Chattering Position-Control for Robot Manipulators with Elastic and Frictional Joints

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Abstract:- For the present paper, we developed an output feedback chattering position regulator for n-degrees-of-freedom (DOF) robot manipulators with elastic and frictional joints. A stability analysis of the closed-loop dynamic system in question was developed within the framework of Lyapunov functions. Performance issues related to the chattering regulator are illustrated in numerical simulations and experimental study applied to a 2-DOF and a 1-DOF robot manipulator respectively.

Key-Words:-Elastic joints, Coulomb friction, Robot manipulators, Chattering control.

1 Introduction

Most industrial robot manipulators have gearboxes or chains to increase the transmitted torque generated by an actuator. Although gearboxes increase the transmission ratio, the elasticity in the joints cannot be neglected. It must be pointed out that the introduction of joint flexibility in the robot modeling increases the order of the equation of motion with respect to the direct drive manipulators. Moreover, information available for feedback can be provided by position sensors placed in the actuator side only. Furthermore, friction effects are also present in manipulators; therefore, they should be considered in the control design to achieve a better performance in the desired positioning of the manipulator.

Motivated by these problems, we developed a chattering position regulator to stabilize, around a desired position, a robot manipulator with elastic joints affected also by Coulomb and viscous friction. The chattering control is composed by a PD control law (reported by Tomei [6] for frictionless systems) augmented with a position-dependent discontinuous part useful to overcome the friction effects. In the analysis is assumed that joint positions are only available for feedback. The velocity will be estimated through a stable first order filter [2]. A stability analysis of the closed-loop dynamic system in question was developed within the framework of Lyapunov functions. The equation of motion along with the proposed controller does not generate sliding motions anywhere except the origin.

This paper is organized as follows: Section 2 introduces the dynamic model of the manipulator with frictional and elastic joints. Section 3 defines the objective control and introduces the chattering position regulator along with its stability analysis. Section 4 provides a simulation study for a 2-DOF robot manipulator with friction using the controller described in Section 3. To complement the study, experimental results made for a 1-DOF manipulator are presented in section 5. Finally, Section 6 establishes conclusions.

The following definition will be used throughout the paper. The norm $\|x\|_2$, with $x \in \mathbb{R}^n$, denotes the Euclidean norm and $\|x\|_1 = |x_1| + \ldots + |x_n|$ stands for the sum norm. The minimum and maximum eigenvalue of a matrix $A \in \mathbb{R}^{n\times n}$ is denoted by $\lambda_{\min}\{A\}$ and $\lambda_{\max}\{A\}$ respectively. The vector $\mathrm{sgn}(x)$ is given by $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \ldots, \mathrm{sgn}(x_n)]^T$ where the signum function is defined as

$$\mathrm{sgn}(y) = \begin{cases} 1 & \text{if} \quad y > 0, \\ (-1,1) & \text{if} \quad y = 0, \\ -1 & \text{if} \quad y < 0, \end{cases} \qquad \forall\, y \in \mathbb{R}. \quad (1)$$

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2 Dynamic Model

The model that describes the dynamics of the n-degrees-of-freedom manipulator with elastic joints is given by [6]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + K(q - q_a) + F(\dot{q}) = 0 \tag{2}$$

$$J\ddot{q}_a - K(q - q_a) = \tau \tag{3}$$

where q is the $n \times 1$ vector of joint displacements of the manipulator, q_a is the $n \times 1$ vector of actuator angular positions, τ is the $n \times 1$ vector of applied joint torques, $K > 0$ represents the joint stiffness, $M(q)$ is the positive definite inertia matrix, $C(q,\dot{q})\dot{q}$ is the vector of centripetal and Coriolis forces, $g(q)$ is the vector of gravitational torques, $J > 0$ is the actuator inertia matrix and $F(\dot{q})$ is the vector of Coulomb and viscous friction forces governed by

$$F(\dot{q}) = F_C \text{sgn}(\dot{q}) + F_v \dot{q} \tag{4}$$

where F_C and F_v are $n \times n$ symmetric positive definite matrices representing the Coulomb and viscous friction coefficients respectively. Throughout, the precise meaning of solutions of the differential equation (2)-(3) with discontinuous functions $F(\dot{q})$ and τ is defined in the Filippov sense [4] as for the solutions of a certain differential inclusion with a multi-valued right-hand side.

