Business Process Modeling Applications Using Decision Process Petri Nets

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Abstract: This paper presents a formal framework for business process modeling and analysis using partially ordered transition Decision Process Petri nets (DPPN). The advantage of this approach is its ability to represent the dynamic behavior of the business process. The business process model is supported by an information technology strategic planning (ITSP) model and methodology. The modeling methodology is based in business strategy transformation. High-level business strategies are refined up to the point when they can be described in terms of the activities needed to achieve a certain tactical business strategy given only in terms of goals and strategies. At this point, partially ordered DPPN are used for business process representation and analysis. DPPN extends the place-transitions Petri net theoretic approach including the Markov decision process. In this sense, DPPN corresponds to a series of strategies which guide the selection of actions that lead to a final (decision) state. By taking into account different possible courses of action the overall utility of each strategy is considered. The utility function of each business process is represented by a Lyapunov like function. Conditions of equilibrium and stability for the DPPN are analyzed. For illustration purposes, two examples are presented.

Key-words: Business Process, information technology strategic planning, DPPN, strategies and goals, Lyapunov.

1 Introduction

This paper introduces a modeling paradigm for developing business process representation via a decision process Petri net. It is supported by an ITSP model and methodology, which integrates the business/organizational strategic visions and the information technology (IT) strategic vision in a resulting unified vision (related works in ITSP model are presented in [7] and [1]).

The method is based in business strategy decomposition. High-level business strategies are refined up to the point when they reach a tactical business strategy level, described only in terms of goals and strategies. The importance of being able to clearly link the business processes with the business strategy is highlighted by the concept of business reengineering [5]. The notion of business strategy decomposition is adopted to represent the process of business strategy refinement. Activities are considered as operationalizations of goals and are applied in accordance with the strategies needed to achieve these goals. Thus, the decomposition process results in a set of primitive actions such as "order a product". Strategies are expressions that define valid state transitions in the business process. In fact, strategies specify the event occurrences and they represent either integrity rules or control operations. Since the business strategy decomposition determines actions sequence applications, a process can be ordered introducing a partial ordered relation. It is important to note that any business process ultimately ends, because real processes are finite.

Partially ordered transitions DPPN are used for business process representation, taking advantage of the well-know properties of this approach namely, formal semantic, graphical display and wide acceptance by practitioners. A decision process Petri net model of a business process gives a specific and unambiguous description of the behavior of the process. Its solid mathematical foundation has resulted in different analysis methods and tools. Despite of the formal background, Petri net models are easy to understand.

The rest of the paper is structured in the following manner. The next section presents the necessary mathematical background and terminology needed to understand the rest of the paper. Section 3, describes the basic formalism of the ITSP model and the methodology. Section 4, discusses the issues associated to the business process model method. Section 5, presents two application examples. Finally, section 6, concludes the paper by giving future research directions.

2 Preliminaries

2.1 DPPN ([2], [10])

Definition 1 A Decision Process Petri net is a 7-tuple $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ where

- $P = \{p_0, p_1, p_2, ..., p_m\}$ is a finite set of places,
- $Q = \{q_1, q_2, ..., q_n\}$ is a finite set of transitions,
- $F \subset I \cup O$ is a set of arcs where $I \subset (P \times Q)$ and $O \subset (Q \times P)$ such that $P \cap Q = \emptyset$ and $P \cup Q \neq \emptyset$,
- $W: F \to N_1^+$ is a weight function,
- $M_0: P \to N$ is the initial marking,
- π : $I \to \mathbb{R}_+$ is a routing policy representing the probability of choosing a particular transition (routing arc), such that for each $p \in P$, $\sum_{(p,q_j):q_j \text{ varying over } Q} \pi((p,q_j)) = 1,$
- $U: P \to \mathbb{R}_+$ is a utility function.



Figure 1a. Routing policy case 1



Figure 1b. Routing policy case 2

In figures 1.a and 1.b we have represented partial routing policies π :

- case 1. In figure 1.a the probability that q_1 generates a transition from state p_1 to p_2 is 1/3. But, because q_1 transition to state p_2 has two arcs, the probability to generate a transition from state p_1 to p_2 is increased to 2/3.
- case 2. In figure 1.b we set by convention for the probability that q_1 generates a transition from state p_1 to p_2 is 1/3 (1/6 plus 1/6). However, because q_1 transition to state p_2 has only one arc, the probability to generate a transition from state p_1 to p_2 is decreased to 1/6.

