Component supervision by sensor placement on bond-graph model

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Abstract : The component supervision procedure consists of the comparison of the actual behaviour with the reference behaviour which describes the normal mode. The analytical redundancy has to find relations between known variables of the system. These relations are satisfied in the normal mode and not satisfied in presence of a failure. A (0-1) table where these relations are reported on rows and the signature fault of the variable to be monitored on columns, is said the signature failures table. This paper deals with combinatorial aspect of detectors placement on bond graph junction nodes. A new method is proposed to avoid the exploration of all the combinations using causal path properties. This is shown through an example.

Key-words : Supervision, Sensors placement, Bond graphs, Causal paths, Modelling.

1 Introduction

In the past, automated production systems have aided operators in controlling the process in order to improve the quality of the finished product, the safety and the industrial units efficiency. The main objective was to increase the productivity by establishing performing commands. Recently, another challenge has appeared, it concerns the diagnosis procedures automation using an intelligent control. The algorithm is based on the principle of: the comparison between the real behavior of the process and a reference behaviour provided by a model under normal operation. The supervision (ability to detect and to isolate faults) of the system depends mainly on the implemented instrumentation architecture[1].

Two sensor placement methodologies have been developed, depending on the kind of knowledge used to describe the process : model based (the model is given under analytical form), and the non model-based (the knowledge is given under rules, tables, pattern recognition, ...).

The innovative interest of the present paper is the use only one representation (bond graph tool) for modelling and sensor placement thanks it's behavioural, structural and causal properties. Indeed, using bond graph, the physical sensor position is explicitly displayed in the graphical model as well as it has a physical meaning. These sensor positions are represented by binary variables, if we have n junctions then we have 2n possibilities to affect virtual detectors. Use of covering causal path through the graph, the optimal sensor placement for diagnosability is systematiccally deduced with any need of calculation. After an outline of the main model based approaches, brief presentation of bond graph tool is done in the first of the second part while the last part deals with the new optimal sensor strategy presentation. The developed methodology is illustrated through a pedagogical example; finally, a conclusion remarks are left to the end.

2 Structural analysis

The advantage of the structural analysis approach is the fact that it can only keep information about constraints acting on variables. This allows to take into account system non linearities and many kind of representations: rules, tables,...etc.

The first step of the procedure consists to generate a subset of equations called analytical redundancy relations (ARRs) which express the difference between the model behaviour and the actual reference given by data provided by the sensor. These relations, whose numerical evaluation leads to residuals vanish when the behavior of the system is conform to the model, are constituted by only known variables. Different approaches have been developed to generate residuals, based on graph theory [11], bond graph theory[2]...etc.

The set of *ARRs* is represented in a binary table. The columns of this table are called failure signatures (in our case are considered only components faults which may affect C and R bond graph elements). A "1" entry in the *ith* row and the *jth* column of the table indicates that the residual r_i is sensitive to the *jth* fault.

The structure *sr* of a residual "*r*" is constituted by the variables occurring in this residual. Each *kth* variable to

be monitored is associated with its Φ_k relation; it can be defined by :

$$sr(r,\Phi_k)=\begin{cases}1 \text{ if and only if } \Phi_k \text{ occurs in ARR } r\\ 0 \qquad\qquad \text{otherwise}\end{cases} \quad (1)$$

where Φ is the set of constraints deduced from a bond graph model (see [9]):

$$\Phi = \{\Phi_i\}_{i=1}^{No} \cup \{\Phi_j\}_{j=1}^{N_1}$$

The vector of failure signature sd_i associated at the relation Φ_i is defined by :

$$sd_i=[sr(r_1,\Phi_i),\ sr(r_2,\Phi_i),...,\ sr(r_k,\Phi_i),...,\ sr(r_m,\Phi_i)]^t \quad (2)$$

where :

$$sr(r_k,\Phi_i)=\begin{cases}1 \text{ if the residual } r_k \text{ use the relation } \Phi_i\\ 0 \qquad\qquad \text{otherwise}\end{cases} \quad (3)$$

