First-order Interval Type-2 TSK Fuzzy Logic Systems Using a Hybrid Learning Algorithm

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Abstract: - This article presents a new learning methodology based on a hybrid algorithm for interval type-2 TSK fuzzy logic systems (FLS). Using input-output data pairs during the forward pass of the training process, the interval type-2 TSK FLS output is calculated and the consequent parameters are estimated by recursive least-squares (RLS) method. In the backward pass, the error propagates backward, and the antecedent parameters are estimated by back-propagation (BP) method. The proposed hybrid methodology was used to construct an interval type-2 TSK fuzzy model capable of approximating the behaviour of the steel strip temperature as it is being rolled in an industrial Hot Strip Mill (HSM) and used to predict the transfer bar surface temperature at the finishing Scale Breaker (SB) entry zone. Comparative results show the advantage of the hybrid learning method RLS-BP over BP.

Key-Words: - Interval type-2 TKS fuzzy inference systems; Interval type-2 TSK neuro-fuzzy systems; Hybrid learning; Uncertain rule-based TKS fuzzy logic systems; Temperature modelling and prediction.

1 Introduction

When a membership value cannot be accommodated in a set as 0 or 1, fuzzy sets of type-1 are used. Similarly, when the membership grade cannot be determined even as a crisp number in [0, 1], fuzzy sets of type-2 are used. Interval type-2 fuzzy logic systems (FLS) constitute an emerging technology. In [1] one-pass and back-propagation (BP) methods are presented as interval type-2 Mamdani FLS learning methods but only BP method for interval type-2 Takagi-Sugeno-Kang (TSK) FLS. When BP method is used, none of antecedent and consequent parameters of the type-2 FLS are fixed at the starting of the training process; they are tuned using exclusively steepest descent method. Recursive least-squares (RLS) is not presented as a type-2 FLS learning method. In [1], there are explained three basic reasons that prevent the use of RLS on both Mamdani and TSK type-2 FLS systems.

The hybrid algorithm for interval type-2 Mamdani FLS has been already presented [3, 4] with two combinations of learning methods: RLS-BP and REFIL-BP.

The aim of this work is to present and discuss a hybrid learning algorithm for interval type-2 TSK FLS' antecedent and consequent parameters estimation during training process using input-output data pairs. Interval type-2 TSK FLS output is calculated during forward pass and consequent' parameters are estimated using RLS [2]. During the backward pass, the error propagates backward and the antecedent parameters are estimated using the BP method. The proposed algorithm is evaluated using an interval type-2 TSK FLS inference system, which predicts the transfer bar surface temperature at Hot Strip Mill (HSM) Finishing Scale Breaker (SB) entry zone.

2 Problem Formulation

Most of the industrial processes are highly uncertain, non-linear, time varying and non-stationary [3, 5], having very complex mathematical representations. Interval Type-2 TSK FLS take easily the random and systematic components of type A or B standard uncertainty [6] of industrial measurements. The non-linearities are handled by

FLS as identifiers and universal approximators of nonlinear dynamic systems [7, 8]. Stationary and non-stationary additive noise is modelled as a Gaussian function centred at the measurement value. In stationary additive noise the standard deviation takes a single value, whereas in non-stationary additive noise the standard deviation varies over an interval of values [1]. Such characteristics make interval type-2 TSK FLS a very powerful inference system to model and control industrial processes. Considering the appropriateness of the hybrid learning method in ANFIS type-1 systems [9, 10] and in type-2 Mamdani FLS [3, 4], it is convenient to consider an equivalent hybrid learning algorithm for interval type-2 TSK FLS.

In [1] only back-propagation (BP) algorithm is presented as interval type-2 TSK FLS learning method. To the best knowledge of the authors, the hybrid learning method has not been reported in type-2 TSK FLS.

Only the BP learning method for interval type-2 TSK FLS has been proposed in the literature and it is used as a benchmark algorithm for parameter estimation or systems identification on interval type-2 TSK FLS [1]. One of the main contributions of this work is to implement a new hybrid learning algorithm for interval type-2 TSK FLS using RLS [2] together with BP algorithm.