Definition 2 The utility function U with respect a Decision Process Petri net $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ is represented by the equation

$$U_k^{q_j}(p_i) = \begin{cases} U_k(p_0) & if \ i = 0, k = 0\\ L(\alpha) & if \ i > 0, k = 0 \& \ i \ge 0, k > 0 \end{cases}$$
(1)

where

$$\alpha = \begin{bmatrix} \sum_{h \in \eta_{ij_0}} \Psi(p_h, q_{j_0}, p_i) * U_k^{q_{j_0}}(p_h), \\ \sum_{h \in \eta_{ij_1}} \Psi(p_h, q_{j_1}, p_i) * U_k^{q_{j_1}}(p_h), ..., \\ \sum_{h \in \eta_{ij_f}} \Psi(p_h, q_{j_f}, p_i) * U_k^{q_{j_f}}(p_h) \end{bmatrix}$$
(2)

the function $L : D \subseteq \mathbb{R}^n_+ \to \mathbb{R}_+$ is a Lyapunov like function which optimizes the utility through all possible transitions (i.e. trough all the possible trajectories defined by the different $q_j s$), D is the decision set formed by the j's; $0 \le j \le f$ of all those possible transitions $(q_j$ $p_i) \in O$, $\Psi(p_h, q_j, p_i) = \pi(p_h, q_j) * \frac{FN(q_j, p_i)}{FN(p_h, q_j)}$, η_{ij} is the index sequence of the list of previous places to p_i through transition q_j , p_h $(h \in \eta_{ij})$ is a specific previous place of p_i through transition q_j .

2.1.1 DPPN Mark-Dynamic Properties

We will identify the mark-dynamic properties of the DPPN as those properties related with the PN.

Definition 3 An equilibrium point with respect a Decision Process Petri net $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ is a place $p^* \in P$ such that $M_l(p^*) = S < \infty, \forall l \ge k$ and p^* is the last place of the net.

Theorem 4 The Decision Process Petri net DPPN = $\{P, Q, F, W, M_0, \pi, U\}$ is uniformly practically stable iff there exists a Φ strictly positive m vector such that $\Delta v = u^T A \Phi \leq 0$.

2.1.2 DPPN Trajectory-Dynamic Properties

We will identify the trajectory-dynamic properties of the DPPN as those properties related with the utility at each place of the PN.

Definition 5 A final decision point $p_f \in P$ with respect a Decision Process Petri net DPPN = $\{P, Q, F, W, M_0, \pi, U\}$ is a place $p \in P$ where the infimum or the minimum is attained, i.e. U(p) = 0 or U(p) = C.

Definition 6 An optimum point $p^{\triangle} \in P$ with respect a Decision Process Petri net DPPN = $\{P, Q, F, W, M_0, \pi, U\}$ is a final decision point $p_f \in P$ where the best choice is selected 'according to some criteria'. **Proposition 7** Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a Decision Process Petri net and let $p^{\triangle} \in P$ an optimum point. Then $U(p^{\triangle}) \leq U(p), \forall p \in P$ such that $p \leq_U p^{\triangle}$.

Theorem 8 The Decision Process Petri net $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ is uniformly practically stable iff $U(p_{i+1}) - U(p_i) \leq 0.$

Definition 9 A strategy with respect a Decision Process Petri net $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ is identified by σ and consists of the routing policy transition sequence represented in the DPPN graph model such that some point $p \in P$ is reached.

Definition 10 An optimum strategy with respect a Decision Process Petri net DPPN = $\{P, Q, F, W, M_0, \pi, U\}$ is identified by σ^{\triangle} and consists of the routing policy transition sequence represented in the DPPN graph model such that an optimum point $p^{\triangle} \in P$ is reached.

Equivalently we can represent (1, 2) as follows:

$$U_k^{\sigma_{hj}}(p_i) = \begin{cases} U_k(p_0) & if \ i = 0, k = 0\\ L(\alpha) \ if \ i > 0, k = 0 \& i \ge 0, k > 0 \end{cases}$$
(3)

$$\alpha = \begin{bmatrix} \sum_{h \in \eta_{ij_0}} \sigma_{hj_0}(p_i) * U_k^{\sigma_{hj_0}}(p_h), \\ \sum_{h \in \eta_{ij_1}} \sigma_{hj_1}(p_i) * U_k^{\sigma_{hj_1}}(p_h), ..., \\ \sum_{h \in \eta_{ij_f}} \sigma_{hj_f}(p_i) * U_k^{\sigma_{hj_f}}(p_h) \end{bmatrix}$$
(4)

where $\sigma_{hj}(p_i) = \Psi(p_h, q_j, p_i)$. The rest is as previous defined.

2.1.3 Convergence of the DPPN Mark-Dynamic and Trajectory-Dynamic Properties

Theorem 11 Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a Decision Process Petri net. If $p^* \in P$ is an equilibrium point then it is a final decision point.

Theorem 12 Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a finite and non-blocking Decision Process Petri net (unless $p \in P$ is an equilibrium point). If $p_f \in P$ is a final decision point then it is an equilibrium point.

Corollary 13 Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a finite and non-blocking Decision Process Petri net (unless $p \in P$ is an equilibrium point). Then, an optimum point $p^{\Delta} \in P$ is an equilibrium point. **Definition 14** Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a Decision Process Petri net. A trajectory ω is an (finite or infinite) ordered subsequence of places $p_{\varsigma(1)} \leq U_k$ $p_{\varsigma(2)} \leq U_k \dots \leq U_k p_{\varsigma(n)} \leq U_k \dots$ such that a given strategy σ holds.