The failure which may affect the component i is detectable if and only if the vector sd_i is different of zero, and is isolable if and only if it is detectable and

$$\forall \Phi_j \in \Phi \,/\, i \neq j \ \ \exists\, r_k \in Res \,/\, sr(r_k,\Phi_i) \oplus sr(r_k,\Phi_j) = 1.$$

3 Bond-graph approach

The invention of bond graphs is attached to the need of a common language to model system involving different energetic domains. This tool was developed since 1961 at MIT, Boston, USA by Paynter ([10]). More recent references are ([3], [4], [13]), and ([14]). The model can be introduced in a graphical form and simulated using special software ([7]).

During the ten last years, a bond graph is used not only for modelling but for control and monitoring analysis as well, because of it's structural and causal properties. A bond graph model is a graph which describes power exchange in physical systems. This is done through power lines called "bond" which in turn contains two variables : effort and flow variables. The vertices of the graph are of two kinds, basic elements (I, C, R, TF, GY) and junction nodes (0, 1).

3.1 Variables Used in Bond Graph Modelling

The variables used in bond graphs are: power variables, efforts e(t), and flows f(t). The power P(t) exchanged by two plant items is the product of an effort and a flow :P(t) = e(t) × f(t) and energy variables, the momentum p(t), and the displacement q(t).

Symbol	Definition	Name
Se $\xrightarrow{e\ f}$	$e = e(t)$	Source of effort
Sf $\xrightarrow{e\ f}$	$f = f(t)$	Source of flow
$\xrightarrow{e\ f}$ R	$\Phi_R(e,f)=0$	Resistance
$\xrightarrow{e\ f}$ C	$\Phi_C(e,q)=0$ $\Phi_C(e,\int f(t)dt=0$	Capacitance
$\xrightarrow{e\ f}$ I	$\Phi_I(f,p)=0$ $\Phi I(f,\int e(t)dt=0$	Inertance
① TF :m ②	$e_1 = m.e_2$ $f_2 = m.f_1$	Transformer
① GY :r ②	$e_1 = r.f_2$ $e_2 = r.f_1$	Gyrator
② ① 0 ③	$f_1 + f_2 - f_3 = 0$ $e_1 = e_2 = e_3$	Common effort junction
② ① 1 ③	$e_1 + e_2 - e_3 = 0$ $f_1 = f_2 = f_3$	Common flow junction

Table1 The basic bond graph elements

There are nine basic bond graph elements, separated into four categories, according to their energy characteristics. These elements and definitions are summarized in figure 1.

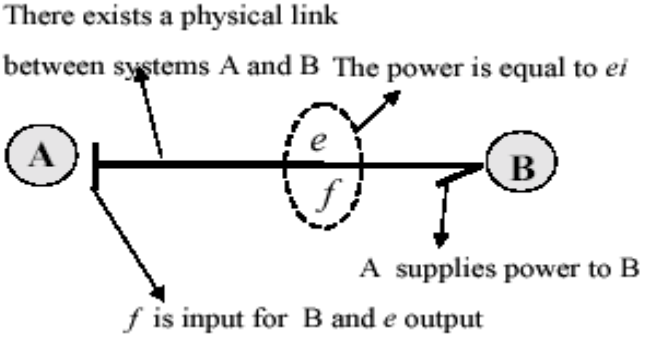

Fig. 1 Informations displayed by the bond graph representation.

As shown on the figure 1, the bond graph symbol gives us four informations: the existence of physical link between two systems by the bond, the type of power (electric, mechanical...) by the power variables, the power direction by the half arrow and the causality by the stroke.