According to Mendel [1], there are three basic points that prevent the use of RLS for consequent parameter estimation in interval type-2 TSK FLS:

1. The starting point for the least-squares method to design an interval FLS is a type-1 Fuzzy Basis Function (FBF) expansion. No such FBF expansion exists for an interval type-2 TSK FLS. Since an interval type-2 TSK FLS output $y(\mathbf{x})$ can be expressed as:

$$y(\mathbf{x}) = \frac{1}{2}\left[\mathbf{y}_l^T \mathbf{p}_l(\mathbf{x}) + \mathbf{y}_r^T \mathbf{p}_r(\mathbf{x})\right] \qquad (1)$$

with $i = 1,2,\ldots M$ ordered rules, it looks like a least-squares method can be used to tune the parameters in $\mathbf{y}_l^T$ (matrix transpose of M left-points y_l^i of consequent centroids) and $\mathbf{y}_r^T$ (matrix transpose of M right-points y_r^i of consequent centroids). Unfortunately, this is incorrect. The problem is that, in order to know the FBF expansion $\mathbf{p}_l(\mathbf{x})$ and $\mathbf{p}_r(\mathbf{x})$, each y_l^i and y_r^i (the M left-points and right points of interval consequent centroids) must be known first. Because at initial conditions there are no numerical values for those elements, hence, the FBF $\mathbf{p}_l(\mathbf{x})$ and $\mathbf{p}_r(\mathbf{x})$ cannot be calculated. This situation does not occur for type-1 FBF expansion.

2. Although y_l and y_r can be expressed in terms of their lower ($\underline{f}^i$) and upper ($\overline{f}^i$) M firing sets as:

$$y_l = y_l\left(\overline{f}^1,\ldots,\overline{f}^L,\underline{f}^{L+1},\ldots,\underline{f}^M, y_l^1,\ldots,y_l^M\right) \quad (2)$$

$$y_r = y_r\left(\underline{f}^1,\ldots,\underline{f}^R,\overline{f}^{R+1},\ldots,\overline{f}^M, y_r^1,\ldots,y_r^M\right) \quad (3)$$

the values of L and R are not known in advance. L is the index to the rule-ordered FBF expansions at which y_l is a minimum, and R is the index at which y_r is a maximum. Once the points L and R are known, (1) is very useful to organize and describe the calculations of y_l and y_r.

3. The next problem deals with the re-ordering of y_l^i and y_r^i. The type-1 FBF expansions have always had an inherent rule ordering associated with them; i.e., rules $R^1, R^2,\ldots,R^M$ always established the first, second,…, and *Mth* FBF. This order is lost and it is necessary to restore it for later use.

Although convergence of the proposed method has been tested in practice, a mathematical proof is still needed in general for hybrid learning algorithms.

A second but very important purpose of this paper is to propose an application methodology based on interval type-2 TSK FLS and the hybrid learning method mentioned above to HSM temperature prediction. Interval type-2 TSK FLS is suitable for industrial modelling and control applications. Although temperature prediction is a critical issue in a HSM the problem has not been fully addressed by fuzzy logic control systems [1, 3, 4].

3 Problem Solution

3.1 Type-2 FLS

A type-2 fuzzy set, denoted by $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$ and $0 \le \mu_{\tilde{A}}(x,u) \le 1$:

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \qquad (4)$$

This means that at a specific value of x, say x', there is no longer a single value as for the type-1 membership function (u'); instead, the type-2 membership function takes on a set of values named the primary membership of x', $u \in J_x \subseteq [0,1]$. It is possible to assign an amplitude distribution to all of these points. This amplitude is named a secondary grade of type-2 fuzzy set. When the values of secondary grade are the same and equal to 1, there is the case of an interval type-2 membership function [1, 11, 12, 13, 14].

3.2 Using RLS Learning Algorithm in Interval Type-2 TSK FLS

In interval type-2 TSK FLS [1], the learning algorithm is BP during backward pass for antecedent and consequent parameters estimation as shown in Table 1.

Table 1
One Pass in Learning Procedure for Interval Type-2 TSK FLS

	Forward Pass	Backward Pass
Antecedent Parameters	Fixed	BP
Consequent Parameters	Fixed	BP

In the proposed hybrid algorithm, RLS is used during forward pass for consequent parameters tuning, and BP during backward pass for antecedent parameters tuning, as shown in Table 2.