Definition 15 Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a Decision Process Petri net. An optimum trajectory ω is an (finite or infinite) ordered subsequence of places $p_{\varsigma(1)} \leq_{U_k^{\triangle}} p_{\varsigma(2)} \leq_{U_k^{\triangle}} \dots \leq_{U_k^{\triangle}} p_{\varsigma(n)} \leq_{U_k^{\triangle}} \dots$ such that the optimum strategy σ^{\triangle} holds.

Theorem 16 Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a non-blocking Decision Process Petri net (unless $p \in P$ is an equilibrium point) then we have that:

$$U_k^{\Delta}(p^{\Delta}) \le U_k(p), \quad \forall \sigma, \sigma^{\Delta}$$

Corollary 17 Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a non blocking Decision Process Petri net (unless $p \in P$ is an equilibrium point) and let σ^{Δ} an optimum strategy. Set $L = \min_{i=1,...,|\alpha|} \{\alpha_i\}$ then, $U_k^{\Delta}(p)$ is equal to:

$\sigma^{\Delta}_{0j_m}(p_{\varsigma(0)})$	$\sigma_{1j_m}^{\Delta}(p_{\varsigma(0)})$	 $\sigma_{nj_m}^{\Delta}(p_{\varsigma(0)})$	$U_k(p_0)$
$\sigma_{0j_n}^{\Delta}(p_{\varsigma(1)})$	$\sigma_{1j_n}^{\Delta}(p_{\varsigma(1)})$	 $\sigma_{nj_n}^{\Delta}(p_{\varsigma(1)})$	$U_k(p_1)$
$\sigma^{\Delta}_{0j_v}(p_{\varsigma(i)})$	$\sigma_{1j_v}^{\Delta}(p_{\varsigma(i)})$	 $\sigma_{nj_v}^{\Delta}(p_{\varsigma(i)})$	$U_k(p_i)$
		 	<u> </u>
<u></u>	σ^{\triangle}		
	0		(5)

where p is a vector whose elements are those places which belong to the optimum trajectory ω given by $p_0 \leq p_{\varsigma(1)} \leq U_k p_{\varsigma(2)} \leq U_k \dots \leq U_k p_{\varsigma(n)} \leq U_k \dots$ which converges to p^{\triangle} .

Definition 18 A Decision Process Petri net $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ is said to be symmetric if it is possible to decompose it into some finite number (greater that 1) of sub-Petri nets in such a way that there exists a bijection ψ between all the sub-Petri nets such that

$$(p,q) \in I \Leftrightarrow (\psi(p),\psi(q)) \in I \text{ and } (q,p) \in O$$

 $\Leftrightarrow (\psi(q),\psi(p)) \in O$
for all of the sub-Petri nets.

Corollary 19 Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a non blocking (unless p is an equilibrium point) symmetric Decision Process Petri net and let σ^{\triangle} be an optimum .strategy. Set $L = \min_{i=1,...,|\alpha|} \{\alpha_i\}$ then,

$$\sigma^{\triangle} U < \sigma U \qquad \forall \sigma, \sigma^{\triangle}$$

where the σ and σ^{Δ} are represented by a matrix and U is represented by a vector.

2.1.4 Optimum Trajectory Planning

Given a non-blocking (unless $p \in P$ is an equilibrium point) Decision Process Petri net $DPPN = \{P, Q, F, W, M_0, \pi, U\}$, the optimum trajectory planning consists in finding the firing transition sequence u such that the optimum target state M_t with the optimum point is achieved. The target state M_t belong to the reachability set $R(M_0)$, and satisfies that it is the last and final task processed by the DPPN with some fixed starting state M_0 with utility U_0 .

Theorem 20 The optimum trajectory planning problem is solvable.



Figure 2. ITSP Model

3 ITSP conceptual model and methodology ([4])

In the model represented in figure 2, the real world is composed by entities representing physical things (people, governments, enterprises, etc.) these entities are related in terms of goals, beliefs, etc. Entities under events generation change the environmental conditions. They take particular strategic positions through the network of relationships with other entities, where they play different roles. The model is based on three fundamental concepts: interaction, adaptation and evolution.

The interaction concept represents the dynamic behavior of the environment, leading to the incorporation or rejection of beliefs and facts related with environment conditions. Interactions are established by the relationships between the roles that each entity plays in the domain of application. The behavior of the environment is induced by the interaction of the entities. The adaptation includes business strategies using a logic inference method, which uses beliefs and facts in order to generate new business strategies. This is a dynamic process where old business strategies are replaced by those corresponding with the present environmental state. In the real world, there are always assumptions that, if they are proven to be unfounded, they are easily corrected. The environmental changes always take place in the curse of events that invalidate previous states. On the other hand, non-monotonic reasoning shows an opposite fact to this problem. It simply allows the retraction of 'truth' whenever contradictions arise by forcing the incorporation of new beliefs.

