Heat transfer and dynamics of the droplet on a superheated surface

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Abstract: - The conditions governing the collapse of a "Leidenfrost drop", e.g. a liquid drop supported by a vapour film on a heated surface was studied analytically. Robust analytical model for the phenomenon has been developed and numerical simulation has been done. The model was represented by two second-order non-linear differential equations for the radius of evaporating drop and its distance over the heated surface. The results obtained have shown the high-frequent oscillations of the drop over the hot plate until complete evaporation of the drop occurs. In contrast with existing precise complex models, the mathematical model developed was simple and could be used for the qualitative estimation of different parameters and quantitative estimation of the integral behaviours of the drop such as time for complete drop evaporation. The effects of surfactants on the Leidenfrost phenomenon and its industrial applications were discussed

Key-Words: - Hot Plate; Droplet; Evaporation; Vapour Film; Oscillation; Leidenfrost phenomenon

1 Introduction

During postulated severe accidents at the nuclear power plants (NPP) the high temperature molten core material is expected to encounter water. The melt-water interaction would then involve film boiling.

If the vapour film around discrete melting fragments collapse, the interaction with the water can result in steam explosions. Such events are of potential safety concern, partly due to the dynamic loading on reactor and/or containment structures, and partly due to the generation of fine debris particles, which could have a negative impact on the long-term coolability of the core material.

The situation is typical for many engineering and industrial applications where the drops of water are in contact with some hot surface and a key feature of the drops' evaporation is crucial in estimation of the heat fluxes at the surface and simulation of the cooling processes as a result. Therefore, an understanding of the peculiarities of a film-boiling phenomenon under such circumstances is of significant importance for many applications.

In this paper, the conditions governing the behaviours of a "Leidenfrost drop", for example, liquid drops supported by a vapour film on a heated surface up till their collapse are studied analytically. And the minimum film boiling temperature is interpreted here, as the minimum surface temperature required sustaining the vapour film. It is usually referred to as the "Leidenfrost temperature" or the "Leidenfrost point". When approaching a superheated surface, a droplet deforms and essentially changes its shape. Only the small droplets and the droplets of a liquid with surfactants (due to increased capillary forces) can keep nearly spherical form.

In general, arbitrary droplet shape can be described by using an infinite number of different spherical harmonics needed for the purpose. A closed theory of a droplet deformation requires, in turn, a special equation to be derived for each of the parameters.

The necessary conclusions in a tractable form are available for some pre-assigned special simple form to a droplet, for instance, a liquid disc of a constant volume. Then the disc radius R characterizes this disc as the only one unknown variable. And the disc thickness is then a function of a disc radius depending on a disc volume.

The addition of surfactants results in a drop, which can keep a spherical form and change the phenomenon dramatically due to decrease of a contact area between the drop and a superheated plate. Under such circumstances, the distance h separating the lower drop surface from the heated surface, which coincides with the vapour interlayer thickness (see Fig.1), is another unknown variable to be studied.

The dynamics and stability of a thin vapour layer moving between a drop placed on the hot plate and the hot plate as well is a key to the Leidenfrost phenomenon. The drop contacts the hot plate and starts the vaporisation process. The vapour is moving between a liquid drop and a plate causing the instability on the free liquid surface.

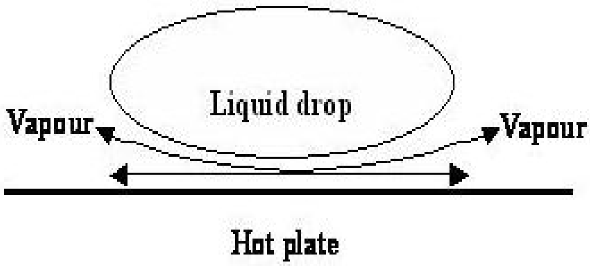

Fig. 1. Schematic process of a drop vaporisation on a hot plate

The modelling of the process is performed in correspondence with a schematic representation of the physical situation given in Fig. 1.

The liquid-vapour interface is prone to the different types of an instability being subjected to an influence of the diverse physical factors and parameters, e.g. temperature of the hot plate, drop size and physical properties (viscosity, surface tension, etc.).

The droplet's size and surface tension predetermine a regime of a vapour flow and a character of the instabilities at the interface. A drop with surfactants will be also subjected to the additional surface forces that are strongly depending on a surfactant concentration, which may vary from a point-to-point being, strictly speaking, chaotic.

The intensity of a vapour flow depends on a plate temperature and on a drop properties (size, density, capillary forces and viscosity), which, in turn, are also subjected to a temperature variation.

In the above-mentioned situation a drop is levitating over a hot plate due to a vapour flow and this, in turn, influences the vapour flow features. As far as a liquid drop interface is prone to various types of instability, the process becomes too complicated.

Thus, in this paper, the engineering robust mathematical model to the phenomenon described is first developed without accounting the drop's surface instabilities.