The dynamic equation (2) has the following property that will be used in the closed-loop stability analysis:

Property 1 ([6]) *The gravitational torque vector $g(q)$ is Lipschitz, that is, there exists a positive constant k_g such that,*

$$\|g(x) - g(y)\| \leq k_g \|x - y\|$$

holds for all $x, y \in \mathbb{R}^n$. Moreover, k_g satisfies

$$k_g \geq \left\| \frac{\partial g(q)}{\partial q} \right\| \geq \lambda_{\max} \left\{ \frac{\partial g(q)}{\partial q} \right\}.$$

Property 2 ([5]) *The matrix $C(q,\dot{q})$ is chosen such that the relation*

$$\dot{q}^T \left[\dot{M}(q) - 2C(q,\dot{q}) \right] \dot{q} = 0,$$

holds for all q, $\dot{q} \in \mathbb{R}^n$.

3 Chattering Control Design

The objective control is defined as follows: Given the desired constant position $q_d = cte$ for all $t > 0$, the control problem is to design a chattering control law τ such that the robot joint positions $q(t)$ approach to the desired position $q_d \in \mathbb{R}^n$ asymptotically, that is,

$$\lim_{t \to \infty} \|q(t) - q_d\| = 0. \tag{5}$$

The following control law is proposed to achieve the objective control (5):

$$\tau = g(q_d) - k_P e_a - k_D \dot{z} - k_s \text{sgn}(e_a), \tag{6}$$

$$\dot{z} = -Lz + k_D e_a \tag{7}$$

where $e_a = q_a - q_{a_d}$ and $e = q - q_d$ are the $n \times 1$ vector of the actuator and manipulator position errors respectively; k_P, k_D, L and $k_s = \text{diag}\{k_{s_i}\}$ are $n \times n$ symmetric positive definite matrices; and q_{a_d} is the actuator's desired position defined by [6]:

$$q_{a_d} = q_d + K^{-1} g(q_d). \tag{8}$$

The equation (7) is a stable first order filter used to estimate the velocity information. The controller consists of a gravitational pre-compensation part, a Proportional-Derivative (PD) part and a switching part designed to stabilize a manipulator with elastic and frictional joints asymptotically around a constant desired position. As can be noted, the proposed chattering regulator (6)-(7) does not need an exact knowledge of friction coefficients (with the only imposed condition: $k_s > F_C$), making it an attractive feature from the physical point of view.

The state space representation of the closed-loop system (2)-(4), (6) in terms of the error $(e, \dot{q}, e_a, \dot{q}_a)^T$ is given by:

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{q} \\ e_a \\ \dot{q}_a \\ z \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M^{-1}(q)[-K(e - e_a) - F(\dot{q})] \\ \dot{q}_a \\ J^{-1}[K(e - e_a) - k_P e_a - k_D \dot{z}] \\ -Lz + k_D e_a \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}(q)[-C(q,\dot{q})\dot{q} + g(q_d) - g(q)] \\ 0 \\ -J^{-1} k_s \text{sgn}(e_a) \\ 0 \end{bmatrix} \tag{9}$$

where $(x^*, \dot{x}^*, z^*) = 0$ $(x = (e, e_a))$ is the unique equilibrium point of (9) if $k_s > F_C$ and $\lambda_{\min}\{P\} \geq$

k_g, where

$$P = \begin{bmatrix} K & -K \\ -K & K + k_P \end{bmatrix}$$

and k_g is defined in Property 1. The main result is summarized in theorem 1.