The evolution is a process in which the business strategy is transformed into operative and IT components (the organizational model, the human resources, the IT model and the planning model). It considers a dynamic application domain which integrates the business/organizational strategic visions and the IT strategic vision in a resulting unified vision.



Figure 3. ITSP Methodology

The evolution process is represented by an inverse pyramid where business strategy represents the "axioms" of the archetype of the organization's. These axioms are considered as true i.e., fundamental principles, in virtue that they are congruent with the reality of the environment. In every case, the ITSP tries to be in contact with the real world in order to give to its construction, logical coherence. The organization propositions [7] (the organization model, the human resources model the IT model –the IT strategy [8]– and the planning model –[3]–) are deduced from the axioms through a logic inference method. Thus, every proposition is true if it can be deduced from the axioms.

The ITSP methodology (figure 3) is organized in fifteen modules which are divided in four phases, and conceived in two visions. In addition, it is concerned with creating a business\organizational vision, which provides the critical information inputs and, it also forms the foundations for later stages of planning. As well as this, it creates a vision of the IT, which exploits new technological solutions and it improves the enterprise situation.

4 Business Process Modeling and Partially Ordered Transition DPPN

In business process modeling, high-level business strategies are refined up to the point when they reach a tactical business strategy level, described only in terms of goals and strategies¹.

Business strategy decomposition represents a hierarchy of objective/decision-points, varying from the high-level business strategy with the maximum longterm impact to the more refined operational business strategy (goal, strategy) with relative short-term impact.

The business strategy refinement process concludes when a resulting business strategy can be transformed into an executable action. In this sense, the nodes found in the lowest levels of the business strategy decomposition tree are usually mapped into actions.

A business process is regarded as a set of activities. Activities are considered as operationalizations of goals and are applied in accordance with the strategies to achieve the goals. Strategies determine the legal sequentially movements that can be made from any activity to another. The structure of each node in the business strategy decomposition is a complex object, defined by the ordered pair goal-strategy.

For completeness let us recall some basic notations of ordering. Given a poset (X, \preceq) a successor of an element $x \in X$ is an element y such that $x \preceq y$, but $x \neq y$ and there is no third element u between x and y. x is a predecessor of y if y is a successor of x. In symbols, for any $x \in X$

Successors of $x: y \in suc(x)$ iff $x \neq y, x \leq y$ and $\forall u: x \leq u \leq y \Longrightarrow (u = x) \lor (u = y)$

Predecessors of $x: y \in pre(x)$ iff $y \neq x, y \leq x$ and $\forall u: y \leq u \leq x \Longrightarrow (u = y) \lor (u = x)$

The graph of the ordering is the graph whose vertices are the points in X and each pair (x, y) where y is a successor of x determines an edge. The graph corresponding to the ordering " \leq " defined is a directed acyclic graph (DAG).

The minimal elements are those with no predecessors, i.e. nodes with null inner degree in the DAG. The maximal elements are those with no successors, i.e. nodes with null outer degree in the DAG. In this ordering the conditions with no input transitions correspond to the minimal elements, and the conditions with no output transitions correspond to the maximal elements.

Since the business strategy decomposition determines actions sequence applications, a process can be ordered as follows. Let X be a process and $x, y \in X$ two activities.

Definition 21 We say that the activity y "depends on" the activity x, and we denoted it by $x \leq y$, if the corresponding decomposed node of x is upper than that of yin the business strategy decomposition tree.

Property 22 Clearly, " \preceq " establishes a partial ordering.

The partial order concept guarantees that the nodes found in the lowest levels of the business strategy decomposition tree, are already partially ordered and ready to be mapped into what next, is defined to be a partially ordered DPPN.

Definition 23 A partially ordered transition Decision Process Petri net is a duple $(DPPN, \preceq)$ where DPPNis a Decision Process Petri net and \preceq_q is the partial order defined on the elements of the set of transitions Q such that the following conditions hold:

- $q_1 \prec_q q_2$ iff $q_1 \preceq_q q_2$ and $\neg (q_2 \preceq_q q_1)$
- $q_1 \sim_q q_2$ iff $q_1 \preceq_q q_2$ and $q_2 \preceq_q q_1$

Note that the order of the DPPN is the order established by the "depends on" relationship (see the definition of \leq).

Events are actions which take place in a process. The occurrence of these events is controlled in part by the state of the process. The state of a process can be described as a set of conditions. The minimal elements of the net are those conditions associated to the initial marking. Since events are actions, they may occur, for an event to occur, it may be necessary that certain preconditions hold. Each transition has associated a strategy that determines the preconditions to hold or not and may cause post-conditions to become true.