The key of bond graph modelling is the representation (by a bond) of power with elements acting between

these variables and junction structures to put the system together. As shown in figure 2(a), the power exchanged between two process plants A and B is indicated by a bond and is the product of two variables - a potential variable (i.e. pressure, electrical potential, temperature, chemical potential, force, etc.) called effort (e) and a current variable (volume flow, current, entropy flow, velocity, molar flow, ...) referred to as flow (f).
The bond graph is an advantageous modelling tool because it exhibits both the structure and the behavior of the studied system. The main advantage regard with monitoring property is that sensor placement has a physical meaning. As opposed to the classical description using equation model based, the sensor location corresponding to a physical placement on the process is explicitly displayed on the bond graph model. Indeed, sensor placement on ”0” and ”1” junction corresponds to a physical component (tank, pump,...) where occurs the energy conservation. In bond graph, two types of sensors are used : the effort detectors *De* (pressure sensor, temperature sensor, ...) and the flow detectors *Df* (mass flow, volume flow). An effort detector *De* is placed in the ”0” junction and measures the energy (displacement) stored by the physical component associated with this junction. A flow detector *Df* is located in ”1” junction while it measures the flow through this junction.
In ([6]), the bond graph modelling is used not only in order to determine the faults causes of faults, but also in order to quantify the effects of component fault using the qualitative description of the system.

3.2 Causality

One important structural property of the bond graph is the causality concept. Indeed, the determination of causes and effects in the process plant is directly deduced from the graphical representation. To organize the component constitutive laws into sets of differential equations, we need to make a series of cause and effect decisions. The cause-effect relations for efforts and flows are represented in opposite directions. A single mark on a bond, called ”causal stroke” indicates how *e* and *f* simultaneously are determined causally on a bond. In the bond graph, it is denoted by the cross-stroke on the right indicating that the effort acts to the right, the side of the cross-stroke while the flow is in the reverse direction. As example in figure 2(a), the assigned causality means that process plant A imposes the effort on B. In the corresponding block diagram given by figure 2(b), the direction of action is indicated by an arrow on each connection as illustrated. The working convention is as follows: if the power is positive (power is flowing from A to B), then the bond graph arrow points from A to B. Independently of the causality, the direction of the positive power is indicated by the half-arrow on the bond. Four examples of causality may appear as shown in figure 2(c), and there exists a systematic procedure assigning causality to Bond-graph model [5].

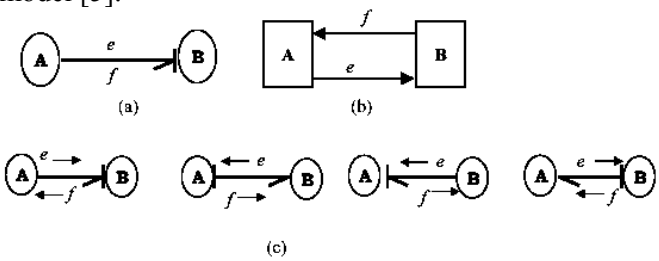

Fig. 2 Bond graph representation and causality

3.3 Causal path

A causal path is a series of links which have the same causal orientation. A close causal path is called causal loop. Depending on the causality, the followed variable is effort or flow. To change the variable, one can pass through a *GY* passive element, or return through an active element (*I, C* or *R*). For example, in figure 3, the covering causal path from control flow source *Sf* to effort sensor *De* leads to : *1 - 2 - C - 2 - De*, and covering causal path from control effort source *Se* to flow sensor *Df* leads to : *6 - 5*

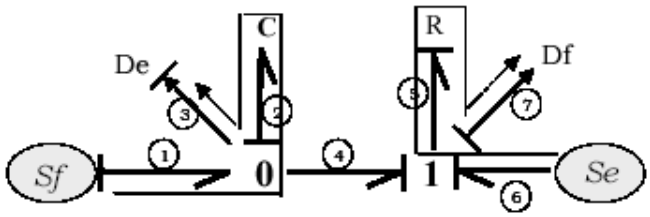

Fig. 3 Causal path example

In ([12]) is proposed a method to generate *ARRs* from bond graph model using covering causal path. The ”0” and ”1” junctions are connecting elements submitted respectively to common effort and common flow and an algebraic sum of the efforts (respectively flow) vanishes. The goal is to study all the causal paths relating the considered junction to the sources and the sensors. This method is interesting because it generates as much relations as the number of ”0” and ”1” junctions. It does not need the calculus of the bond graph determinant and the different causal loop gains (as opposed to transfer function approach).

