# STRUCTURAL EQUATION MODELS: GOODNESS-OF-FIT QUALITY AND ALTERNATIVE MODELS

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Abstract:- The present work has been carried out to summarize some of the last tendencies of the analysis of structural equations models (SEM). The application of the software LISREL has helped in the solution of this models, however, the great quantity of goodness-of-fit measures which do not have a statistical test has caused that the use of the alternative models and the comparison of goodness-of-fit measures among them, could be the best solution to know the effectiveness of the proposed model.

*Key Words*:- Structural Equation Models, SEM, goodness-of-fit tests, alternative models.

# 1. Introduction

 ${
m The\ structural\ equation\ models\ are\ sets\ of}$ linear equations used to specify phenomena in terms of their presumed cause-andeffect variables. In their most general form, the models allow for variables that cannot be measure directly. Also, it provides the most appropriate and efficient estimate for series of estimates of simultaneous equations by means of multiple regressions. It is characterized by two basic components: the structural pattern and the measure pattern. The structural model is the guide model that connects independent variables with dependent variables. In this case, the theory will allow to distinguish which independent variables would predict each

dependent variable. The measure model, on the other hand, allows to use several variables or indicators that cannot be measured directly (the dependent variable can be a concept represented by an additive scale), and it is possible to evaluate the contribution of each scale item, as well as to incorporate the reliability of each one, in the estimate of the dependent and independent variables [1]. The software for the selection, fitting and evaluation of structural equation models was developed by Jreskog and Srbom and at the moment it is broadly, well-known and diffused as system LIS-REL (Line Structural Relationships) [2]. These models are particularly helpful in decisive problems of firm profitability, management, the organizational behavior, and in other branches of sciences, as biology, health and even genetics. Also they have been particularly useful in the social and behavioral sciences to study the relationship between social status and achievement, the efficacy of social action programs and other interesting mechanisms [3], [4] and [5].

The reasons of their wide use are due to that they provides a direct method to analyze simultaneously multiple relationships with a great statistical effectiveness and to their capacity to evaluate the relationships in an exhaustive way, providing a transition from the exploratory analysis to the confirmatory analysis; that is to say, it allows to focus a problem with a more systematic and holistic perspective. However, the decision of which one is the best model to describe an specific situation still remains something to solve. Alternative or rival models is a choice to compare the results among them and select the best.

#### 2. The LISREL Model

The overall LISREL model might be expressed in function of eight matrices; two that defines the structural equations, two that defines the correspondence of indicators and constructos, one for the correlation of exogenous constructs, one for the correlation of endogenous constructs and two that detail the correlated errors in the endogenous and exogenous variables measurement. Such matrices are used to form the basic equations of both models.

Using the notation of [6], the LISREL model is given by the following equations: Structural model:

$$\eta_{(m \times 1)} = B_{(m \times m)} \eta_{(m \times 1)} + \Gamma_{(n \times n)} \xi_{(n \times 1)} + \zeta_{(m \times 1)}$$
(1)

Measure models:

$$\mathbf{Y}_{(p\times 1)} = \Lambda_{y_{(p\times m)}} \eta_{(m\times 1)} + \epsilon_{(p\times 1)} \\
\mathbf{X}_{(q\times 1)} = \Lambda_{x_{(q\times m)}} \xi_{(q\times 1)} + \delta_{(q\times 1)}$$
(2)

where  $\mathbf{Y}_{(p \times 1)}$  and  $\mathbf{X}_{(q \times 1)}$  denotes the endogenus and exogenus constructs respectively, with

$$\begin{aligned} \mathbf{E}(\zeta) &= \mathbf{0}, \quad \mathbf{Cov}(\zeta) = \Psi \\ \mathbf{E}(\epsilon) &= \mathbf{0}, \quad \mathbf{Cov}(\epsilon) = \Theta_{\epsilon} \\ \mathbf{E}(\delta) &= \mathbf{0}, \quad \mathbf{Cov}(\delta) = \Theta_{\delta} \end{aligned} \right\}, \quad (3) \end{aligned}$$