Proposition 24 Let us suppose that all the condition of theorem 3.6 are satisfied and let us suppose that the DPPN is a partially ordered Decision Process Petri net. Then, equation (5) reduces to

1	0		 		$U_k(p_0)$
$\sigma_{0j_m}^{\Delta}(p_{\varsigma(1)})$	0	0	 		$U_k(p_1)$
	$\sigma_{0j_n}^{\Delta}(p_{\varsigma(2)})$	0	 		
			 0		$U_k(p_i)$
			 $\sigma_{nj_v}^{\Delta}(p_{\varsigma(n)})$	0	
<u> </u>		-			
	a				0
					(6)

 $^{^{1}}$ For simplification, we decompose the business strategy in goals and strategies, which we consider is adequate from an operational point of view

Proof. It follows from corollary 19 and the fact that the Decision Process Petri net is partially ordered. ■

Objectives	Goals	Strategies	CSFs
O1 Achieve Market	G1 Reduce Operation Costs	S1 Improvements in Marketing	C1 Improve Cash Flow
Leadership position (M)	5% (O1, O2)	Practices (O3, O4, M, A)	Management (O1, G3)
O2 Improve Service Quality	G2 Achieve 30% Market Participation (O1 O4 M)		C7 Growth Through Acquisition (O1, S2)
(1, 0, 10, 11, 11, 1, 1, 11)	runcipation (01, 04, m)	S2 Penetration into New Markets (O1, O2, G3)	C3 Improve Human Resources Training (O4)
O3 Gain Competitive	G3 Improve Stockholders		
Advantage (M)	Gains 15% (O1, O5, M)		C5 Improve Client Service (O2, G4, S3)
04 Enhance Company Image	G4 Introduce IS/IT Solutions	S3 Improvements in New	C4 Introduce Strategic





5 Application examples

The aim of this section is to present two business process application examples, represented by the partially order (DPPN), where the optimum strategy and its correspondent tracking are described.

Example 25 Let us consider an insurance broker agency. As a broker, the agency sells policies for different companies. The main products are life and automobile policies. For selling and advertising the insurance company obtains detailed information from potential customers (C), and from private and governmental agencies (A). This information is distributed between the company's agents (AG) which contact potential clients via phone and try to set up a conference call; however, they also have their own sources of information. At the interview, the agent examines the client's current insurance coverage and tries to find an opportunity for a policy that will best fit the customer's needs. Before obtaining an insurance policy, the new client suffers an identity investigation. In the case of a life insurance, the client has, in addition, to approve a physical examination test in an accredited hospital (H). In the case that the investi*qation is positive both parts sign a policy and keep a copy* of the contract. If during the investigation irregularities are found, the agent is informed, who meets with the client in order to find new options. The insurance policy is in effect when the client makes the first insurance premium payment. Every policy carries with a schedule of premiums, which varies with the type and coverage. Each policy provides a commission for the agency. The commission varies with the insurance company, policy type and coverage. The insurance company management (M) defines the commissions politic, which varies from agency to agency. The agency splits the commission received for each policy with the agent who sold it; the rate depends on the seniority of the agent. Once a policy has been sold, the agency submits premium bills to the client, collects payment and sends the payment, minus it commission, to the insurance company. If a client fails to pay premiums, the agent who sold the policy is informed. so that he can contact the client. Claims can be made on insurance policies as specified in the policy itself. Clients or beneficiaries (B) contact the agent to file such claims. Life insurance claims may be made by the beneficiaries on the death of the insured. In both cases, the insurance company sends an adjustor (AD) to legitimate the claim and arrange the final insurance details. For an automobile insurance policy, claims are made when the car is involved in an accident, damaged or stolen. For simplification, we will consider just the organizational strategy of the insurance company. Let us construct the organizational strategy like in [8]. In the business strategy decomposition tree each node has a complex structure as follows: objective, goal, strategy, critical success factors (CSF).

The complete business strategy decomposition is out of the scope of this article. However, we are following the decomposition process presented in [9]. The decomposition is shown in figure 4. The business strategy decomposition tree is shown in figure 5.

Next, the partially ordered DPPN net model $(DPPN, \preceq)$ is constructed by mapping the activities in the business strategy decomposition tree (figure 5). Notice that the goals are represented by the places while the transitions represent the activities. The partially ordered DPPN (figure 6) has the following specifications:

Places:

 P_0 : order requested,

 P_1 : investigated client

 P_2 : examined client health

 P_3 : rejected order

 P_4 : declined order

 $P_5\colon$ consented physical examination and admitted antecedents

 P_6 : authorized policy P_7 : delivered policy

Transitions:

 q_1 : investigate client antecedent

- q_2 : review physical condition
- q_3 : deny physical examination
- q_4 : refuse antecedents
- q_5 : accept order
- q_6 : sign contract
- q_7 : send life policy

Define the Lyapunov like function L in terms of the Entropy $H(p_i) = -p_i \ln p_i$ as $L = \max_{i=1,\dots,|\alpha|} (-\alpha_i \ln \alpha_i)$

then.