Theorem 1 *Let the mechanical manipulator (2)-(3) be driven by the control law (6)-(7). Then, the equilibrium point* $(x^*, \dot{x}^*, z^*) = 0$ *of the closed-loop system is asymptotically stable.*

Proof. In order to conclude asymptotical stability of the equilibrium point, let us introduce the following Lyapunov function for the closed-loop system (9):

$$\begin{aligned} V =& \frac{1}{2}x^T P x + \frac{1}{2}\dot{q}^T M(e + q_d)\dot{q} + \frac{1}{2}\dot{q}_a^T J \dot{q}_a \\ &+ U(e + q_d) - U(q_d) - g(q_d)^T e \\ &+ (k_D e_a - Lz)^T (k_D e_a - Lz) + \sum_{i=1}^{n} k_{s_i} |e_{a_i}|, \end{aligned}$$

where $U(\cdot)$ denotes the potential energy.

The time derivative of V along the solution of (9) yields

$$\dot{V}(x, \dot{x}, z) =$$

$$\begin{aligned} =& e^T K \dot{q} - \dot{q}^T K e_a - e^T K \dot{q}_a + e_a^T (K + k_P)\dot{q}_a \\ &+ \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{q}_a^T J \ddot{q}_a + \dot{U}(e + q_d) \\ &- g(q_d)^T \dot{q} + (k_D e_a - Lz)^T (k_D \dot{q}_a - L\dot{z}) \\ &+ \dot{q}_a^T k_s \mathrm{sgn}(e_a) \\ =& e^T K \dot{q} - \dot{q}^T K e_a - e^T K \dot{q}_a + e_a^T (K + k_P)\dot{q}_a \\ &+ \dot{q}^T \left[-K(e - e_a) - F_C \mathrm{sgn}(\dot{q}) - F_v \dot{q} - C(q, \dot{q})\dot{q}\right] \\ &+ \dot{q}^T g(q_d) - \dot{q}^T g(q) + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{U}(e + q_d) \\ &+ \dot{q}_a^T \left[K(e - e_a) - k_P e_a - k_D \dot{z} - k_s \mathrm{sgn}(e_a)\right] \\ &- g(q_d)^T \dot{q} + (k_D e_a - Lz)^T (k_D \dot{q}_a - L\dot{z}) \\ &+ \dot{q}_a^T k_s \mathrm{sgn}(e_a). \end{aligned}$$

Taking into account Property 2 and by virtue of

$$\dot{U}(q) = \frac{\partial U(q)}{\partial q}^T \dot{q} = g(q)^T \dot{q},$$

one obtains

$$\begin{aligned} \dot{V} =& -\dot{q}^T F_C \mathrm{sgn}(\dot{q}) - \dot{q}^T F_v \dot{q} \\ &- (k_D e_a - Lz)^T L (k_D e_a - Lz) \\ \leq& -\lambda_{\min}\{F_C\}\|\dot{q}\|_1 - \lambda_{\min}\{F_v\}\|\dot{q}\|_2^2 \\ &- (k_D e_a - Lz)^T L (k_D e_a - Lz) \leq 0, \end{aligned}$$

which is negative semidefinite. Note that the closed-loop system is autonomous. Therefore it is possible to conclude asymptotical stability of the equilibrium point by invoking the extended invariance principle [1]. Let us introduce the set S defined by,

$$\begin{aligned} S &= \{(x, \dot{x}, z) \in \mathbb{R}^{5n} | \dot{V}(x, \dot{x}, z) = 0\} \\ &= \{(x, 0, z) \in \mathbb{R}^{5n} | \dot{V}(x, \dot{x}, z) = 0\}. \end{aligned} \tag{10}$$

To obtain the largest invariant set that belongs to S, note that

$$\dot{x}(t) \equiv 0 \Rightarrow \ddot{x}(t) \equiv 0.$$

Thus, from the closed loop systems (9), we get

$$\begin{aligned} 0 &= -K(e - e_a) - F_C \mathrm{sgn}(0) + g(q_d) - g(e + q_d) \\ 0 &= Ke - (K + k_P)e_a - k_s \mathrm{sgn}(e_a) \end{aligned} \tag{11}$$

where $(e, e_a) = (0, 0)$ is a solution of the set of equations (11). To guarantee that $e = 0$ is a unique solution of (11), it is necessary to satisfy $\lambda_{\min}\{P\} > k_g$. Furthermore, adding the two equations in (11) with $e = 0$ we have

$$e_a = -k_P^{-1}\left[k_s \mathrm{sgn}(e_a) + F_C \mathrm{sgn}(0)\right] \tag{12}$$

where $e_a = 0$ is a unique solution of (12) if and only if $k_s > F_C$ ([1]). Consequently $z = 0$. In conclusion, the origin $(x, \dot{x}, z) = 0 \in \mathbb{R}^{5n}$ is the largest invariant set in S. Thus we can conclude asymptotical stability of the origin. ■