4 Technical specification for sensor location

The goal of these sections is to provide an optimal sensor placement method on the bond graph model in

order to make all components monitorable. We assume that the faults are not multiple and may affect only components.

Let given a bond graph model obtained from physical process. We suppose that the sensors are not placed yet on the bond graph model.

Let x_i and y_j the boolean variables to express the potential sensor placement on the junction nodes such as

$$x_i = \begin{cases} 1 \text{ if the } i^{th} \text{ sensor is placed on the } i^{th} \text{ "0" junction} \\ 0 \text{ otherwise} \end{cases}$$
$$y_j = \begin{cases} 1 \text{ if the } j^{th} \text{ sensor is placed on the } j^{th} \text{ "1" junction} \\ 0 \text{ otherwise} \end{cases} \quad (4)$$

Let :

N_0 number of "0" junctions

N_1 number of "1" junctions

n_i number of bonds around the *ith*"0" junction ($i = 1,N_0$)

m_j number of bonds around the *jth*"1" junction (j = $1,N_1$)

In the following, "*f*" and "*e*" denote flow and effort vector respectively :

Equations of the *ith* "0" junction are :

$$\begin{cases} \sum_{k=1}^{n_i} a_k f_k = 0 \text{ where } a_k = \begin{cases} 1 \text{ if the half-arrow is toward junction} \\ -1 \qquad \text{otherwise} \end{cases} \\ e_k = e_{Ci} \quad \text{where} \quad k = 1, n_i - 1 \end{cases} \quad (5)$$

Equations of the jth "1" junction are :

$$\begin{cases} \sum_{l=1}^{m_j} a_l e_l = 0 \text{ where } a_l = \begin{cases} 1 \text{ if the half-arrow is toward junction} \\ -1 \qquad \text{otherwise} \end{cases} \\ f_l = f_{Rj} \quad \text{where} \quad l = 1, m_j - 1 \end{cases} \quad (6)$$

Based on causal properties of the bond graph modelling, the unknown variables can be calculated using covering causal paths (this methodology is developed in [8]). For the "0" and the "1" junction (figure 4).

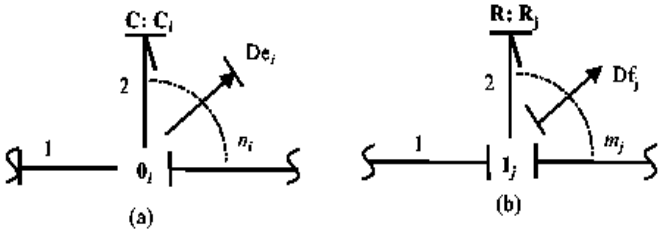

Fig. 5. Junction structure constraints

the unknown variable (based on fixed causality) is calculated as follows :

$$\begin{cases} f_{Ci} = \Phi_{Ci}[s\{(1-x_i)e_{Ci} + x_i De_i\}] \; ; \; i = 1, N_0 \\ e_{Ci} = \frac{1}{s}(1-x_i)\Phi_{Ci}^{-1}(f_{Ci}) + x_i De_i \end{cases} \quad (7)$$

Where s denotes the Laplace variable for a linear system.

$$\begin{cases} e_{Rj} = \Phi_{Rj}[(1-y_j)f_{Rj} + y_j Df_j]) \; ; \; j = 1, N_1 \\ f_{Rj} = (1-y_j)\Phi_{Rj}^{-1}(e_{Rj}) + y_j Df_j \end{cases} \quad (8)$$

Now, we will study, through an example, the combinatorial problematic of sensor placement. We have shown that to each sensor position, a binary variable (0 - 1) can be associated. The optimal solution is then a binary vector in which 010 value indicates that a sensor has to be placed.