As an initial state of the system under this investigation, it is assumed that a drop is placed on a hot plate. And a hot plate has the temperature that is substantially higher than the temperature of the drop, which supposed to be close to the saturation temperature. Then depending on the temperature difference and on the initial drop/plate contact area, the vaporisation process and the drop oscillations are analysed.

2 The effect of surfactants on the Leidenfrost temperature

2.1 Leidenfrost phenomenon

J.G. Leidenfrost made the first observations of a film boiling of the discrete water drops on the hot surfaces in 1756.

Numerous studies of the Leidenfrost phenomenon have been published since then [1-12] but most attempts to obtain and correlate experimental data and to formulate the heat transfer and fluid dynamic mechanisms involved are comparably recent.

Reviews of those studies are given in [1-3]. Becker and Lindland [4] reported that in two metallurgical factories in Norway, which produce granulates of copper and ferrous alloys by pouring hot melts into water, surfactants have successfully been used for years to prevent steam explosions. Their hypothesis to explain this phenomenon was that the surfactant molecules act to stabilise the interface between the vapour film and the water. They suggested that surfactants could possibly suppress or prevent the steam explosions, which might occur in the core melt accidents.

2.2 "Leidenfrost temperature"

Wennerstrom et al. [5] studied experimentally the conditions governing a collapse of the "Leidenfrost drop" on a heated surface. In this case, the minimum film boiling temperature was interpreted, as the minimum surface temperature required sustaining the vapour film. It is usually referred to as the "Leidenfrost temperature" or the "Leidenfrost point".

The Leidenfrost temperatures for the water drops with and without surfactants placed on a heated horizontal plate have been measured at the atmospheric pressure using the two different methods. In the first of them, the drop evaporation times have been measured with the plate temperature kept constant.

This procedure was repeated for the different plate temperatures, and the Leidenfrost temperature was defined as the plate temperature that gave the maximum evaporation time. In the second method, the plate temperature was allowed to cool down until the drop collapsed. In this case, the Leidenfrost temperature was defined as the plate temperature prior to the collapse.

2.3 Influence of the surfactants on the Leidenfrost phenomenon

The addition of surfactants leads to a small decrease (2-5%) in the Leidenfrost temperature using the first method. When measuring the Leidenfrost temperature for small drops using the second method, no effect of surfactants added to the water could be found, but for larger drops the addition of surfactants resulted in a reduction (5-10%) in the Leidenfrost temperature.

This observation could be connected to the Marangoni effect, if it is assumed that the Rayleigh-Taylor instability is responsible for the collapse of the Leidenfrost drop.

It was also observed that a presence of the surfactants in water eliminated the oscillations of the small drops at the plate temperatures well above the Leidenfrost temperature. The experimental results obtained using the second method showed that the Leidenfrost temperature for the discrete drops increases with a diameter.

The effect of surfactants on the melt-water interactions, namely their influence on the minimum film boiling temperature, was pointed out by Baker et al. [6], and in the above-mentioned paper [5], for example.

2.4 The dynamic Leidenfrost phenomenon

The method used in Buyevich et al. [7,8] is based on giving consideration to the liquid disc deformation (see Fig.2) using the disc momentum and the energy conservation. Total liquid disc energy includes:

- the potential energy of surface tension
- the disc kinetic energy due to the translational motion of the disc as a whole and due to the axisymmetric flow inside the disc owing to its deformation.

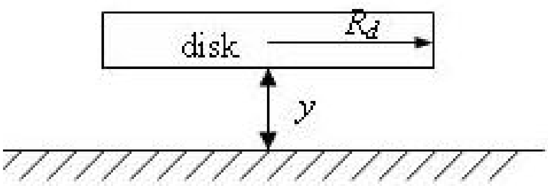

Fig. 2. Liquid disk schematic simplified model

The variation of the total energy during a small time interval *dt* identically equals the work of the repulsive force that is exerted at the lower disc boundary by pressure in the intervening vapor interlayer accomplished on corresponding displacement *dy*. This equality leads to one equation for the liquid disc's evolution.

The other needed equation directly follows from the Newton's second law. These equations determine both unknown variables, R_d and y.

The closing relation for the governing equations was an expression for a force f that owes its origin to excessive vapor pressure within the interlayer and retards the droplet as it comes closer to the surface. Such an expression follows from solving the hydrodynamic problem for vapor flow within the planar interlayer with a vapor source at the lower disc boundary.

Source intensity results from the requirement that all the heat transferred to this boundary from the hot surface is spent on evaporation. The authors supposed that:

- flow and heat transfer inside the vapor interlayer are quasistationary. This is approximately true if characteristic time scales y^2/v and y^2/a (v and a are vapour kinematic viscosity and heat diffusivity, respectively) are much smaller than the relevant time scale of the droplet motion near the surface, which is of the order of $y/|dy/dt|$;
- impinging droplets are heated up to the liquid boiling point.