 $\zeta$ ,  $\epsilon$  and  $\delta$  are mutually uncorrelated;  $\mathbf{Cov}(\xi) = \Phi$ ;  $\zeta$  is uncorrelated with  $\xi$ ;  $\epsilon$  is uncorrelated with  $\eta$ ;  $\delta$  is uncorrelated with  $\xi$ ; B has zeros in the diagonal; and the matrix (I - B) is non singular. Besides the previous suppositions, we consider that:  $\mathbf{E}(\xi) = 0$  and  $\mathbf{E}(\eta) = 0$ .

The quantities and in (1) are the cause-effect variables, respectively; and generally, they are not directly observed. They are sometimes called latent variables. The quantities  $\mathbf{Y}$  and  $\mathbf{X}$  in (2) are such that they are linearly related to  $\eta$  and  $\xi$  through the coefficient matrices  $\mathbf{Y}$  and  $\mathbf{X}$ , and  $\Lambda_{\mathbf{X}}$  these variables can be measured. Their observed values constitute the data. Equations (2) are sometimes called the measurement equations. Path diagrams are useful aids for formulating structural models, since they indicate the direction and nature of the casualty, forcing the investigator to think about the problem.

#### 3. Covariance Structure

As  $\xi$  and  $\eta$  are not observed, the LIS-REL model cannot be verify directly. However, like in factor analysis, the model and assumptions imply a certain covariance structure, which can be checked. Define the data vector:  $[\mathbf{Y}', \mathbf{X}']'$ , then:

$$\mathbf{Cov} = \Sigma_{(p+q)(p+q)} = (\mathbf{Y}/\mathbf{X}) = \begin{pmatrix} \sum_{11_{p\times p}} \vdots \sum_{12_{p\times q}} \\ \dots & \vdots & \dots \\ \sum_{21_{q\times p}} \vdots & \sum_{22_{q\times q}} \end{pmatrix} = \begin{pmatrix} \mathbf{Cov}(Y) & \vdots & \mathbf{Cov}(Y, X) \\ \dots & \vdots & \dots \\ \mathbf{Cov}(X, Y) & \vdots & \mathbf{Cov}(X) \end{pmatrix}$$
(4)

where, with  $\mathbf{B} = \mathbf{0}$  to simplify the discussion:

$$\begin{array}{lll} \mathbf{Cov}(\mathbf{Y}) &=& \mathbf{E}(\mathbf{Y}\mathbf{Y}') \\ &=& \mathbf{\Lambda}_{\mathbf{y}} \, \mathbf{Cov}(\eta) \, \mathbf{\Lambda}_{\mathbf{y}}' + \mathbf{\Theta}_{\epsilon} \\ &=& \mathbf{\Lambda}_{\mathbf{y}} \, (\mathbf{\Gamma} \, \mathbf{\Phi} \mathbf{\Gamma}' + \mathbf{\Psi}) \mathbf{\Lambda}_{\mathbf{y}}' + \mathbf{\Theta}_{\epsilon} \end{array}$$

$$\begin{array}{lll} \mathbf{Cov}(\mathbf{X}) &=& \mathbf{E}(\mathbf{X}\mathbf{X}') \\ &=& \mathbf{\Lambda}_{\mathbf{x}} \, \mathbf{Cov}(\xi) \, \mathbf{\Lambda}_{\mathbf{x}}' + \mathbf{\Theta}_{\delta} \\ &=& \mathbf{\Lambda}_{\mathbf{x}} \, \mathbf{\Phi} \, \mathbf{\Lambda}_{\mathbf{x}}' + \mathbf{\Theta}_{\delta} \end{array}$$