4.1 Example

Consider an hydraulic process given in figure 5

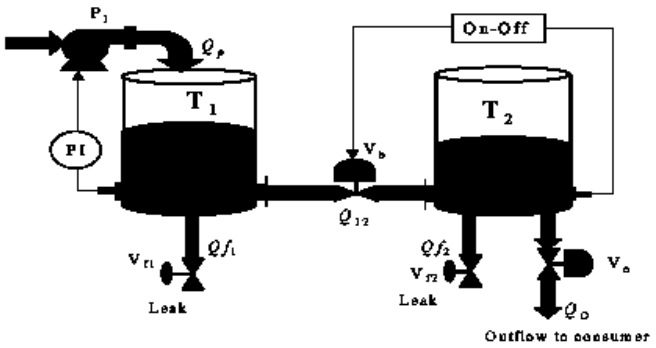

Fig. 5 Thermofluid system with two tanks

The main aim of the two tanks is to provide a continuous water flow Q_o to a consumer. The process consists of two tanks T_1 and T_2 connected by a pipe. Tank T_1 is filled by a pump P_1 up to a nominal water level of h_1 = 0.5m. The water level in this tank T_1 is controlled by a P_1 level controller acting to the inlet flow Q_p provided by the pump.

The water flow Q_{12} between the tanks T_1 and T_2 can be controlled by a valve V_b using an "ON/OFF" controller in order to keep the water level h_2 at the medium level (0.09m ≤ h_2 ≤ 0.11m) in tank T_2. The quantity of water outflow Q_o to a consumer is simulated by the valve V_o position. In our case, V_o is opened in nominal regime. The valves Vf_1 and Vf_2 can be used to simulate a leakage respectively in tank T_1 and T_2. In the faultless mode Vf_1 and Vf_2 are closed. The connection pipe between the tanks is placed at the bottom of the tanks (pipe with valve V_b).

The corresponding bond graph model is shown by figure 6.

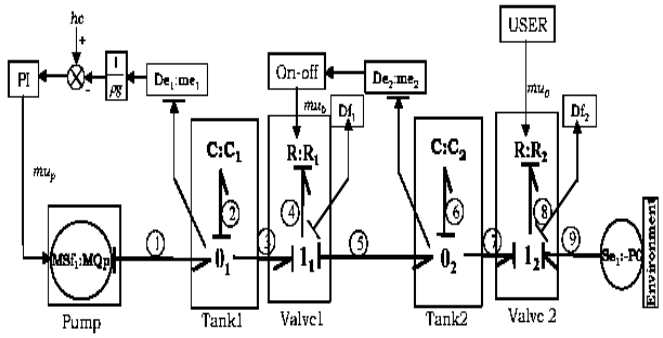

Fig. 6 Bond graph model of thermofluid system with virtual detectors

This model is composed by :
2 junctions 0: 0_1 and 0_2, 2 junctions 1: 1_1 and 1_2, 4 components: C_1, R_1, C_2, and R_2 , 2 sources : Sf_1 and Se_1
The places of the sensors are virtual.
The problematic is that we want to place a minimal number of sensors to supervise 4 components. We suppose that sensors and sources are not affected by faults. The set of known variables is K = {Sf_1, Se_1, De_1, Df_1, De_2, Df_2} and the set of unknown variables is X = {e_2, f_2, e_4, f_4, e_6, f_6, e_8, f_8}. For our example, the equations in junctions are given by
For 0_1 junction we have :

$$\begin{cases} f_1 - f_2 - f_3 = 0 \quad \text{and} \quad e_1 = e_2 = e_3 \\ f_1 = Sf \ \text{ and } \ f_{C1} = f_2 = \Phi_{C1}[s\{(1-x_1)e_2 + x_1 De_1\}] \\ e_2 = e_{C1} = \frac{1}{s}(1-x_1)\Phi_{C1}^{-1}(f_2) + x_1 De_1 \end{cases} \quad (9)$$