 $U_{k=0}(p_0) = 1$ $U_{k=0}^{\sigma_{hj}}(p_1) = L[\sigma_{01}(p_1) * U_{k=0}^{\sigma_{01}}(p_0)] = L[7/10 * 1] = \max H[7/10 * 1] = 0.249$

 $U_{k=0}^{\sigma_{hj}}(p_2) = L[\sigma_{02}(p_2) * U_{k=0}^{\sigma_{02}}(p_0)] = L[3/10 * 1] =$ $\max H[3/10 * 1] = 0.361$

 $U_{k=0}^{\sigma_{hj}}(p5) = L[\sigma_{15}(p_5) * U_{k=0}^{\sigma_{15}}(p_1) + \sigma_{25}(p_5) * U_{k=0}^{\sigma_{25}}(p_2)] = \max H[4/10 * 0.249 + 4/10 * 0.361] = 0.286$ $U_{k=0}^{\sigma_{hj}}(p_6) = L[\sigma_{56}(p_6) * U_{k=0}^{\sigma_{56}}(p_5)] = L[1 * 0.286] =$ $\max H[0.286] = 0.358$

 $U_{k=0}^{\check{\sigma}_{hj}}(p_7) = L[\sigma_{67}(p_7) * U_{k=0}^{\sigma_{67}}(p_6)] = L[2 * 0.358] = \max H[2 * 0.358] = 0.239$

the firing transition vector is u, which tracks the optimum strategy σ^{Δ} for selling a policy, is given by transitions q_1, q_2, q_5, q_6 and q_7 as follows

aı —	1	1	0	0	1	1	1		
<i>u</i> –	q_1	q_2	q_3	q_4	q_5	q_6	q_7		

In this case the strategy is optimum because the selling is obtained. In case that the policy is rejected the operative costs are supported by the insurance company.

Alternatively, we have

 $U_{k=0}^{\sigma_{hj}}(p_3) = L[\sigma_{23}(p_3) * U_{k=0}^{\sigma_{23}}(p_2)] = L[8/10 * 0.361] = \max H[8/10 * 0.361] = 0.358$ $U_{k=0}^{\sigma_{hj}}(p_4) = L[\sigma_{14}(p_4) * U_{k=0}^{\sigma_{14}}(p_1)] = L[8/10 * 0.361] = 0.358$

 $0.249 = \max H[8/10 * 0.249] = 0.321$



Example 26 Continuing with the insurance broker agency we have that in case of a car accident, the insurance company depends on the adjustor appraisal to evaluate the damages. To maintain company profitability the adjustor must evaluate the case so that only the minimal necessary repairs will be considered. In this sense, the adjustor evaluation is expected to be in favor of the insurance company because of his dependence on the latter. However, the adjustor must be careful, because the insurance company wants to offer a good service in order to keep the client. As a result, the automobile owner depends on the appraisal of the adjustor for an appropriate accident evaluation. The automobile owner can also be assisted by an authorized garage to obtain a fair evaluation of the car's damage. Notice that, the garage must satisfy both the client and the insurance company, given that the garage income depends on the car owner and on the insurance company. In case that the accident includes physical damage, the client and passengers must be directed to an accredited hospital for medical treatment. Three different strategies can be presented to manage a car accident in order to optimize the company's profitability ([6]). To improve the operation cost, small accidents can be directly evaluated by the adjustor or the authorized garage, and reported to the insurance company. Accidents of considerable size must be managed centrally by the insurance company. The partially ordered DPPN (figure 7) has the following specifications:



Places:

 P_0 : claim settled,

 P_1 : handled accident info centrally

 P_2 : handled accident info by authorized garage

<u></u>	1	0	()	1	0	0	1	1	1	0	0	0	1	1	0) () ()	0	1	0	0	0]
	q_1	q	2	<i>4</i> 3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}	q_{12}	$ q_1 $.3 q	14 9	115 9	116	117	q_{18}	q_{19}	q_{20}	q_{21}	q_{22}	
	Figure 8																								
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/		0	1	0	0	1	0	0	0) 1	. 1	L (0	0	0	1	1	0	0	0	1	0	()
u =		q_1	q_2	q_3	q_4	q_5	q_6	$q_{\overline{q}}$	q	8 q	99	10 9	111	q ₁₂	q_{13}	q_{14}	q_{15}	q_{16}	q_{17}	q_1	$_8 q_1$	q	$_{20} q$	21 6	<i>]</i> 22

Figure 9

<i>"</i> ″_	0	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	1
<i>u</i> –	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}	q_{12}	q_{13}	q_{14}	q_{15}	q_{16}	q_{17}	q_{18}	q_{19}	q_{20}	q_{21}	q_{22}
	Figure 10																					