They have derived the set of two strongly non-linear dynamic equations for the dimensionless droplet radius and distance from the hot plate:

$$2\left(1+\frac{8}{3x^6}\right)\frac{d^2x}{d\tau^2}-\frac{16}{x^7}\left(\frac{dx}{d\tau}\right)^2+x-\frac{1}{x^2}=\frac{x}{\eta^4}, \tag{1}$$

$$\frac{d^2\eta}{d\tau^2}-\frac{2}{x^3}\frac{d^2x}{d\tau^2}+\frac{6}{x^4}\left(\frac{dx}{d\tau}\right)^2=\frac{1}{8}\frac{x^4}{\eta^4},$$

where the dimensionless variables are: $x=R_d/R_0$, $\eta=y/L_h$, $\tau=t/L_t$ (L_h, L_t are the charcteristic length and time scales, respectively [8]). The numerical solution of the equation array (1) has shown an oscillating character of the drop on a hot surface.

3 Mathematical modelling of the Leidenfrost phenomenon

When a drop without surfactants is placed on the plate, it is deformed due to the gravitational forces acting on it. A drop with the surfactants is able to keep a spherical form due to an increase of the surface forces. Thus, depending on a surfactant concentration the drop form may vary from some kind of a disk to a spherical one. The initial drop's mass is computed as $m_d = 4/3\pi R_d \rho_d$, where R_d, ρ_d are the drop's radius and its density, respectively.

The most important temperatures in the case considered are the following: T_p, T_v, and T_d, which correspond to the hot plate, vapour between the drop and plate and the initial drop's temperature, respectively.

For the comparably small droplets and big plate, the bottom drop temperature is assumed to be approximately constant and kept at the saturation point as far as a heating of a small droplet is faster than its evaporation.

Then considering a levitating drop due to a vapour pressure, first the case of a small capillary force (absence of surfactants) may be considered when the capillary forces are not able to keep a spherical form of the drop.

In contrast, the drop with surfactants, by the same weight, keeps its spherical form and therefore is prone to a smaller heat flux from the plate being at the same distance over a plate but having a smaller cross section.

From the other point of view, because of a smaller cross section, a spherical drop has to be more in contact with a hot plate and due to this have more evaporation. This is why the drops with surfactants and the drops without them are quite different as concern to their oscillation over a hot plate due to the drops' evaporation.

Thus, the drop is levitating over the hot plate due to a vapour pressure, which supports the drop weight. The supporting force is equal to $p_v S_d$, where p_v is a vapour pressure, S_d is a bottom surface of the drop when it is deformed, and it is considered a middle cross section of a spherical drop in case of the surfactants or of the small droplets.

3.1 Physical assumptions for the model

Assumed that the plate temperature does not change substantially one can focus on the process of a drop evaporation, which leads up to a complete drop's evaporation.

Thus, the temperatures T_d and T_p are assumed to be constant. Then, due to a change of the drop's mass and instability of its levitation, the drop may oscillate over the plate. Now the vapour pressure can be estimated by the Clausius-Clapeyron equation:

$$\frac{d\ln(p_v / p_0)}{dT_v} = \frac{h_{fv}}{RT_v^2}, \qquad (2)$$

where p_0 is the vapour pressure under the normal conditions (temperature 373K and atmospheric pressure).

The vapour temperature changes in a thin vapour film between the plate and the drop from T_p to T_d (saturation temperature). In the first-order approach, it can be expressed as an average temperature $0.5(T_p + T_d)$ or, more precisely, calculated from the known heat conductivity solution.

3.2 Heat transfer process for the drop

The heat flux from the plate to the drop is expressed as

$$q = -k_v \frac{\Delta T}{y}, \qquad (3)$$

where $\Delta T = T_p - T_d$, and k_v is the heat conductivity of vapour.

For the drop without surfactants, as reported in the literature, y can be few times less at the edge of the drop comparing to the centre but the average gap's thickness can be adopted for computations. Now the pressure force acting on the drop is expressed as $p_v S_b$, where S_b is the bottom surface area of the drop.

For the drops with surfactants and for the small droplets, it is a half of the drop's surface open for the direct heat flux from a hot surface.

3.2.1 Vapour pressure calculation for a levitating drop

The vapour pressure p_v is computed from the equation (2) using the above-mentioned approximate temperature $0.5(T_p + T_d)$ for the vapour film flow.

If the drop is levitating without an oscillation, then this pressure supports the weight of a drop: $mg = p_v S_b$. But a drop is not symmetrical indeed and evaporation is not ideally regular; therefore a drop can oscillate over a hot plate due to the diverse above-mentioned perturbations and the Rayleigh-Taylor instability of a bottom surface of a drop (liquid over vapour represents the typical Rayleigh-Taylor instability). The capillary forces suppress this instability aiming to a minimal drop's surface, which is spherical in case of a substantial surface tension.

3.2.2 The drop's evaporation process

The other important question has concern to a drop's evaporation and to a consequent decrease of a drop's mass up to a drop's disappearance. Thus, the process is quite complex due to unstable multiphase flow and heat transfer with the alternating mass and unstable interface between a liquid and a vapour.