4 Simulation Results

The performance of the controllers was studied by simulations. In the simulations, a two-links manipulator was required to move from the origin $q_1(0) = q_2(0) = 0$ to the desired position $q_{d_1} = q_{d_2} = \pi$ rad. The motion of the 2-DOF manipulator is governed by (2) where

$$M(q) = \begin{bmatrix} 8.77 + 1.02\cos q_2 & 0.76 + 0.51\cos q_2 \\ 0.76 + 0.51\cos q_2 & 0.62 \end{bmatrix},$$

$$C(\cdot) = \begin{bmatrix} -0.51\sin(q_2)\dot{q}_2 & -0.51\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ 0.51\sin(q_2)\dot{q}_1 & 0 \end{bmatrix},$$

$$g(q) = 9.8 \left[\begin{array}{c} 7.6 \sin q_1 + 0.63 \sin(q_1 + q_2) \\ 0.63 \sin(q_1 + q_2) \end{array} \right],$$

and

$$F(\dot{q}) = \left[\begin{array}{c} 5\mathrm{sgn}(\dot{q}_1) + 1\dot{q}_1 \\ 5\mathrm{sgn}(\dot{q}_2) + 1\dot{q}_2 \end{array} \right]$$

were taken from [2, 3]. The regulator and compensator gains were selected as follows:

$$k_P = \left[\begin{array}{cc} 100 & 0 \\ 0 & 60 \end{array} \right], \qquad k_D = \left[\begin{array}{cc} 20 & 0 \\ 0 & 20 \end{array} \right],$$

$$k_s = \left[\begin{array}{cc} 20 & 0 \\ 0 & 15 \end{array} \right], \qquad L = \left[\begin{array}{cc} 25 & 0 \\ 0 & 20 \end{array} \right].$$

The initial velocities $\dot{q}(0)$, $\dot{q}_a(0)$ were set to zero in all the simulations.

The resulting joint positions of the closed-loop system (9) are depicted in Figure 1. These figures demonstrate that the chattering controller asymptotically stabilizes the manipulator around the desired position, thus satisfying the objective control (5). Figure 2 shows that the actuator angular position errors converge to zero. Figure 3 presents the input torque, which illustrates the chattering phenomena.

For the sake of comparison, we simulated the regulator with no switching part, as was proposed in [6], to drive the manipulator to the desired position. Figure 4 shows that the regulator drives the manipulator to a wrong position.

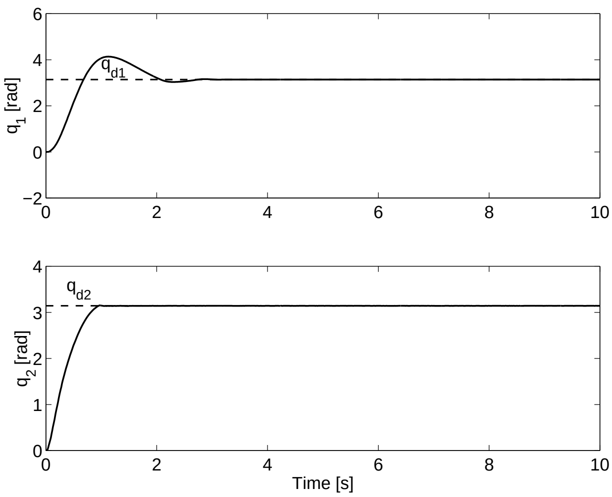

Fig. 1: Joint positions of the manipulator.