 P_3 : handled accident info by adjustor

 P_4 : verified policy covering centrally

 $\mathbb{P}_5:$ verified policy covering by authorized garage

 P_6 : verified policy covering by adjustor

 P_7 : corroborated accident details

 P_8 : evaluated damage centrally

 P_9 : got medical treatment cost

 P_{10} : determined accident in range

 P_{11} : send info to be handle centrally

 P_{12} : got accident info by adjustor

 P_{13} : assessed client antecedents

 P_{14} : determined accident covering centrally

 $P_{15}:$ got accident info by authorized garage

 P_{16} : evaluated damage by authorized garage

 P_{17} : determined accident in range

 P_{18} : send info to be handle centrally

 P_{19} : adjusted policy and made covering offer centrally

 P_{20} : made covering offer by authorized garage

 P_{21} : evaluated damage by adjustor

 P_{22} : made covering offer by adjustor

Transitions:

 q_1 : handle accident info centrally

 q_2 : handle accident info by authorized garage

 q_3 : handle accident info by adjustor

 q_4 : verify policy covering centrally

 q_5 : verify policy covering by authorized garage

 q_6 : verify policy covering by adjustor

 q_7 : corroborate accident details

 q_8 : evaluate damage

 q_9 : get medical treatment cost

 q_{10} : determine accident in range

 q_{11} : send info to be handle centrally

 q_{12} : get accident info by adjustor

 q_{13} : assess client antecedents

 q_{14} : determine accident covering centrally

 q_{15} : get accident info by authorized garage

 q_{16} : evaluate damage by authorized garage

 q_{17} : determine accident in range

 q_{18} : send info to be handle centrally

 q_{19} : adjust policy and make covering offer centrally

 q_{20} : make covering offer by authorized garage q_{21} : evaluated damage by adjustor

 q_{22} : make covering offer by adjustor

Define the Lyapunov like function L in terms of the Entropy $H(p_i) = -p_i \ln p_i$ as $L = \max_{i=1,...,|\alpha|} (-\alpha_i \ln \alpha_i)$ then,

I) The optimum strategy σ^{\triangle} for accidents of considerable size that must be manage centrally by the assurance company is represented by

$$U_{k=0}(p_0) = 1$$

 $U_{k=0}^{\sigma_{hj}}(p_1) = L[\sigma_{01}(p_1) * U_{k=0}^{\sigma_{01}}(p_0)] = L[1/3 * 1] = \max H[2/3 * 1] = 0.270$

 $U_{k=0}^{\sigma_{hj}}(p_4) = L[\sigma_{14}(p_4) * U_{k=0}^{\sigma_{14}}(p_1)] = L[1 * 0.270] = \max H[0.270 * 1] = 0.353$

 $U_{k=0}^{\sigma_{hj}}(p_7) = L[\sigma_{47}(p_7) * U_{k=0}^{\sigma_{47}}(p_4)] = L[2/5 * 0.353] = \max H[2/5 * 0.353] = 0.276$

 $U_{k=0}^{\sigma_{hj}}(p_8) = L[\sigma_{48}(p_8) * U_{k=0}^{\sigma_{48}}(p_4)] = L[1/5 * 0.353] = \max H[1/5 * 0.353] = 0.187$

 $U_{k=0}^{\sigma_{hj}}(p_9) = L[\sigma_{49}(p_9) * U_{k=0}^{\sigma_{49}}(p_4)] = L[2/5 * 0.353] = \max H[2/5 * 0.353] = 0.276$

 $U_{k=0}^{\sigma_{hj}}(p_{13}) = L[\sigma_{7,13}(p_{13}) * U_{k=0}^{\sigma_{7,13}}(p_7)] = L[1 * 0.276] = \max H[1 * 0.276] = 0.355$

 $U_{k=0}^{\sigma_{hj}}(p_{14}) = L[\sigma_{8,14}(p_{14}) * U_{k=0}^{\sigma_{8,14}}(p_8) + \sigma_{9,14}(p_{14}) * U_{k=0}^{\sigma_{9,14}}(p_9)] = L[1/2 * 0.187 + 1/2 * 0.276] = \max H[1/2 * 0.187 + 1/2 * 0.276] = 0.338$

 $U_{k=0}^{\sigma_{hj}}(p_{19}) = L[\sigma_{13,19}(p_{19}) * U_{k=0}^{\sigma_{13,19}}(p_{13}) + \sigma_{14,19}(p_{19}) * U_{k=0}^{\sigma_{14,19}}(p_{14})] = L[6/20 * 0.355 + 8/20 * 0.338] = \max H[6/20 * 0.355 + 8/20 * 0.338] = 0.343$

the firing transition vector is shown in figure 8.