Table 2
Two Passes in Hybrid Learning Procedure for Interval Type-2 TSK FLS

	Forward Pass	Backward Pass
Antecedent Parameters	Fixed	BP
Consequent Parameters	RLS	Fixed

3.3 Adaptive RLS-BP Hybrid Learning Algorithm

The hybrid training method is based on the initial conditions of consequent parameters: y_l^i and y_r^i. It is presented as in [1]: Given N input-output training data pairs, the hybrid training algorithm for E training epochs, should minimize the error function:

$$e^{(t)} = \frac{1}{2}\left[f_{s2}\left(\mathbf{x}^{(t)}\right) - y^{(t)}\right]^2 \quad (5)$$

where $f_{s2}\left(\mathbf{x}^{(t)}\right)$ is the defuzzified output. The following paragraph describes the RLS-BP hybrid algorithm:

1. Initialize all parameters in the antecedent and consequent membership functions.

2. Set the counter, ep of the training epoch to zero; i.e., $ep \equiv 0$.

3. Set the counter, t of the training data to unity; i.e., $t \equiv 1$.

4. Apply the input $\mathbf{x}^{(t)}$ to the interval type-2 TSK FLS and compute the total firing interval and consequent for each rule; i.e. compute $\underline{f}^i$ and $\overline{f}^i$.

5. Compute y_l and y_r using the iterative method described in [1]. Establish L and R, so y_l and y_r can be expressed as:

$$y_l = y_l\left(\overline{f}^1,\ldots,\overline{f}^L,\underline{f}^{L+1},\ldots,\underline{f}^M,y_l^1,\ldots,y_l^M\right)$$
$$y_r = y_r\left(\underline{f}^1,\ldots,\underline{f}^R,\overline{f}^{R+1},\ldots,\overline{f}^M,y_r^1,\ldots,y_r^M\right)$$

6. Compute the defuzzified output, $f_{TSK,2}\left(\mathbf{x}^{(t)}\right)$.

7. Determine the explicit dependence of y_l and y_r on membership functions. Because L and R obtained in step 5 usually change from one t-iteration to the next, the dependence of y_l and y_r on membership functions will change also from one t-iteration to the next.

8. Test each component of $\mathbf{x}^{(t)}$ to determine the active branches: the lower and upper values of membership functions of each rule.

9. Tune the parameters of the active branches of the consequent using RLS algorithm [2].

10. Tune the parameters of the active branches of the antecedent's membership functions using the BP algorithm.

11. Set $t \equiv t+1$. If $t \equiv N+1$, go to step 12; otherwise, go to step 4.

12. Set $ep \equiv ep+1$. If $ep \equiv E$, STOP; otherwise go to step 3.

4 Application to Transfer Bar Surface Temperature Prediction

4.1 Hot Strip Mill (HSM)

Because of the complexities and uncertainties involved in rolling operations, the development of mathematical theories has been largely restricted to two-dimensional models applicable to heat losing in flat rolling operations. Fig. 1 shows a simplified diagram of a HSM.

Quality assurance in HSM processes lies mostly in the effective use of control and automation techniques. The most critical stage in the HSM process occurs in the Finishing Mill (FM). Actually, there are several mathematical model-based systems for setting up the FM. There is a model-based set-up system [15] that calculates the FM working references needed to obtain gauge, width and temperature at the FM exit stands.

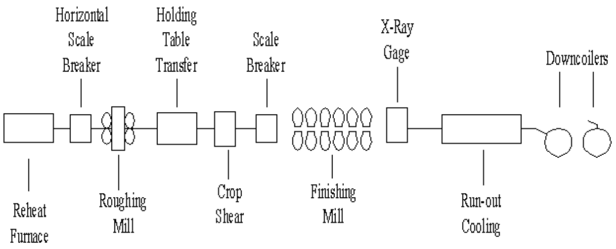

Fig. 1 Typical Hot Strip Mill (HSM)

The errors in the gauge of the transfer bar are absorbed in the first two FM stands and therefore have a little effect on the target exit gauge. It is very important for the model to know the FM entry temperature accurately. A temperature error will propagate through the entire FM.

The inputs of the interval type-2 TSK FLS model used to predict the SB entry temperature are the surface temperature of the transfer bar and the time required by the transfer bar head to reach the SB entry zone. Currently, the surface temperature is measured using a pyrometer located at the Roughing Mill (RM) exit side. This measurement is affected by noise produced by the transfer bar scale growth, environment water steam, pyrometer location, calibration, resolution and repeatability. The head end transfer bar travelling time is calculated by mathematical modelling using FM estimated thread speed. This estimation is associated with the inherent modelling uncertainty.