With account of the above-mentioned, the Newton's second law for the drop's dynamics is as follows:

$$\frac{d(mv)}{dt} = (p_v - p_a)S_b - mg,$$

or

$$\frac{d}{dt}\left(m\frac{dy}{dt}\right) = (p_v - p_a)S_b - mg, \qquad (4)$$

where $v = dy/dt$ is a velocity of the drop, S_b is a variable bottom drop's surface (due to evaporation and instability), *m* is a variable mass of the drop.

In the differential equation (4) thus obtained, except unknown function *y*, there are two other interconnected unknowns *m(t)* and $S_b(t)$. Thus, first of all, one needs to determine these two unknowns to close the equation (4).

From the energy balance can be got that an increase of a potential energy for the levitating drop is approximately equal to a heat flux from the plate to the drop:

$$g\frac{d(my)}{dt} = -\beta\frac{k_v(T_p - T_d)}{y}S_b, \qquad (5)$$

where the heat flux is projected on the middle cross section of the drop so that $S_b = \pi D^2/4$, $\beta \le 1$ is a proportionality coefficient.

The thermal energy is assumed to be converted to the potential energy of a levitating drop. But as far as the conversion from the thermal to potential energy will never be 100%, conversion efficiency is attached to the right hand side of the equation (5) as proportionality. The value of this coefficient β is subject to our further investigation. Here it is assumed that heat is spent completely on a drop's evaporation because it is going by a constant temperature.

Then, in turn, the heat of evaporation is transformed into a potential energy of the levitating drop through the vapour flow completely so that the limit case $\beta = 1$ is considered.

Actually some energy is lost on a dissipation due to a liquid flow inside the drop and due a vapour flow, etc., which is neglected in the first approach. Thus, this ideal model gives the overestimated drop's evaporation times.

3.2.3 Calculation of the drop's vapour mass flow

A vapour mass flow rate must be taken into account as

$$\frac{dm_v}{dt} = \pi D y \rho_v U_v, \qquad (6)$$

where the velocity of vapour flow U_v can be estimated with the Bernoulli equation:

$$U_v = \sqrt{2\frac{p_v - p_a}{\rho_v}}. \qquad (7)$$

Here p_a is the atmospheric pressure outside the vapour film.

The distance from the drop to the plate is considered as an averaged value because for the spherical drop it is obviously variable and for the deformed drop (say, big enough or without surfactants) there are experimental observations [9], which show that the drop is like a cup (at the edges of the drop a vapour film between the drop and plate is nearly seven times thinner than at the centre).

As far as a vapour is produced due to the drop's evaporation, it can be represented as

$$\frac{dm_v}{dt} = -\frac{dm}{dt}, \qquad (8)$$

where from follows after some calculation, with accounting the above-mentioned:

$$\frac{dR_d^2}{dt} = -\frac{y}{\rho_f}\sqrt{2\rho_v(p_v - p_a)}. \qquad (9)$$

This is an equation for the evolution of the radius R_d of a vaporising drop in time. Then the Clausius-Clapeyron relationship yields

$$Q_v = h_{fv} \frac{dm_v}{dt} , \qquad (10)$$

where from with account of the above-mentioned yields

$$h_{fv} \frac{dm_v}{dt} = -g \frac{d(my)}{dt} , \qquad (11)$$

and further, with account of the equations (6), (7) follows

$$g \frac{d(my)}{dt} = -h_{fv} \pi D y \rho_v \sqrt{2 \frac{p_v - p_a}{\rho_v}} . \qquad (12)$$

3.3 Mathematical model of the process

The equation (12) can be transformed to the following one:

$$g \frac{d(my)}{ydt} = -h_{fv} \pi D \sqrt{2\rho_v (p_v - p_a)}$$
$$= -2h_{fv} \sqrt{2\pi S_b \rho_v (p_v - p_a)} , \qquad (13)$$

which gives the relation between the drop's mass and its bottom surface area.

Now substituting (5) into (13) results in the following simple relation between the mass of a drop (or size, bottom surface area) and its distance from the hot plate:

$$\beta \frac{k_v (T_p - T_d)}{y^2} \sqrt{S_b} = 2h_{fv} \sqrt{2\pi \rho_v (p_v - p_a)} . \qquad (14)$$

The equation (14) shows that the drop is moving to the plate with a loose of mass due to vaporization and $m=0$ ($S_b=0$) in a limit approach by $y=0$.

The less the distance, the more intensive the heat flux from a plate to a drop, and consequently the less the drop due to its evaporation. Moreover, the mass decreases as a parabolic function of a distance from the plate.

The equation (14) yileds the following relation between the bottom surface area of a drop and the distance from the drop to the hot plate

$$S_b = 8\pi h_{fv}^2 \frac{\rho_v (p_v - p_a)}{\beta^2 k_v^2 (T_p - T_d)^2} y^4 . \qquad (15)$$

3.3.1 Statement of the basic equations and boundary conditions

The equations (14), (15) can be used for the approximate estimation of the parameters because they were obtained based on a simplification of the process, which is complex in dynamics.