For 1_1 junction we have :

$$\begin{cases} e_3 - e_4 - e_5 = 0 \quad \text{and} \quad f_3 = f_4 = f_5 \\ e_{R1} = e_4 = \Phi_{R1}[(1-y_1)f_4 + y_1 Df_1] \\ f_4 = f_{R1} = (1-y_1)\Phi_{R1}^{-1}(e_4) + y_1 Df_1 \end{cases} \quad (10)$$

For 0_2 junction we have :

$$\begin{cases} f_5 - f_6 - f_7 = 0 \quad \text{and} \quad e_5 = e_6 = e_7 \\ f_{C2} = f_6 = \Phi_{C2}[s\{(1-x_2)e_6 + x_2 De_2\}] \\ e_6 = e_{C2} = \frac{1}{s}(1-x_2)\Phi_{C2}^{-1}(f_6) + x_2 De_2 \end{cases} \quad (11)$$

For 1_2 junction we have :

$$\begin{cases} e_7 - e_8 - e_9 = 0 \quad \text{and} \quad f_7 = f_8 = f_9 \\ e_{R2} = e_8 = \Phi_{R2}[(1-y_2)f_8 + y_2 Df_2] \\ f_8 = f_{R2} = (1-y_2)\Phi_{R2}^{-1}(e_8) + y_2 Df_2 \end{cases} \quad (12)$$

From equations of junctions we obtain the following system :

$$\begin{cases} Sf_1 - \Phi_{C1}[s\{(1-x_1)e_2 + x_1 De_1\}] - (1-y_1)\,\Phi_{R1}^{-1}(e_4) - y_1 Df_1 = 0 \\ \frac{1}{s}(1-x_1)\Phi_{C1}^{-1}(f_2) + x_1 De_1 - \Phi_{R1}[(1-y_1)f_4 + y_1 Df_1] \\ -\frac{1}{s}(1-x_2)\Phi_{C2}^{-1}(f_6) - x_2 De_2 = 0 \\ (1-y_1)\Phi_{R1}^{-1}(e_4) + y_1 Df_1 - \Phi_{C2}[s\{(1-x_2)e_6 + x_2 De_2\}] \\ -(1-y_2)\Phi_{R2}^{-1}(e_8) - y_2 Df_2 = 0 \\ \frac{1}{s}(1-x_2)\Phi_{C2}^{-1}(f_6) + x_2 De_2 - \Phi_{R2}[(1-y_2)f_8 + y_2 Df_2] - Se_1 = 0 \end{cases} \quad (13)$$

From the binary variables x_i (i = 1, 2,) and y_j (j = 1, 2) we can determine the final structure of the monitorable system. Two 3-sensor placement combinations provide the monitorability of the 4 components.
For $[x_1 y_1 x_2 y_2]^t = [1\ 1\ 0\ 1\ ]^t$ the residuals are given by :

$$r_1 = Sf_1 - \Phi_{C1}[sDe_1] - Df_1 \quad (14)$$

$$r_2 = De_1 - \Phi_{R1}[Df_1] - \frac{1}{s}\Phi_{C2}^{-1}(Df_1 - Df_2) \quad (15)$$

$$r_3 = \Phi_{R2}[\,Df_2] - \frac{1}{s}\Phi_{C2}^{-1}(Df_1 - Df_2) - Se_1 \quad (16)$$

The residuals structures table is given by table 2:

	Φ_{C1}	Φ_{R1}	Φ_{C2}	Φ_{R2}
r_1	1	0	0	0
r_2	0	1	1	0
r_3	0	0	1	1

Table 2 Signature faults using three sensors

The fault signatures are different from each other and not equal to zero, then the components C_1, R_1, C_2 and R_2 are monitorable.
For $[x_1 y_1 x_2 y_2]^t = [1\ 0\ 0\ 1\ 1\ ]^t$ the residuals are given by :

$$r_1 = Sf_1 - \Phi_{C1}[sDe_1] - \Phi_{R1}^{-1}(De_1 - De_2) \quad (17)$$

$$r_2 = \Phi_{C2}[sDe_2] + Df_2 - \Phi_{R1}^{-1}(De_1 - De_2) \quad (18)$$

$$r_3 = Se_1 - De_2 - \Phi_{R2}(Df_2) \quad (19)$$

The residuals structures table is given by table 3:

	Φ_{C1}	Φ_{R1}	Φ_{C2}	Φ_{R2}
r_1	1	1	0	0
r_2	0	1	1	0
r_3	0	0	0	1

Table 3 Signature faults using four sensors

The fault signatures are different from each other and not equal to zero, then the components C_1, R_1, C_2 and R_2 are monitorable. The question arises whether we are able to supervise this system by only three sensors? and what are the combinations which provide this result?
By exploring all the combinations (see table 4), we remark that only two of the 3-sensor placement combination provide the monitorability of the 4 components.
All results for 2 and 3-sensor placement combinations are summarized on the following table (table 4) :

Nber of sensors	Combinations	Monitorables	Detectables
3	1101	C_1, R_1, C_2, R_2	ϕ
3	1011	C_1, R_1, C_2, R_2	ϕ
3	1110	C_1, R_1	C_2, R_2
3	0111	C_2, R_2	C_1, R_1
2	0101	C_1, R_1	C_2 , R_2
2	1010	C_1, R_1	R_2, C_2
2	1100	C_1, R_1	ϕ
2	1001	R_2	C_1 , R_1
2	0011	R_2	R_1, C_2
2	0110	ϕ	C_1, R_1, C_2, R_2

Table 4: All 2 and 3-sensor placement results

5 Algorithm sensor placement

The innovative interest of the presented work consists in assigning detectors on junction nodes of bond graph without generating explicitly the ARRs.
The original idea consists on analyzing the placement combination, which is a binary vector, in order to obtain straightforwardly the lines of the signature table, which is in turn a set of binary vectors.
Intuitively, this method is like heuristic method which includes a set of rules applied to the combinations. Suppose in the first stage that the model does not contain causality loops.
It is useless to generate the analytical redundancy relations because the vector of sensors placement combination leads directly to the structure of residuals [Φ_{C1}, Φ_{R1}, Φ_{C2}, Φ_{R2}].
This method is as follows:
For each ’1’ in the sensor placement combination vector, we associate one residual structure by applying the following rules. Thus the number of obtained residuals is equal to the number of ’1’ in the sensor placement combination vector.

1) When we have two adjacent ’1’ in the sensor placement vector, we place ’1’ on the residual structure position and all the other positions get ’0’ .
Schematically:
[1 1 φ φ] = [1 0 0 0] where φ = 0 or 1
2) If ’1’ is adjacent to ’0’, we conserve the position of ’1’ and we replace ’0’ by ’1’:
[1 0...] = [1 1...]
[0 1...] = [1 1...]
3) If ’1’follows or is followed by two consecutive ’0’ then we conserve the’1’ place and we replace ’0’ by ’1’:
[1 0 0..] = [1 1 1..]
[0 0 1..] = [1 1 1..]
4) If we have two ’0’ followed by ’1’, the last ’0’ becomes ’1’ (the last ’1’ is conserved) :
[1 0 0 1] = [1 0 1 1]

Proposition 1: For any simple bond graph model, the rules (1) to (4) generate residuals structure with any need of analytical calculation.