$$\begin{aligned} \mathbf{Cov}(\mathbf{Y}, \mathbf{X}) &= \mathbf{E}(\mathbf{Y}\mathbf{X}') \\ &= \mathbf{E}\left(\mathbf{\Lambda}_{\mathbf{y}}\left(\mathbf{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}\right) + \boldsymbol{\epsilon}\right) \\ &\quad \left(\mathbf{\Lambda}_{\mathbf{x}}\,\boldsymbol{\xi} + \boldsymbol{\delta}\right)' \\ &= \mathbf{\Lambda}_{\mathbf{y}}\mathbf{\Gamma}\mathbf{\Phi}\mathbf{\Lambda}_{\mathbf{x}}' \\ &= [\mathbf{Cov}(\mathbf{X}, \mathbf{Y})] \end{aligned}$$

The covariances are non-linear functions

of the model parameters  $\Lambda_x, \Lambda_y, \Gamma, \Phi, \Psi, \Theta_\epsilon$ and  $\Theta_\delta$ .

Given "*n*" multivariate observations  $[y'_j, x'_j]', j = 1, 2, ..., n$ , the sample covariance matrix

$$S_{(p+q)(p+q)} = \begin{pmatrix} S_{11_{p\times p}} & \vdots & S_{12_{p\times q}} \\ \dots & \vdots & \dots \\ S_{21_{q\times p}} & \vdots & S_{22_{q\times q}} \end{pmatrix}$$

can be constructed and partitioned conformable to  $\Sigma$ . The information in S is used to estimate the model parameters. Specifically, we set:

$$\hat{\Sigma} = S,\tag{5}$$

and solve the resultings equations.

#### 4. Estimate

Unfortunately, the equations in (5) often cannot be solved explicitly. An iterative search routine that begins with initial parameter estimates must be used to produce a matrix which closely approximates S. The search routines uses a criterion function that measures the discrepancy between and S. The LISREL program currently uses a least squares and a maximum likelihood criterion to estimate the model parameters. [6] In general, to estimate the model parameters we need more equations than unknowns. Consequently, if "t" is the total number of unknown parameters, "p" and "q" must be such that:

$$t \le (p+q)(p+q+1)/2,$$
 (6)

Condition (6) however, does not guarantee that all parameters can be estimated uniquely. The final model fit must be assessed carefully. Individual parameters esti mates along with the entries in the residual matrix  $S - \hat{\Sigma}$  should be examined. Parameter estimates should have appropriate signs and magnitudes. For example, iterative parameter estimation routines operate over the entire parameter space and may not yield variance estimates that are positive. Entries in the residual matrix should be uniformly small.

#### 5. Model-Fitting Strategy

In linear structural equation models, interest is often centered on the values of parameters and the associated "effects". Predicted values for the variables are not easily obtained unless the model reduces to a variant of the multivariate linear regression model. A useful model-fitting strategy consists of the following [7]:

- 1. If possible, generate parameter estimates using several criteria (for example, least square, maximum likelihood) and compare them keeping in mind if the signs and magnitudes are consistent, if the variances estimates are positive and if the residual matrices  $S - \hat{\Sigma}$  are similar.
- 2. To carry out the analyses as much with S as with R, the sample correlation matrix to know the effect of standardization the observable variables on the outcome.
- 3. Split large data sets in half and perform Steps 1 and 2 on each half to compare the solutions with each other and with the result for the complete data set to check solution stability.

#### 6. Goodness-of-fit

The evaluation of the overall goodness-offit for the structural equation models is not as direct as in other multivariate techniques. SEM does not have a statistical test that describes the best strength of the predictions of the model. In their place, many goodness-of-fit measures have been developed to evaluate the results since three perspectives: overall fitting, comparative fitting regarding a base model and the parsimony of the model. Nevertheless, except the statistical Chi-square, the rest do not have an associated statistical contrast [8]. However, the prevalent thought maintains that the strongest test in any proposed model is gotten by means of the comparison of the model with others. This makes that the formalized comparison process among the alternative or rival models would be considered as the strictest test in the theory.

