Stabilization Around Periodic Orbits for Planar Vertical Takeoff and Landing Aircrafts

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Abstract:- In the present paper, the problem of stabilization around periodic orbits for PVTOL (Planar Vertical Takeoff and Landing) aircrafts is considered. Using the reduced dynamic model for PVTOL aircrafts, a control law is obtained for tracking control of one state variable of the system, where a modified version of the Van der Pol system was employed as our source of periodic signals (reference signals to follow). The other state variables were forced to be bounded. The final controller obtained was then applied to the no-reduced dynamic model of PVTOL aircrafts. Simulation results shown the performance of our controller.

Key-Words:- Orbital Stabilization, Flight Control, Non linear Oscillators.

1 Introduction

Recently, trajectory tracking control and configuration stabilization of planar vertical takeoff and landing (PVTOL) aircrafts has been studied (see [3], and references cited in it). This system offers a challenging example for nonlinear control studies ([2]). The PVTOL dynamic model captures the essential features for convectional aircraft with fixed wing. A simplified dynamic model of this system can be obtained by assuming no coupling between rolling moment and lateral acceleration. Using this model, we construed a dynamic controller for the stabilization problem around periodic orbits. The controller obtained was then applied to the original dynamic model to study, numerically, the performance of our design. The source of periodic orbits is a modified Van der Pol oscillator which has periodic signals very close to sinusoidal signals (see [1]). The main interest of the present paper is to design a control law such that the plane follows some kind of periodic signals. So, the objective is to navigate the plane along almost periodic movements. One motivation for this could be a military tactic for programming missile evasion. The novelty of this control law is the programming of periodic movements in the aircraft.

The PVTOL model is a dynamic model that has a minimum number of states and inputs but retains the most important features of aircrafts, especially for the Harrier one ([4], page 472). Figure one shows the prototype PVTOL system, where x, y is the center of mass of the aircraft, and the angle θ is the relative angle with respect to the x-axis. The reference $x = 0$ is not necessary the sea level (see Figure 10.15 in [4]).This model is as follows ([4], page 472):

$$\begin{aligned} \ddot{x} &= -sin(\theta)u_1 + \epsilon cos(\theta)u_2 \\ \ddot{y} &= cos(\theta)u_1 + \epsilon sin(\theta)u_2 - 1 \\ \ddot{\theta} &= u_2 \end{aligned} \quad (1)$$

where $'-1'$ is the gravitational acceleration and ϵ is the small positive coefficient giving the coupling between the rolling moment and the lateral acceleration of the aircraft. The control inputs u_1 and u_2 are the thrust and the rolling moment, respectively. $\dot{x}$, $\dot{y}$, and $\dot{\theta}$ are the corresponding velocities.

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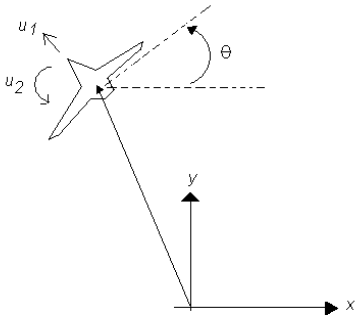

Fig. 1 PVTOL aircraft.

For the propose of our control design, the PVTOL dynamic model (1) can be simplified, by letting $\epsilon = 0$, which corresponds for the no-coupling case between rolling moment and lateral acceleration ([4]), to:

$$\begin{aligned}\ddot{x}_m &= -sin(\theta)u_1 \\ \ddot{y}_m &= cos(\theta)u_1 - 1 \\ \ddot{\theta} &= u_2.\end{aligned} \quad (2)$$

2 Problem Statement

The modified Van der Pol equation has a general representation given by the following second order scalar nonlinear differential equation:

$$\ddot{v} + \varepsilon[(v - v_0)^2 + \frac{\dot{v}^2}{\mu^2} - \rho^2]\dot{v} + \mu^2(v - v_0) = 0. \quad (3)$$

This system possesses a periodic solution (a limit cycle) that attracts every other solutions except the unique equilibrium point $(v, \dot{v}) = (v_0, 0)$. So, the parameter ρ controls the amplitude of this limit cycle, the parameter μ controls its frequency, and the parameter ε controls the speed of the limit cycle transients (see [1] for details). In this respect, v_0 can be interpreted as the DC component of the signal $v(t)$.

Phase portrait of equation (3) is shown in Figure two for the parameter values $\varepsilon = 0.1$, $\rho^2 = 10$, $v_0 = 0$, and $\mu^2 = 1$.