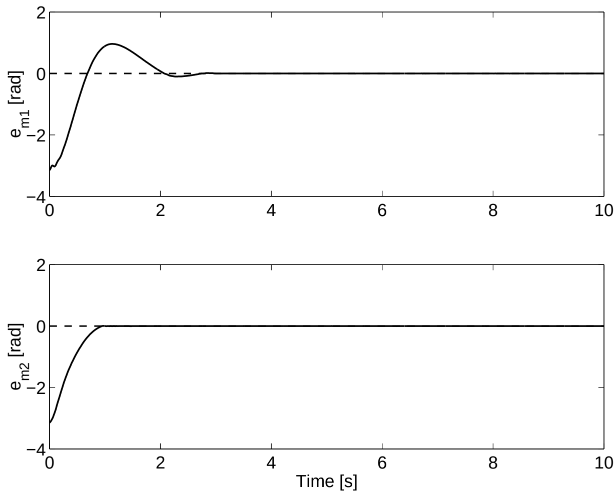

Fig. 2: Actuator position errors.

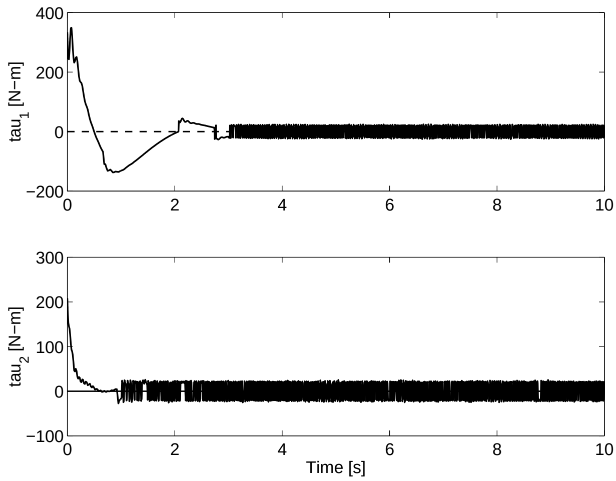

Fig. 3: Input torque.

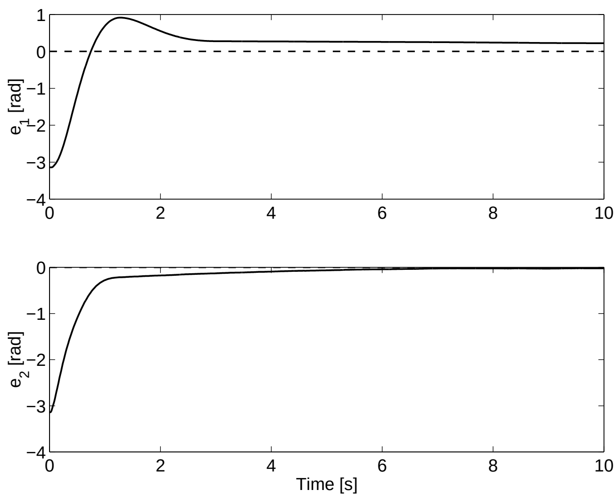

Fig. 4: Joint position errors for regulator with no switching part ($k_s = 0$).

5 Experimental study

5.1 Experimental test bench

Experimental setup, installed in the Robotics & Control Laboratory of CITEDI-IPN, involves a DC motor linked to a 1-DOF arm through a gear train. Figure 5 shows a schematic diagram of the test bench. The gear reduction ratio is of 65.5:1 and it is the main source of friction. The maximum allowable torque is 1.24 N-m. The ISA Bus servo I/O card from the company *Servo To Go* allows one to control the servomotor in real time. Resolution of the encoder is 2000 ppr. A high resolution potentiometer has been placed in the load side to support the results. A linear power amplifier is installed in the servomotor accepting control signals from the D/A converter in the range of ± 10 volts.