In case of a spherical drop, accounting the equation (14) or (15), the momentum equation for the drop can be got from the equation (4). Then, accounting the equation (9), yields

$$\frac{d^2 y}{dt^2} - \frac{3\sqrt{2\rho_v (p_v - p_a)}}{2R_d^2 \rho_f} y \frac{dy}{dt} =$$
$$= 6R\rho_v T_v h_{fv}^2 \frac{\rho_v (p_v - p_a)}{\rho_f \beta^2 k_v^2 (T_p - T_d)^2} \frac{y^4}{R_d^3} - g ,$$
$$\frac{dR_d^2}{dt} = -\frac{y}{\rho_f} \sqrt{2\rho_v (p_v - p_a)} . \qquad (16)$$

The second-order non-linear differential equation array (16) thus obtained is slightly simpler than the equation array (1) of Buyevich at al. [7,8] for the modelling of impinging evaporating drops on the hot surface. Nevertheless the equation array (16) has also the similar non-linear oscillating solutions.

3.3.2 The characteristic length and characteristic velocity for the oscillating drop

Now the characteristic length is introduced as the initial vapour film thickness between the drop and the plate determined by the plate's and the drop's parameters, and the characteristic velocity is computed by the pressure difference between a vapour at the initial state and an atmosphere (the dynamic vapour influence to the drop).

Then the Cauchy problem for the differential equation array (16) is stated as follows:

$$t=0, \quad R_d = R_0, \quad y = y_0 = \sqrt{\frac{R_0 \beta k_v (T_p - T_d)}{2\sqrt{2} h_{fv} \rho_v u_0}} , \qquad (17)$$

$$\frac{dy}{dt} = u_0 = \sqrt{\frac{p_v - p_a}{\rho_v}}.$$

where R_0 is the initial radius of a drop.

The equation array (16) must be solved with the initial conditions (17).

3.3.3 The characteristic time for the process

The characteristic time for the process is taken as y_0 / u_0 so that

$$t_0 = \sqrt{\frac{R_0 \beta k_v (T_p - T_d)\sqrt{\rho_v}}{2\sqrt{2} h_{fv} (p_v - p_a)^{3/2}}}. \qquad (18)$$

3.4. Dimensionless formulation of the model (the Cauchy problem)

With the introduced characteristic scales of the process (17), (18), the Cauchy problem (16), (17) is transformed to the following dimensionless form:

$$\frac{d^2Y}{d\tau^2} - \frac{3\rho_v}{\sqrt{2}\rho_f}\frac{Y}{X^2}\frac{dY}{d\tau} = \frac{3\rho_v(1+\overline{p}_a)}{4\rho_f}\left(\overline{R}_0\right)^2\frac{Y^4}{X^3} - \frac{1}{\mathrm{Fr}},$$

$$\frac{dX^2}{d\tau} = -\sqrt{2}\frac{\rho_v}{\rho_f}Y; \qquad (19)$$

$$\tau = 0, \quad X = \overline{R}_0, \quad Y = 1, \quad dY/d\tau = 1. \qquad (20)$$

Here are:

$$\overline{R}_0 = R_0 / y_0 = 2\sqrt{2}(u_0 / w_v), \quad Y = y / y_0,$$

$$X = R_d / y_0, \quad \tau = t / t_0, \quad \overline{p}_a = p_a / (\rho_v u_0^2).$$

3.4.1 The Froude number and the drop's evaporation rate

The Froude number Fr and the drop's evaporation rate w_v are expressed as

$$\mathrm{Fr} = \frac{u_0^2}{g y_0} = \frac{(p_v - p_a)^{5/2}\sqrt{2\sqrt{2}h_{fv}}}{\rho_v^{3/4} g \sqrt{R_0 \beta k_v (T_p - T_d)}},$$

$$w_v = \frac{\beta k_v (T_p - T_d)}{h_{fv} \rho_v y_0}. \qquad (21)$$

3.4.2 Analysis of the dimensionless mathematical model

Now according to the above-mentioned and (17), (18), (21), the ratio of evaporation rate to the characteristic velocity of a vapour flow is proportional to the ratio of a drop radius to the characteristic vapour film thickness. This allows expressing the characteristic scales introduced with (17), (18) in the more compact form:

$$y_0 = \frac{R_0}{2\sqrt{2}}\left(\frac{w_v}{u_0}\right),$$

$$t_0 = \frac{1}{2\sqrt{2}}\left(\frac{R_0}{u_0}\right)\left(\frac{w_v}{u_0}\right). \qquad (22)$$

Analysis of the dimensionless mathematical model (19), (20) thus obtained shows that the process studied is completely determined by the density ratio (vapour to liquid), the ratio of the dynamic vapour head to the evaporation rate (or, the same, the ratio of a drop's radius to a vapour film thickness) and the Froude number.