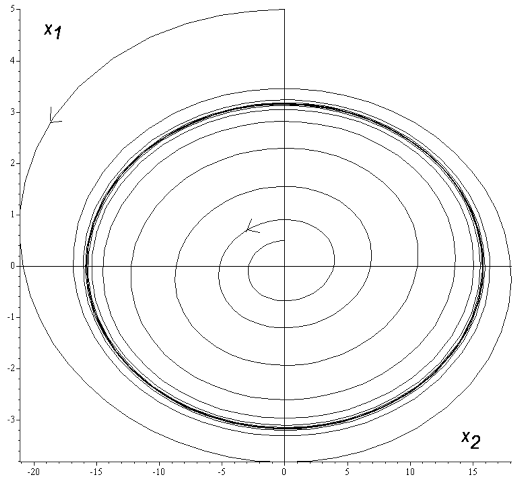

Fig. 2 Phase portrait: $x_1 = v$ versus $x_2 = \dot{v}$.

Our objective control is, given the two output equations:

$$y_1 = \theta - v, \quad (4)$$

and

$$y_2 = x_m^2 + y_m^2 - r^2, \quad (5)$$

where r is a positive constant, find u_1 and u_2 such that

$$\lim_{t\to\infty} y_1 = 0, \quad (6)$$

and

$$\lim_{t\to\infty} y_2 = 0. \quad (7)$$

The second time derivatives, with respect to time, of (4) and (5), produce:

$$\ddot{y}_1 = \ddot{\theta} - \ddot{v} = u_2 - \ddot{v} \quad (8)$$

and

$$\ddot{y}_2 = 2x_m\ddot{x}_m + 2\dot{x}_m^2 + 2y_m\ddot{y}_m^2 + 2\dot{y}_m^2. \quad (9)$$

Doing the following assignments:

$$\ddot{y}_1 = -a_1\dot{y}_1 - a_2y_1 \quad (10)$$

and

$$\ddot{y}_2 = -b_1\dot{y}_2 - b_2y_2; \quad (11)$$

from (8), (9), and (2), the following control laws are obtained:

$$u_2 = -a_1\dot{y}_1 - a_2y_1 + \ddot{v} \quad (12)$$

and

$$u_1 = \frac{-b_1\dot{y}_2 - b_2 y_2 - 2(\dot{x}_m^2 + \dot{y}_m^2 - y_m)}{-2x_m sin(\theta) + 2y_m cos(\theta)}, \quad (13)$$

where $\ddot{v}$ is obtained from (3). If the constant parameters a_1, a_2, b_1, and b_2 are positives, then systems (10) and (11) are asymptotically stables. So, these two last equations solve our control objective. Observe that the objective in equation (7) means that the plot x_m versus y_m is a circle of radius r.

3 Simulation Experiments

Simulation experiments were programmed using system (1) with $\epsilon = 0.01$; i.e., $x_m = x$ and $y_m = y$. This value of ϵ corresponds to a typical Harrier aircraft ([4], page 477). To avoid singularities in (13), the next modification was used:

$$b_x = -2x_m sin(\theta) + 2y_m cos(\theta) \quad (14)$$

$$a_x = -b_1\dot{y}_2 - b_2 y_2 - 2(\dot{x}_m^2 + \dot{y}_m^2 - y_m) \quad (15)$$

and

$$u_1 = \frac{a_x}{c_x},$$

where $c_x = 0.1$ if $|b_x| < 0.1$ and $c_x = b_x$ otherwise. System (3) was programmed with $\varepsilon = 0.1$, $\rho = 1$, $\mu = 5$, and $v_0 = 0$. The other parameters were: $b_1 = b_2 = a_1 = a_2 = 100$, and $r = 1$. Using zero initial conditions for all variables but $v(0) = 0.01$, the simulation results are shown below.

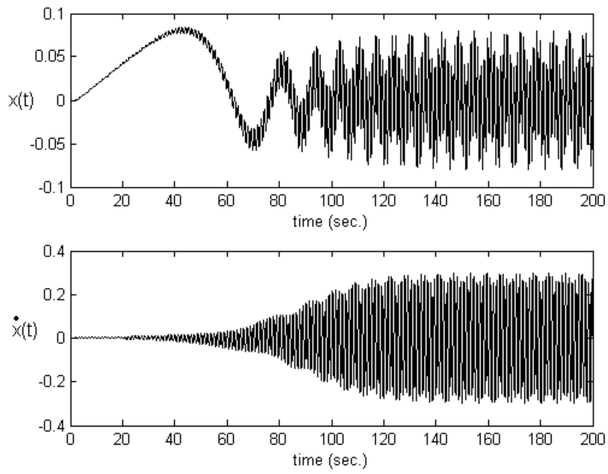

Fig. 3 Simulation results: a)Top: $x(t)$, b)Bottom: $\dot{x}(t)$.