Proof: These rules of generating signatures table are due to the fact that the existence of causal paths between junctions where sensor is placed and an other where it is absent. The first property is the fact that the number of residuals is equal to the number of ’1’ in the combination. Indeed, if the combination contains a ’1’, a detector is placed at this junction (rule1) disclosing the E element in the residual $\Phi_E( ) =$.
These rules indicate in fact how to get ARRs structure from the bond graph model using the causal paths properties. If a variable can not be eliminated at a junction node, then we use the causal path to go to the next junction and then the component attached to this junction will appear in the structure of the residual. Each component E_i has a possibly vector-values, parameter θ_i which determine the exact fault state of the component. For instance, the notation:
Φ_{C2} $[s\{(1 - x_2)e_6 + x_2De_2\}]$ indicates that, if x_2 equals to 1, then the detector De_2 is placed and we stop using the variable component; elsewhere, this latter variable is used in the next adjacent junction. To examine causal paths, the presence of a ’0’ adjacent to ’1’ indicates the absence of detector at the ’0’ location and impose the use of constitutive equation of element present at this junction, which impose its occurence in residual structure (rule2).
The apparition of ’1’ after ’0’ stops substitution and so eliminates the corresponding element (and their pursuers) in the residual structure (rule 3); the application of this rule is made in both senses (right and left).
However, if two ’0’ or more are consecutive, the substitution in the equations of effort and flow implies the use of the element equation attached at the second junction where detector is absent instead of using the first equation.

This reasoning is repeated if we have several consecutive '0', only the last '0'(before '1') arises the corresponding element (rule 4). In the case of causality loop (a closed causal path contain only junction nodes), it is sufficient to apply rules at extremes locations of the combination that become adjacent points.
These heuristic rules allow to deduce sensors placement strategy on simple bond-graph.

Proposition 2: The optimal sensor placement on simple junction structure is that which alternate sensors locations.

Proof: Reasoning by induction on sensors number. For n = 1, it is trivial, one sensor supervise one component. For n > 1 components: E_1, E_2,...,E_n, we would need k detectors, $k \leq n$ for monitoring the model.
From bond-graph, we have shown that we can generate one residual each time a sensor is placed at a node junction so we need k such that $2^k > n$ to have various combinations.
We can start with k = n + 1 if n is odd and k = n if n is even.
For n = 2, we have [1 0] and [0 1] shown by figure 7.
Which leads to only one residual and we cannot supervise both the components E_1 and E_2.

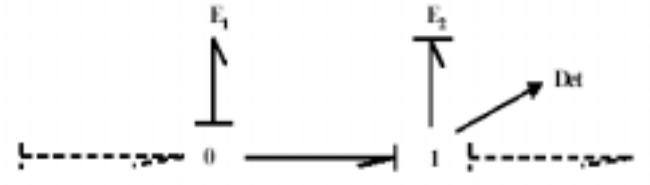

Fig. 7 Case of two different junctions representing the combination [0 1]

For n = 3, it is easy to verify that only the combination [1 0 1] allows to supervise 3 components and give two residuals by applying the second rule:
1 1 0 and 0 1 1 that are sufficient for monitoring. We suppose that the property is true for n, and we show that it remains true for n + 1.
1st case : if n is odd then the optimal combination is that with two extreme 1 and an alternation of 0 and 1.
2sd case : if n is even then the couple [1 0] or [0 1] is duplicated as many times as necessary, applying the rules 2 and 3, this leads to the following signature table :

Φ_1	Φ_2	Φ_3	...	Φ_n
1	1	0	...	0
0	1	1	10...	0
:	:	:	::::	:
0	0	0	...01	1

Table 5 Signature faults using n sensors

In this table, it is clear that all the signatures are different from each other. If we add one component at the column *n + 1*, make it monitorable, it is sufficient to add one residual where only the component relations of E_n and E_{n+1} appear, so we must add one detector after the last 0 if *n* is even and do nothing if *n* is odd.

6 Conclusion

In general, the sensor placement using model based approaches requires both model and analytical redundancy relations. In this paper, it is shown how bond graph methodology is a powerful tool for supervision analysis, especially in generating residuals structure without need to compute ARRs. From this point of view, the method is inspired from operational research technics avoiding the computational process of ARRs. The proposed heuristic rules are efficient ones allowing to get directly the residual's structure if we know the sensor placement combination. They circumvent the solution set exploration and operates transformations directly on the sensor placement combination in order to built the faults signature table. Furthermore, we have proved that the optimal placement corresponds to the alternating detector places in the bond graph junction structure. Further works are concerned with application of combinatorial research methodologies for more complex bond graph models (multi-ports models, causal loops models,...).

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