cooperation between company 1 and 2. Since there are many uncertainty factors during the cooperation, each player is not willing to offer all its resources to this specific cooperation. In another word, they only supply part of their resources. In order to reduce risk and get more payoffs, when the players take part in this cooperation, they only know the determining participation levels and the participation levels that they do not participate. For example, the company 1 has 10000 units of resources, the determining participation level is 3000 units, and 2000 units are not devoted to cooperation. Namely, the true membership grade of the player 1 is 0.3 = 3000/ 10000, and the false membership grade of the player 1 is 0.2 = 2000/10000. In such a way, a vague set is interpreted. Consider a vague coalition U defined by

$$U = \sum_{i\in\{1,2,3\}} [t_U(i), 1 - f_U(i)]/i$$

$$= [0.3, 0.6]/1 + [0.2, 0.3]/2 + [0.6, 0.8]/3.$$

If the crisp coalition values are given by table 1

Table 1. The fuzzy payoffs of the crisp coalitions (millions of dollars)

S_0	$v_0(S_0)$	S_0	$v_0(S_0)$
{1}	1	{1,3}	3
{2}	2	{2,3}	5
{3}	1	{1,2,3}	10
{1,2}	6		

From table 1, we know that when the company 1 and 2 cooperate with all their resources, then their payoff is 6 millions of dollars.

When the relationship between the values of the fuzzy coalitions and that of their associated crisp coalitions as given in (4) and (5). Namely, this fuzzy game belongs to $G_V^O(N)$. From (6), we get the player true Shapley values are

$$\varphi_1^O(t_U, v_O) = 0.42\,,\ \varphi_2^O(t_U, v_O) = 0.64\,,$$

$$\varphi_3^O(t_U, v_O) = 0.84\,.$$

From (7), we get the player upper Shapley values are

$$\varphi_1^O(e^{\mathrm{Supp}U} - f_U, v_O) = \varphi_2^O(e^{\mathrm{Supp}U} - f_U, v_O) = 1.11$$

$$\varphi_3^O(e^{\mathrm{Supp}U} - f_U, v_O) = 1.28\,.$$

Since the associated crisp game is convex, we know that the player Shapley values is a VMPAF and an element in its core.

Furthermore, the player possible payoffs with respect to the Shapley function are [0.42, 1.11], [0.64, 1.11] and [0.84, 1.28].

6 Conclusion

In some cooperative games, the players only know the determination participation levels and the levels that they do not participate. The fuzzy games on vague sets can well solve this situation. For this purpose, we research the fuzzy games on vague sets and discuss the Shapley value for fuzzy games on vague sets. When the given fuzzy games on vague sets are convex, some properties are investigated. Furthermore, we study a special kind of fuzzy games on vague sets. The Shapley value and the core for this kind of fuzzy games are studied.

However, we only study the Shapley value for fuzzy games on vague sets and it will be interesting to study other payoff indices.

7 Acknowledgment

The authors gratefully thank the Chief Editor Prof. Panos Kostarakis and anonymous referees for their valuable comments, which have much improved the paper. This work was supported by the National Natural Science Foundation of China (Nos 70771010, 70801064 and 71071018).

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