4.2 Interval Type-2 TSK Fuzzy Logic System Design

The architecture of the interval type-2 TSK FLS was established in such away that parameters are continuously optimized. The number of rule-antecedents was fixed to two; one for the RM exit surface temperature and the other for transfer bar head travelling time. Each antecedent-input space was divided in three fuzzy sets, fixing the number of rules to nine. Gaussian primary membership functions of uncertain means were chosen for the antecedents. Each rule of the each interval type-2 TSK FLS is characterized by six antecedent membership function parameters (two for left-hand and right-hand bounds of the mean and one for standard deviation, for each of the two antecedent Gaussian membership functions) and six consequent parameters (one for left-hand and one for right-hand end points of each of the three consequent type-1 fuzzy sets), giving a total of twelve parameters per rule.

The resulting interval type-2 TSK FLS uses type-1 singleton fuzzification, join under maximum t-conorm, meet under product t-norm and product implication.

4.3 Noisy Input-Output Training Data Pairs

Noisy input-output data pairs of three different coil types from actual production schedules of an industrial HSM were collected and used as training data. The inputs were the noisy surface temperature measured at the RM exit, and the travelling time measured from the RM exit to the SB entry transfer bar. The output was the noisy transfer bar surface temperature measured at the SB entry.

4.4 Fuzzy Rule Base

The interval type-2 TSK fuzzy rule base consists of a set of IF-THEN rules that represents the model of the system. The interval type-2 TSK FLS has two inputs $x_1 \in X_1$, $x_2 \in X_2$ and one output $y \in Y$, which have a corresponding rule base size of $M = 9$ rules of the form:

$$\begin{aligned} &R^i : IF \quad x_1 is \tilde{F}_1^i \quad and \quad x_2 is \tilde{F}_2^i, THEN \\ &Y^i = C_0^i + C_1^i x_1 + C_2^i x_2 \end{aligned} \tag{6}$$

where Y^i the output of the *ith* rule, is a fuzzy type-1 set, and C_j^i with $i = 1,2,3,...,9$ and $j = 0,1,2$, are the consequent type-1 fuzzy sets.

4.5 Input Membership Function

The primary membership functions for each input of interval type-2 TSK FLS are singletons described by vertical lines at the input value $x_k = x_k^{'}$, the measured value of the input, where $k = 1,2$ (the number of singleton inputs). Not being able to compensate for uncertain measurements.

4.6 Antecedent Membership Functions

The primary membership functions for each antecedent are interval type-2 fuzzy sets described by Gaussian primary membership functions with uncertain means:

$$\mu_k^i(x_k) = \exp\left[-\frac{1}{2}\left[\frac{x_k - m_k^i}{\sigma_k^i} \right]^2 \right] \tag{7}$$

where $m_k^i \in [m_{k1}^i, m_{k2}^i]$ is the uncertain mean, with k =1,2 (the number of antecedents) and i = 1,2,..9 (the number of M rules), and σ_k^i is the standard deviation.

The means of the antecedent fuzzy sets are distributed over the entire input space. The intervals of uncertainty for the means of RM exit temperature antecedent's fuzzy sets were selected as:

$$\begin{aligned}&\lfloor m_{x_1} - 8\sigma_{x_1} - \sigma_{n_1}, m_{x_1} - 8\sigma_{x_1} + \sigma_{n_1} \rfloor\\ &\lfloor m_{x_1} - \sigma_{x_1}, m_{x_1} + \sigma_{x_1} \rfloor\\ &\lfloor m_{x_1} + 8\sigma_{x_1} - \sigma_{n_1}, m_{x_1} + 8\sigma_{x_1} + \sigma_{n_1} \rfloor\end{aligned} \quad (8)$$

and for the transfer bar head end travelling time antecedent's fuzzy sets were selected as:

$$\begin{aligned}&\lfloor m_{x_2} - 8\sigma_{x_2} - \sigma_{n_2}, m_{x_2}, m_{x_2} - 8\sigma_{x_2} + \sigma_{n_2} \rfloor\\ &\lfloor m_{x} - \sigma_{x_1}, m_{x_1} + \sigma_{x_1} \rfloor\\ &\lfloor m_{x_2} + 8\sigma_{x_2} - \sigma_{n_2}, m_{x_2} + 8\sigma_{x_2} + \sigma_{n_2} \rfloor\end{aligned} \quad (9)$$

where m_{x_1} and σ_{x_1} are the mean and standard deviation of input x_1, m_{x_2} and σ_{x_2} are the mean and standard deviation of input x_2, σ_{n_1} is the standard deviation of RM exit temperature noise, and σ_{n_2} is the standard deviation of travelling time noise.

