cos(\omega_1 t + \theta_1)$. Hence, $\dot{x}_2(t) = -\omega_1^2 A_1\cos(\omega_1 t + \theta_1) = -\omega_1^2 x_1(t)$ that leads to the continuous state-space model:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{9}$$

$$y_1(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{10}$$

A discrete state-space model on the form

$$x_{k+1} = Hx_k \tag{11}$$

$$y_{1_k} = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k \tag{12}$$

can be found applying the relationship [14]

$$H = e^{A\Delta t} \approx I + A\Delta t + \frac{1}{2!} A^2 (\Delta t)^2 \tag{13}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & 0 \end{bmatrix} \tag{14}$$

The model (11)-(12) leads to the following ARX representation

$$y_{1_{k+1}} = \left[2 - \omega_1^2 (\Delta t)^2 \right] y_{1_k} - \left[1 + \frac{\omega_1^4 (\Delta t)^4}{4} \right] + \varsigma_{k+1} \tag{15}$$

that will be useful in the fundamental frequency identification described in next section. In equation (15) ς_{k+1} represents measurement noise.

3 Fundamental Frequency Identification

Two methods for fundamental frequency identification are presented here. The first one relies on a RLS algorithm [15] while the second one is an AFPLL (amplitude-frequency-phase-locked loop) [13]. Here the AFPLL is used for the identification of the fundamental frequency.

3.1 RLS Method

In order to apply the RLS algorithm, the equation (15) is represented in the following form:

$$y_{1_{k+1}} = \phi_k^T \theta \tag{16}$$

where

$$\phi_k = \begin{bmatrix} y_{1_k} & y_{1_{k-1}} \end{bmatrix} \tag{17}$$

$$\theta = \begin{bmatrix} \left(2 - \omega_1^2 (\Delta t)^2 \right) & -\left(1 + \frac{\omega_1^4 (\Delta t)^4}{4} \right) \end{bmatrix}^T \tag{18}$$

The RLS algorithm is summarized below.

Initial conditions: $\phi(0)$, $\hat{\theta}(0)$ and $P_{RLS}(-1) = cI$, with $c \in \Re^+$.

At the time $k+1$:

1) Measure $y_{1_{k+1}}$

2) Evaluate the predicted output $y_{1_{k+1}}$, that is,

$$\hat{y}_{1_{k+1}} = \phi_k^T \hat{\theta}_k \tag{19}$$

3) Evaluate the gain

$$K_{RLS_k} = \frac{P_{RLS_{k-1}} \phi_k}{1 + \phi_k^T P_{k-1} \phi_k} \tag{20}$$

4) Update the parametric estimate

$$\hat{\theta}_{k+1} = \hat{\theta}_k + K_{RLS} \left(y_{1_{k+1}} - \hat{y}_{1_{k+1}} \right) \tag{21}$$

5) Update the covariance matrix

$$P_{RLS_k} = \left(I - K_{RLS_k} \phi_k^T \right) P_{RLS_{k-1}} \tag{22}$$

6) Increment k and return to step 1.

The fundamental angular frequency ω_1 is obtained at each sample interval $k+1$ through $\hat{\theta}_{k+1}(1)$, that is,

$$\hat{\theta}_{k+1}(1) = \left(2 - \hat{\omega}_{1_{k+1}}^2 (\Delta t)^2 \right) \tag{23}$$

hence,

$$\hat{\omega}_{1_{k+1}} = \sqrt{\frac{2 - \hat{\theta}_{k+1}(1)}{(\Delta t)^2}} \tag{24}$$

3.2 AFPLL Method

Let A, δ and ω be the amplitude, phase angle and angular frequency of the desired component. Then the following set of equations is used:

$$\dot{A}(t) = \mu_1 e(t) \sin \phi(t) \tag{25}$$

$$\dot{\omega}(t) = \mu_2 e(t) \cos \phi(t) \tag{26}$$

$$\dot{\phi}(t) = \mu_2 \mu_3 e(t) \cos \phi(t) + \omega(t) \tag{27}$$

$$\hat{y}(t) = A(t)\sin\phi(t) \quad (28)$$

$$e(t) = y(t) - \hat{y}(t) \quad (29)$$

where $y(t)$ is measure of the signal. The parameters μ_1, μ_2 and μ_3 are positive numbers that determine the behavior of the algorithm in terms of convergence rate versus accuracy.