# 7. Alternative or rivals models

The best approach for the models evaluation is to compare the proposed model with several rival models that act as alternative explanations of the proposed model. The comparison of the goodness-of-fit measures of these models will allow us to determine if the proposed model is acceptable or not. This comparison is still more important when the statistical Chi-square indicates that significant differences do not exist in the overall fitting of the model. [4] As a matter to analyze the comparison, we will use an example offered for [1], where they work with two alternative models that appear expressed jointly as diagrams of relationships with the proposed model as shown in Fig. 1.







A = Firm / product factorsB = Factors based on pricesC = Factors based on purchase relation-ships

D = Level of use of the product

E =Satisfaction with the firm

Fig. 1. Diagrams of sequence of the estimated model and two rival models [1].

Table 1 compares the three models with the three types of goodness-of-fit measures

Table 1: Comparison of goodness-of-fit measures for the estimated model and te two rival model.

Goodness of	Estimated	Rival 1	Rival 2
fit measures	Model	Model	Model
Absolute fitness measures			
$\chi^2$ Likelihood ratio			
	178.714	174.450	175.397
Degrees of freedom			
	85	82	84
Non centrality parameter			
	93.714	92.450	91.397
Standard non centrality parameter			
	0.689	0.680	0.672
Goodness-of-fit index			
	0.865	0.867	0.866
Residual Mean square			
	0.076	0.074	0.075
Residual Mean Error Approximation			
	0.090	0.091	0.090
Expected Cross Validation Index			
	1.842	1.855	1.833
Fitness Augmented Measures			
Augmented Goodnesss-of-fit Indez			
-	0.810	0.805	0.809
Tucker-Lewis Index			
	0.876	0.873	0.878
Normed Fitness Index			
	0.828	0.832	0.831
Parsimony Fitness Measures			
Parsimony Normed Fitness Index			
	0.670	0.650	0.665
Parsimony Goodness-of-fit Index			
	0.613	0.592	0.606
Normed $\chi^2$	2.103	2.127	2.088
Akaike Information Criteria			
	248.714	250.450	247.397

Fitness Measures we see that the rival 1 model has the lowest values or different to the other two models; however it is necessary to remember that this model has the biggest number of estimated parameters, and therefore, the smallest number of

degrees of freedom. The estimated model does not differ much of both models, although it is not the best taking into account these measures. For the Fitness Augmented Measures, both rival models have a better behavior, however the differences with the estimated model are not important. The Parsimony Fitness Measures have a better behavior for the estimated model than for the rival model except for the Normed 2 and the Akaike Information Criteria. This example illustrates the great value of the comparison with rival models for all structural equation models with the purpose of making sure that the proposed model is really the best available model.

## 8. Conclusions

1. The structural equations modelization combines elements, so much of the regressin multiple as of the factorial analysis. Besides, it allows not only the evaluation of the complex interrelations of dependence, but also to incorporate the effects of the measure error on the structural coefficients at same time.

2. Although SEM is useful in many cases, should be used as a confirmatory matter, leaving the exploratory matter to other multivariate techniques.

3. Before evaluating the measure or structural models, the overall adjustment of the model should be analyzed to verify that is an adequate representation of all the causal relations using the three quality measures: the absolute, augmented and parsimony fitness measures.

4. The techniques developed to evaluate the structural equation models have a confirmatory bias that tends to confirm that the model is adjusted to the data, although, this only has been confirmed that is one of the various acceptable possible models.

5. The most rigorous test is the comparison with alternative models, since they represent the true and different structural hypothetical relations.

6. It would be interesting to compare the confirmatory strategies of modelization and alternative models with the strategy of the development of the model trying to improve by different changes, the measure and structural models.

7. Another important subject to study is related with the estimation process, being the most usual: the direct estimation, jackknife or bootstrap, besides other simulation techniques.

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