The variation of the atmospheric pressure does not influence very much except the case when it is remarkable comparing to the vapour pressure.

4 Numerical simulation of the Leidenfrost phenomenon

Numerical simulation of the process has been done for the parameters given in the Table 1, where p_s is the saturation pressure by a corresponding temperature, H is the enthalpy, p_a =1,0133 bar (1bar=10^5 N/m^2), R =468,383 (m^2/Ks^2). h_{fv} =2000 kJ/kg.

The data presented in the Table 1, show availability for the following approximations:

$$\rho_v = \rho_v^0 (T_v / T_0)^{\alpha}, \quad k_v = k_v^0 (T_v / T_0)^{\beta},$$

$$H = H_0 (T_v / T_0)^{\gamma}, \quad p_s = p_s^0 (T_v / T_0)^{\varepsilon}, \qquad (23)$$

where the first formula in (23) has an accuracy of the same order as the experimental data presented, with α =-0,94567, the second one has the maximal inaccuracy of about 3%, which is also nearly of the same order as the data, with parameter β =1,2828; the accuracy of the third formula is less than 1% except the case of high temperatures when it grows up to 5%, and γ =0,33439.

The last approximation has the lowest accuracy with the deficiency up to 10% and

$$\varepsilon = 12.749 - 8.384(T_v / T_* - 1) - 0.384(T_v / T_* - 1)^2, \quad (24)$$

where T_* =400K, T_0 =373K, and all the parameters taken by this temperature are signed with zero indexes.

Table 1. Physical properties of the water steam depending on a temperature

T_v, K	ρ_v, kg/m^3	h_{fv}, kJ/kg	k_v, W /(mK)	p_v, bar	p_s, bar	u_0, m/s
373	0.58	2.67	0.025	1.013	1.0133	0
400	0.56	2.73	0.026	2.17	2.47	455
423	0.52	2.78	0.029	3.92	4.96	748
450	0.49	2.83	0.031	7.19	9.35	1123
500	0.44	2.93	0.036	18.6	26.47	2000
530	0.42	2.99	0.039	30.1	44.69	
550	0.40	3.03	0.041	40.3	61.34	

If the specific heat of an evaporation does not depend on the temperature, the Clausius-Clapeyron equation (2) gives the following solution with the initial conditions $T_v = T_0$, $p_v = p_0$:

$$p_v = p_0 e^{h_{fv}/R(1/T_0 - 1/T_v)}. \quad (25)$$

The results of the calculations by (25) are presented in the Table 1.

4.1. Numerical simulation of the drop's oscillations over hot plate

The mathematical model thus obtained was used for the numerical simulation in a dimensionless form (19), (20). The results of computer computation are shown in Figs. 3-12.

The Figs 3-5 represent correspondingly the drop's radius, the distance between a plate and a drop and the velocity of a drop in the whole range of the time (τ =0; 0.062) up till the drop's collapse (τ =0.062). Then the other three different time scales are applied, which differ nearly ten times each other, respectively. For example, the Figs 6-8 correspond to the temporal interval (τ =0; 7*10^{-3}).

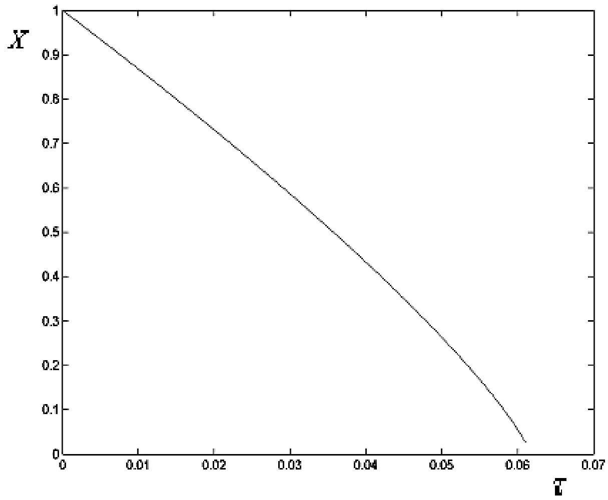

Fig. 3. Decrease of dimensionless drop's radius with time

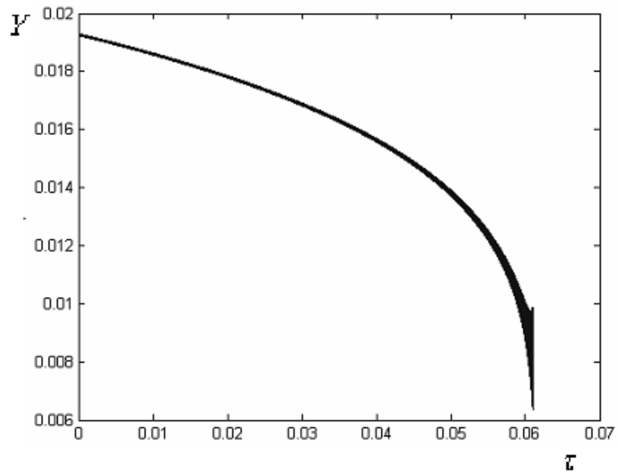

Fig. 4. Dependence of dimensionless distance between the drop and hot surface with time

As shown in Figs 7-12, oscillations of the drop are substantial and must be taken into account for calculation of the heat transfer on the hot surface cooled down by a number of distributed by size ensemble of the drops and in other similar physical situations.