Theorem 1: Let $u(t) = A_0 \sin(\omega_0 t + \delta_0) + g(t)$ where A_0, ω_0 and δ_0 are real constants, and $g(t)$ is an arbitrary T_0-periodic bounded continuous function that has no frequency component at ω_0. For a proper choice of parameters $\{\mu_i, i = 1,2,3\}$, the dynamical system (25)-(29) has a unique periodic orbit $\vartheta(t)$ in (A, ω, ϕ) space in a neighborhood of $\mu_0(t) = A_0 \sin(\omega_0 t + \delta_0)$. This neighborhood is determined by the function $g(t)$ and the parameters μ_1 to μ_3. Moreover, this periodic orbit is asymptotically stable. The orbit coincides with $\mu_0(t)$ when $g(t)$ is zero.

Proof: See [16].

4 Kalman Filter Algorithm

Consider dynamic system represented by the following stochastic model:

$$x_{k+1} = \Phi_k x_k + \Gamma_k \gamma_k \quad (30)$$

$$y_{k+1} = F_k x_k + \upsilon_k \quad (31)$$

$$\dim x_k = n,\ \dim y_k = r,\ \dim \gamma_k = p \quad (32)$$

where γ_k and ς_k are uncorrelated Gaussian white noise sequences with means and covariances as follows:

$$E\{\gamma_i\} = 0,\ E\{\gamma_i \gamma_i^T\} = Q_i \delta_{ij} \quad (33)$$

$$E\{\upsilon_i\} = 0,\ E\{\upsilon_i \upsilon_i^T\} = R_i \delta_{ij} \quad (34)$$

$$E\{\gamma_i \upsilon_j^T\} = 0, E\{\gamma_i x_j^T\} = 0, E\{\upsilon_i x_j^T\} = 0, \forall i, j \quad (35)$$

where $E\{\cdot\}$ denotes expectation and δ_{ij} denotes the Kronecker delta function.

The filtering equations are [17]:

$$\hat{x}_{k+1|k} = \Phi_k \hat{x}_{k|k} \quad (36)$$

$$\hat{x}_{k|k} = \hat{x}_{k+1|k} + K_k\left(y_k - F_k \hat{x}_{k|k-1}\right) \quad (37)$$

where

$$K_k = P_{k|k-1} F_k^T \left(F_k P_{k|k-1} F_k^T + R_k\right)^{-1} \quad (38)$$

$$P_{k|k} = P_{k|k-1} - K_k F_k P_{k|k-1} \quad (39)$$

$$P_{k+1|k} = \Phi_k P_{k|k} \Phi_k^T + \Gamma_k Q_k \Gamma_k^T \quad (40)$$

with given initial conditions $\hat{x}_0$ and $P_{0|-1}$.

We also have

$$P_{k|k-1} \stackrel{\Delta}{=} E\left\{\left(x_k - \hat{x}_{k|k-1}\right)\left(x_k - \hat{x}_{k|k-1}\right)^T\right\} \quad (41)$$

$$P_{k|k} \stackrel{\Delta}{=} E\left\{\left(x_k - \hat{x}_{k|k}\right)\left(x_k - \hat{x}_{k|k}\right)^T\right\} \quad (42)$$

5 Simulation Results

In order to test the reference generation schemes proposed. It was considered a 60 Hz three-phase system (380 V phase to phase) and a 6 pulse three-phase rectifier with a 500 Ω load. The results presented are related to phase A. The sample frequency is 21 kHz and it is considered noise measurements N~(0, 0.005). In Fig. 1 it is presented the measured current of phase A.