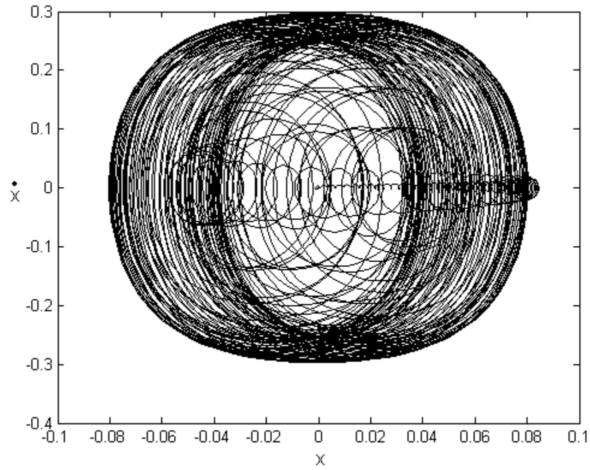

Fig. 4 Simulation results: $\dot{x}(t)$ versus $x(t)$.

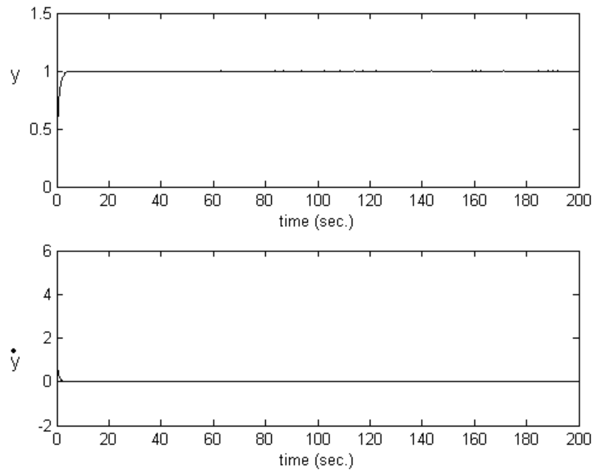

Fig. 5 Simulation results: a)Top: $y(t)$, b)Bottom: $\dot{y}(t)$.

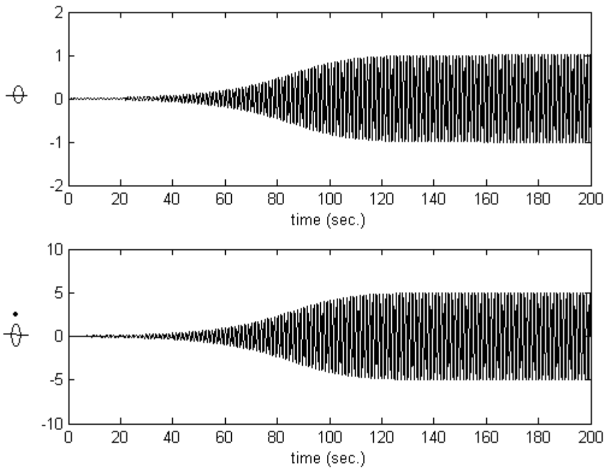

Fig. 6 Simulation results :a)Top: $\theta(t)$, b)Bottom: $\dot{\theta}(t)$.

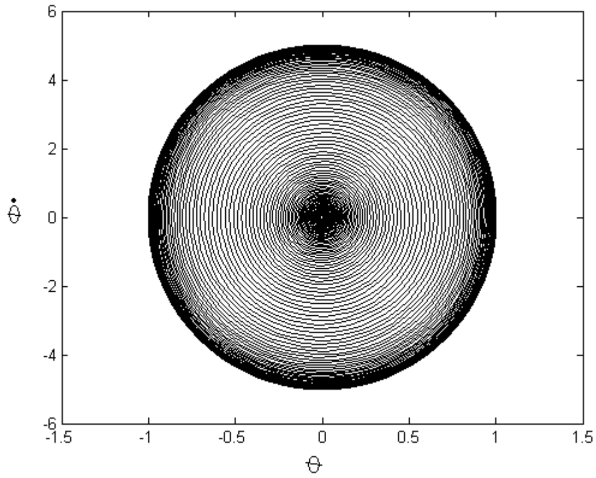

Fig. 7 Simulation results: $\theta(t)$ versus $\dot{\theta}(t)$.

From Figures three and five, the reference position y is regulated meanwhile reference x is a bounded time varying one. This implies that the aircraft is moving in the $x - axis$ with altitude y regulated to a prescribed level. Figure seven shows the convergence to a periodic orbit of the state variables θ and $\dot{\theta}$ (the movement is a spiral starting at the center of the picture).

4 Conclusions

In the present paper, a control design for tracking control of periodic signals in flight control problem is shown. The source of periodic signals is done via a modified Van der Pol oscillator. Perhaps, a motivation for this could be a military tactic for missile evasion.

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