For this case the adjustor or the garage must abort the process becasue the accident is out of their range obtaining that:

 $U_{k=0}^{\sigma_{hj}}(p_{11}) = L[\sigma_{5,11}(p_{11}) * U_{k=0}^{\sigma_{5,11}}(p_{5})] = L[4/5 * 0.367] = \max H[4/5 * 0.367] = 0.359$

 $U_{k=0}^{\sigma_{hj}}(p_{18}) = L[\sigma_{12,18}(p_{18}) * U_{k=0}^{\sigma_{12,18}}(p_{12})] = L[3/4 * 0.367] = \max H[3/4 * 0.367] = 0.355$

concluding

$$U_{k=0}^{\sigma_{hj}}(p_{19}) < U_{k=0}^{\sigma_{hj}}(p_{11}) < U_{k=0}^{\sigma_{hj}}(p_{18})$$

i.e. $U_{k=0}^{\sigma_{hj}}(p_{11}), U_{k=0}^{\sigma_{hj}}(p_{18})$ are more expensive than $U_{k=0}^{\sigma_{hj}}(p_{19}).$

II) The optimum strategy $\sigma^{\prime \Delta}$ for small accidents that must be manage ideally by the garage is represented by

 $U_{k=0}^{\sigma_{hj}}(p_2) = L[\sigma_{02}(p_2) * U_{k=0}^{\sigma_{02}}(p_0)] = L[1/3 * 1] = \max H[1/3 * 1] = 0.366$

 $U_{k=0}^{\sigma_{hj}}(p_5) = L[\sigma_{25}(p_5) * U_{k=0}^{\sigma_{25}}(p_2)] = L[1 * 0.366] = \max H[1 * 0.366] = 0.367$

 $U_{k=0}^{\sigma_{hj}}(p_{10}) = L[\sigma_{5,10}(p_{10}) * U_{k=0}^{\sigma_{5,10}}(p_{5})] = L[1/5 * 0.367] = \max H[1/5 * 0.367] = 0.191$

 $U_{k=0}^{\sigma_{hj}}(p_{15}) = L[\sigma_{10,15}(p_{15}) * U_{k=0}^{\sigma_{10,15}}(p_{10})] = L[6/8 * 0.191] = \max H[6/8 * 0.191] = 0.278$

 $U_{k=0}^{\sigma_{hj}}(p_{16}) = L[\sigma_{10,16}(p_{16}) * U_{k=0}^{\sigma_{10,16}}(p_{10})] = L[1/8 * 0.191] = \max H[1/8 * 0.191] = 0.089$

 $U_{k=0}^{\sigma_{hj}}(p_{20}) \stackrel{=}{=} L[\sigma_{15,20}(p_{20}) * U_{k=0}^{\sigma_{15,20}}(p_{15}) + \sigma_{16,20}(p_{20}) * U_{k=0}^{\sigma_{16,20}}(p_{16})] = L[1/5 * 0.278 + 4/5 * 0.089] = \max H[1/5 * 0.278 + 4/5 * 0.089] = 0.261$

the firing transition vector is shown in figure 9.

Intuitively the result is correct, because the best option for the insurance company is that after a car accident happens, the customer takes the car to the garage and the company does not have to send an adjustor.

III) The strategy σ'' for small accidents that must be manage by the adjustor is represented by

 $U_{k=0}^{\sigma_{h_{j}}}(p_{3}) = L[\sigma_{03}(p_{3}) * U_{k=0}^{\sigma_{03}}(p_{0})] = L[1/3 * 1] = max H[1/3 * 1] = 0.366$

 $U_{k=0}^{\sigma_{hj}}(p_6) = L[\sigma_{36}(p_6) * U_{k=0}^{\sigma_{36}}(p_3)] = L[1 * 0.366] = \max H[1 * 0.366] = 0.367$

 $U_{k=0}^{\sigma_{6,j}}(p_{12}) = L[\sigma_{6,12}(p_{12}) * U_{k=0}^{\sigma_{6,12}}(p_6)] = L[1 * 0.367] = \max H[1 * 0.367] = 0.367$

 $U_{k=0}^{\sigma_{hj}}(p_{17}) = L[\sigma_{12,17}(p_{17}) * U_{k=0}^{\sigma_{12,17}}(p_{12})] = L[1/4 * 0.367] = \max H[1/4 * 0.367] = 0.219$

 $U_{k=0}^{\sigma_{hj}}(p_{21}) = L[\sigma_{17,21}(p_{21}) * U_{k=0}^{\sigma_{17,21}}(p_{17})] = L[1 * 0.219] = \max H[1 * 0.219] = 0.332$

 $U_{k=0}^{\sigma_{hj}}(p_{22}) = L[\sigma_{21,22}(p_{22}) * U_{k=0}^{\sigma_{21,22}}(p_{21})] = L[2 * 0.332] = \max H[2 * 0.332] = 0.271$

The firing transition vector is shown in figure 10.

Notice that since $U_{k=0}^{\sigma_{hj}}(p_{22})$ is strictly bigger than $U_{k=0}^{\sigma_{hj}}(p_{20})$ small accidents must be handled by the garage, whenever it is possible.

6 Conclusions and future work

A formal framework for business process modeling using partially ordered Decision Process Petri nets has been presented. The paper was motivated by the fact that it is essential to combine decision process and business process. The technique presented allows to identify, design and evaluate value adding opportunities for business improvement and reengineering. The business process model was supported by an information technology strategic planning (ITSP) model and methodology. The modeling methodology was based in business strategy transformation. Application examples where decision process properties were shown to hold were addressed.

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