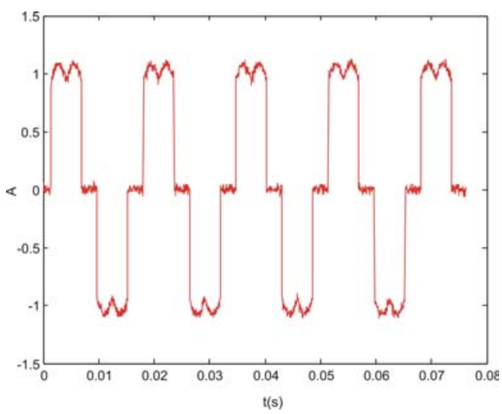

Fig. 1 – Measurement of the current of phase A

The harmonics model considered the odd harmonics from the 1st up to the 31st except the 3rd and its multiples. The Kalman filter variance matrices are: $Q = 0.05 I_{22}$ and $R = 10$. The initial conditions of the Kalman filter are: $\hat{x}_{0|-1} = 0, P_{0|-1} = 10 I_{22}$.

The RLS algorithm was initialized with the following parameters: $\phi_0 = 0$, $P_{RLS_{-1}} = 0.1 I_2$ and $\hat{\theta}_0 = [1.9997091488 \quad \text{-}1.00000002115]^T$. After the transitory, at $t = 0.05\,s$, the covariance matrix P_{RLS}

is reseted always that $trace(P_{RLS}) < 5$. This procedure is to avoid large transient on the start-up of the RLS and to improve the convergence after $t = 0.05\,s$. In the AFPLL method $\mu_1 = 100$, $\mu_2 = 2000$ and $\mu_3 = 0.05$. The initial conditions were $A_0 = 0$, $\phi_0 = 0$ and $\omega_0 = 2 \cdot \pi \cdot 57$.

Fig. 2 shows the convergence of the identified fundamental frequency with RLS. After a transient, the identified value converges to the desired value. Fig. 3 presents the reference signal to compensate the harmonics. This reference leads to Fig. 4 where it is depicted the compensated line current. High order harmonics are not compensated because they are not described in the Kalman filter model and in practice, due limitations on the compensation capacity of the active filters, they can be compensated by means of passive filters.

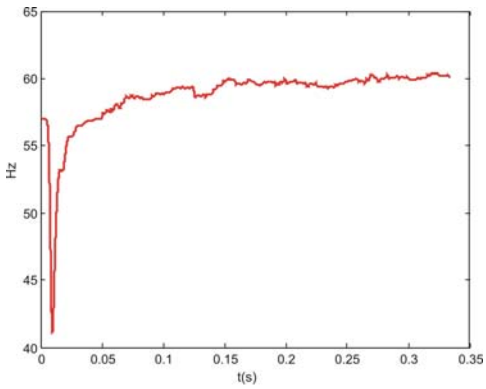

Fig. 2 – Identification of frequency with RLS

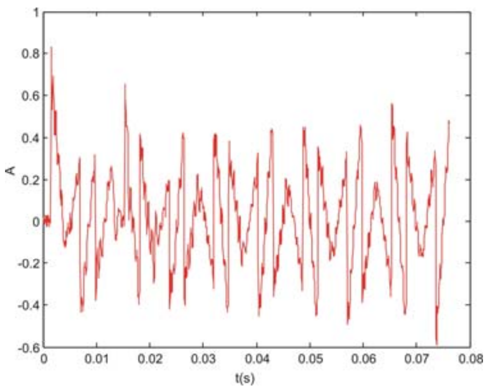

Fig. 3 – References obtained with Kalman filtering and RLS

In Fig. 5 it is depicted the convergence of the identified fundamental frequency with AFPLL. The transient is lower compared to the RLS. Fig. 6 presents the reference signal to compensate the harmonics. This reference leads to Fig. 7 where it is depicted the compensated line current. The results are similar to the Kalman filter with RLS except for the initial transient that is a little bit higher in the Kalman filter with RLS.