Table 3 shows the calculated three intervals values of uncertainty for input x_1.

Table 3
Intervals of Uncertainty of Input x_1

	m_{11} ^oC	m_{12} ^oC	σ_1 ^oC
1	950	952	60
2	1016	1018	60
3	1080	1082	60

Fig. 2 shows the initial membership functions for the antecedent fuzzy sets of input x_1.

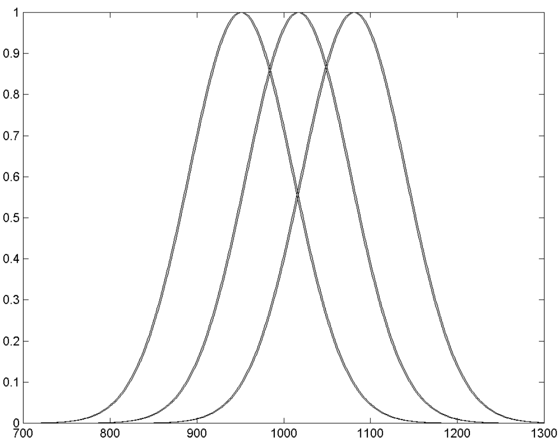

Fig. 2 Membership functions for the antecedent fuzzy sets of input x_1

Table 4 shows the values of the three intervals of uncertainty for input x_2.

Table 4
Intervals of Uncertainty of Input x_2

	m_{21} s	m_{22} s	σ_2 s
1	32	34	10
2	42	44	10
3	56	58	10

Fig. 3 shows the initial membership functions for the antecedent fuzzy sets of input x_2.

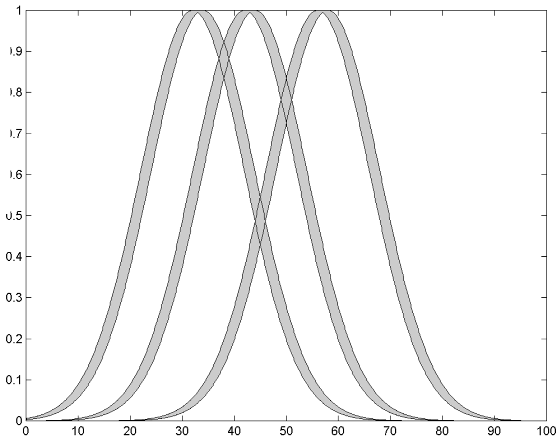

Fig. 3 Membership functions for the antecedent fuzzy sets of input x_2

In this application we have used three different material type coils with different target gage, target width and steel grade as shown in Table 5.

Table 5
Material Type Coils

	Target gage mm	Target width mm	Steel grade SAE/AISI
Coil A	1.95	1104.0	1006
Coil B	5.33	1066.0	1009
Coil C	3.04	939.0	1045

The calculated mean and standard deviation of input x_1 and input x_2 of training data are shown in Table 6.

The standard deviation of temperature noise σ_{n_1} was initially set to 1.0 ^oC and the standard deviation of time noise σ_{n_2} was set to 1.0 s.

Table 6
Calculated Mean and Standard Deviation of x_1 and x_2 inputs

	m_{x_1} ^oC	σ_{x_1} ^oC	m_{x_2} s	σ_{x_2} s
Coil A	1050.0	13.0	39.50	2.41
Coil B	1037.2	22.98	39.67	2.52
Coil C	1022.0	16.78	37.32	3.26

The standard deviations of antecedent's fuzzy sets were chosen for the M = 9 rules to be σ_1^i= 60.0 ^oC and σ_2^i= 10.0 s respectively. The means of the antecedent fuzzy sets were uniformly distributed over the entire input space.