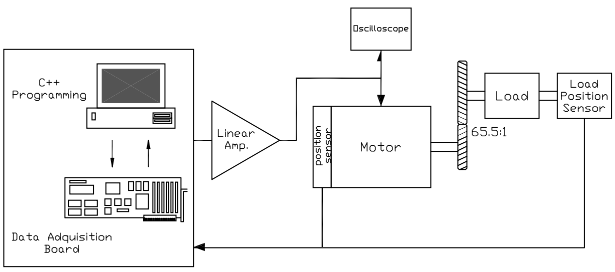

Fig. 5: Experimental test bench.

5.2 Experimental results

The experiment was carried out for the closed loop system (9) with a position sensor located at the motor side thus considering the angular motor position as the only information available for feedback. In the experiment, the load was required to move from the initial static position $q(0) = 0$ to the desired position $q_d = \pi$ rad.

The chattering controller (6) is specified with

$$g(q_d) = 9.8ml\sin(q_d)$$

where $l = 0.15$ m, $m = 0.25$ Kg; $K = 250$ N-m/rad and

$$k_P = 100, \quad k_D = 10, \quad k_s = 10, \quad L = 40.$$

The velocity and compensator were set to zero in the experiment ($\dot{q}(0) = \dot{q}_a(0) = \dot{z}(0) = 0$).

Figure 6 illustrates the joint position of the link and position error of the actuator. Trajectories converge to the equilibrium point as it was shown in the theory. The input torque in Figure 7 shows the chattering effects. It can be seen that the chattering appears when the trajectory converges to the equilibrium point.

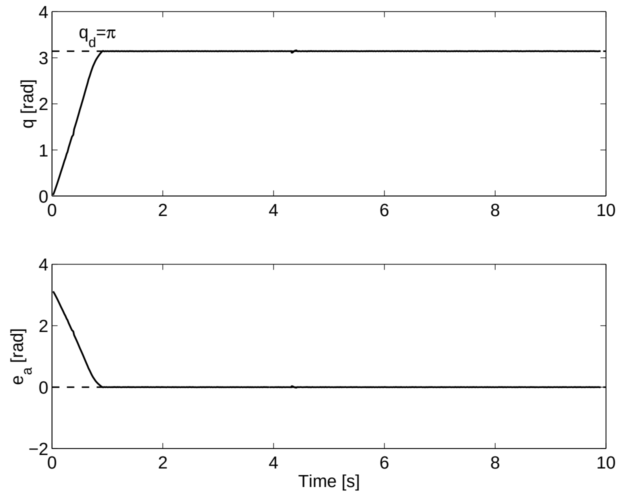

Fig. 6: Joint position of the manipulator and actuator angular position error.

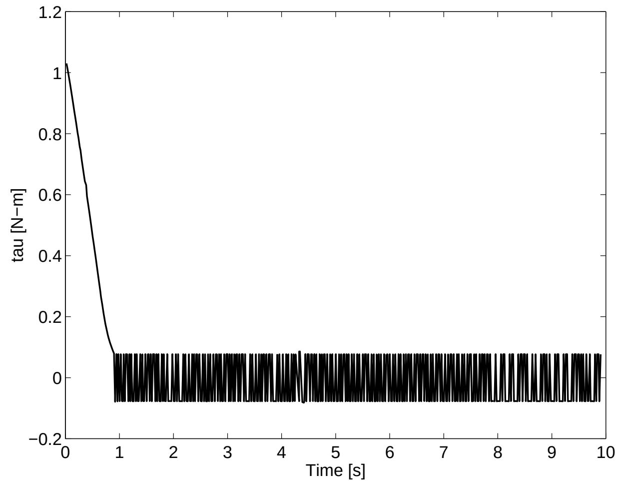

Fig. 7: Input torque.

6 Conclusions

In this paper, we applied an output feedback chattering controller to stabilize a manipulator around a desired position, assuming the presence of friction and elasticity in the joints. The proposed controller does not need an exact knowledge of the Coulomb friction level, making it attractive in engineering applications. Although the controller appears to have an infinite number of switches on a finite time interval, it does not rely on the generation of sliding motion. The invariance principle-based approach for discontinuous systems [1] was used to conclude asymptotical stability. Performance issues of the chattering controller are illustrated in a simulation and experimental study made in a 2-DOF and a 1-DOF manipulator respectively.

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