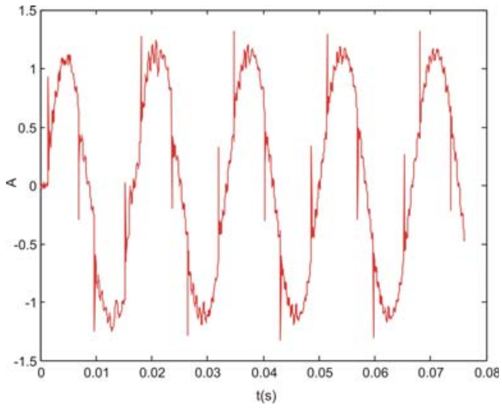

Fig. 4 – Compensated currents with Kalman filtering and RLS

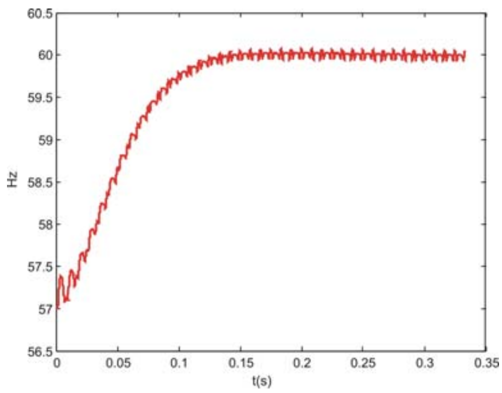

Fig. 5 – Identification of frequency with AFPLL

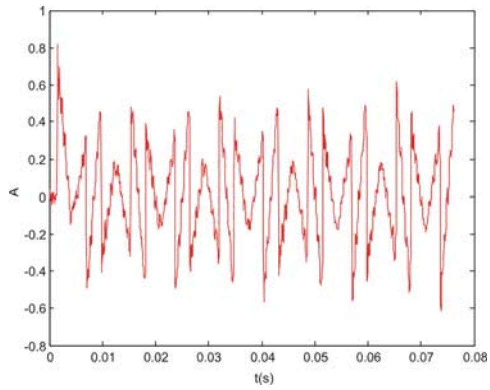

Fig. 6 – References obtained with Kalman filtering and AFPLL

6 Conclusion

This paper presented two schemes for current reference generation for shunt active filters applied for harmonic cancellation under unknown fundamental frequency. Both schemes are based on the Kalman filter and a RLS and AFPLL are proposed to cope with the problem of unknown or variable fundamental frequency. The simulation results show the effectiveness of the method. Both strategies of frequency identification are of simple implementation. Since AFPLL deals with other quantities than frequency, it provides better results compared to the RLS.

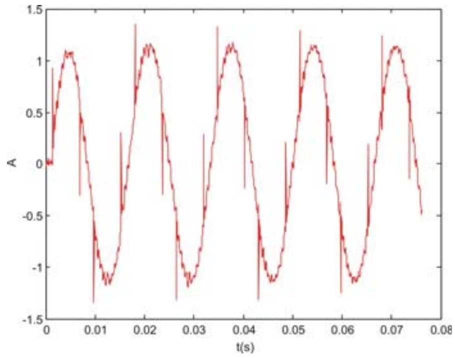

Fig. 7 – Compensated currents with Kalman filtering and AFPLL

In practice high order harmonics are not cancelled by the active filter because of cost or due limitations of the filter. If necessary, they can be treated by passive filters.

Since the Kalman filter deals with stochastic systems, the measurement noise influences are reduced in this strategy. Phase issues that arise in other methods, such as low pass filter or band pass filter, are also avoided. The scheme does not have problems concerning number of samples per period as those that appears when a FFT or DFT strategy is employed.

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