4.7 Consequent Membership Functions

Each consequent is an interval type-1 fuzzy set with $Y^i = [y_l^i, y_r^i]$:

$$y_l^i = \sum_{j=1}^{p} c_j^i x_j + c_0^i - \sum_{j=1}^{p} |x_j| s_j^i - s_0^i \qquad (10)$$

and

$$y_r^i = \sum_{j=1}^{p} c_j^i x_j + c_0^i + \sum_{j=1}^{p} |x_j| s_j^i + s_0^i \qquad (11)$$

where c_j^i denotes de centre (mean) and s_j^i denotes de spread of the consequent centroid C_j^i, with $i = 1,2,3,...,9$ and $j = 0,1,2$. Then y_l^i and y_r^i are the consequent parameters. When only the N input-output $(x^{(1)} : y^{(1)}), (x^{(2)} : y^{(2)}), \ldots, (x^{(N)} : y^{(N)})$ data training pairs are available and there is not data information about the consequents, the initial values for the centroid parameters c_j^i and s_j^i can be chosen arbitrarily in the output space [16, 17]. In this work the initial values of c_j^i were set equal to 0.001 and the initial values of s_j^i equal to 0.0001, for $i = 1,2,3,...,9$ and $j = 0,1,2$.

5 Results

An interval type-2 TSK FLS system named in this paper as Sugeno ANFIS Type-2, was trained and used to predict the SB entry temperature, applying the RM exit measured transfer bar surface temperature and RM exit to SB entry zone travelling time as inputs. For each of the two methods, BP and RLS-BP, we ran fifteen epoch computations; one-hundred eight parameters of nine rules were tuned using eighty-seven, sixty-eight and twenty-eight input-output training data pairs per epoch, for coil type A, type B and type C respectively. The same interval type-2 FLS system was tested using two different learning methods under the same initial conditions, the same input-output training data pairs, and the same input-output checking data pairs.

The performance evaluation for the learning methods was based on the Root Mean-Squared Error (RMSE) benchmarking criteria, as in [1]:

$$RMSE_{TSK,2}(*) = \sqrt{\frac{1}{n} \sum_{k=1}^{n} \left[Y(k) - f_{TSK,2}\left(\mathbf{x}^{(k)} \right) \right]^2} \qquad (12)$$

where $Y(k)$ is the output data from the input-output checking pairs, $RMSE_{TSK,2}(*)$ stands for $RMSE_{TSK,2}(BP)$ and for $RMSE_{TSK,s2}(RLS-BP)$, obtained when applied BP and RLS-BP learning methods to an interval type-2 TSK FLS.

Fig. 4 shows RMSE of the two used interval type-2 TSK FLS systems with fifteen epochs' computations of type C coil. It can be appreciated that the hybrid RLS-BP type-2 TSK has better performance than the BP type-2 TSK. The interval type-2 TSK FLS were very sensitive to learning parameters values.

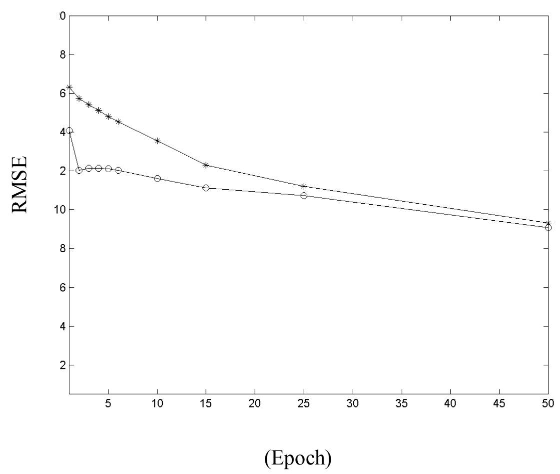

Fig. 4 IntervalType-2 TSK FLS (*) RMSE $_{TSK,2}$ (BP) (o) RMSE $_{TSK,2}$ (RLS-BP)

6 Conclusions

We have presented a new application of interval type-2 TSK FLS using the proposed RLS-BP hybrid learning method. The interval type-2 TSK FLS antecedent membership functions and consequent centroids absorbed the uncertainty introduced by the antecedent and consequent values initially selected, temperature measurements, and inaccurate travelling time estimations. BP, and RLS-BP methods were tested and the modelling results demonstrated the power of the hybrid parameters estimation. RLS-BP achieves better performance over BP method.

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