The heat flux from each of the drops can be computed based on the results obtained for the interval of complete drop evaporation. Afterwards such data are applied for averaging the total heat flux using the data on size distribution of the drops.

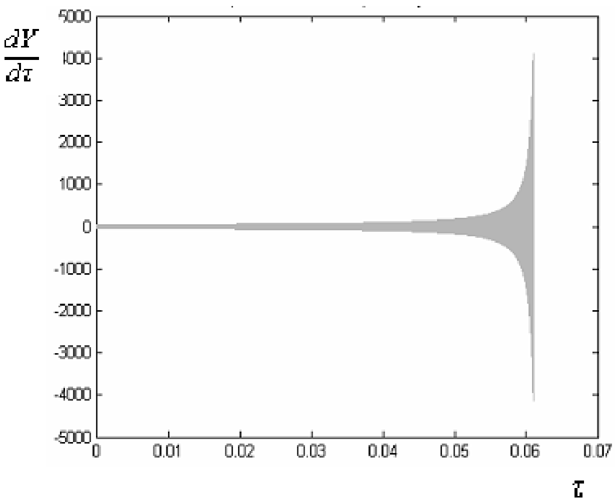

Fig.5. Velocity of a drop oscillation against time

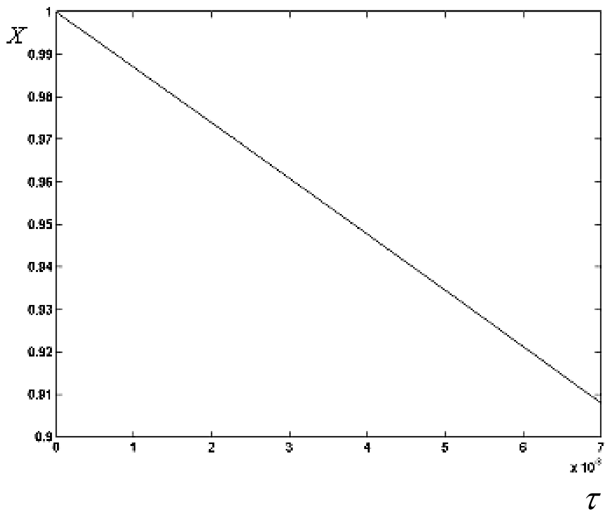

Fig. 6. Drop's radius against time at the initial stage

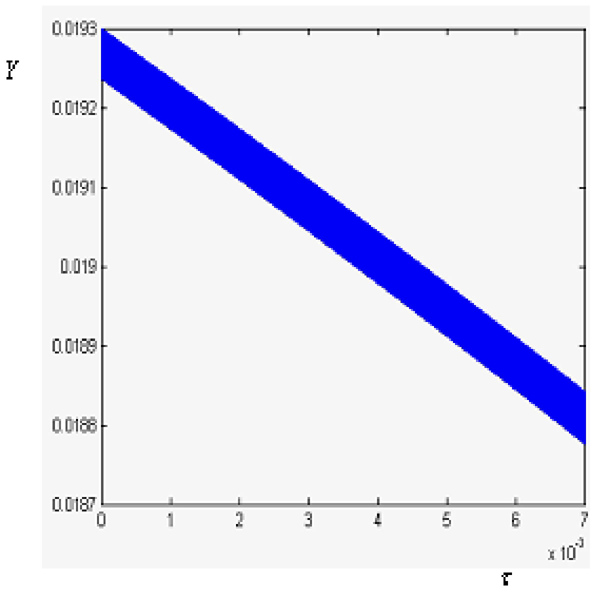

Fig.7. Dimensionless distance drop-to-surface with time

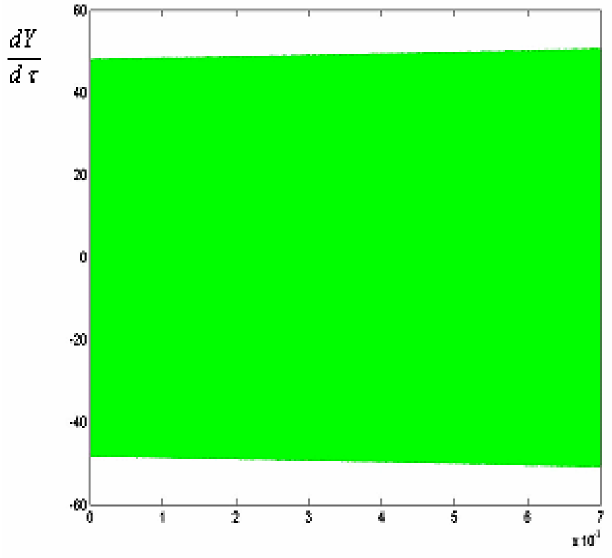

Fig. 8. Velocity of a drop oscillation against time

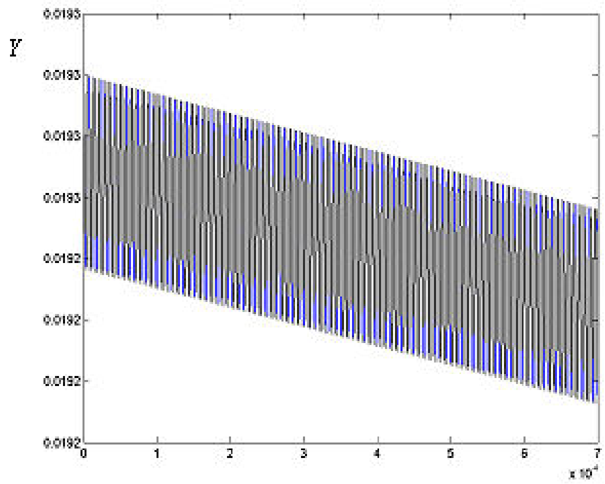

Fig.9. Dimensionless distance drop-surface with time

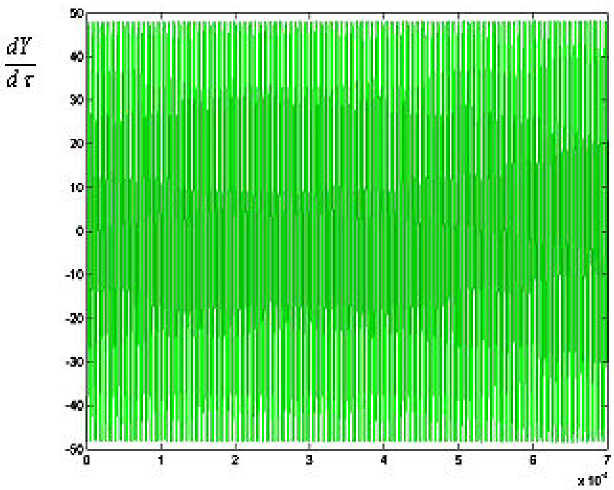

Fig. 10. Velocity of a drop oscillation against time

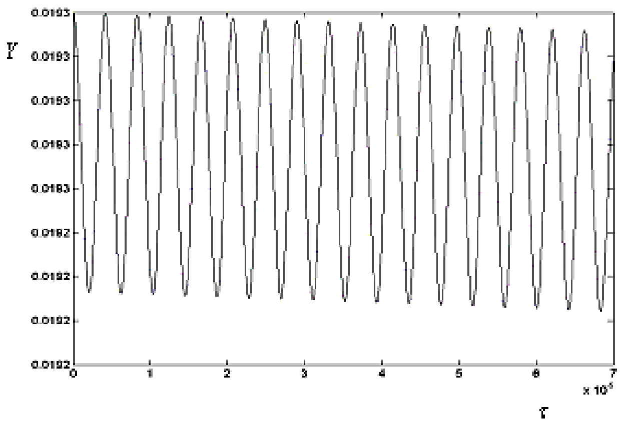

Fig.11. Dimensionless distance between the drop and hot surface with time

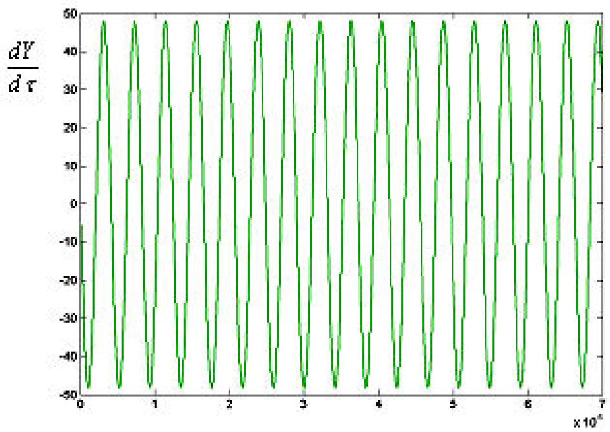

Fig. 12. Velocity of a drop oscillation against time

The drop's radius is a slightly non-linear function of time in any time scale applied, while the film thickness and the velocity of the drop are oscillating that becomes more and more visible in the next two scales, τ =0; 7*10^{-4} and τ =0; 7*10^{-5}, respectively, implemented in the Figs 9, 10 and in the Figs 11, 12.

4.2. Frequency of the drop's oscillations over a hot plate

As shown in the figures, the dimensionless frequency of oscillations is Ω =2.5*10^5, where from the dimensional frequency of the drop oscillations is computed as $\omega = \Omega / t_0$.

4.3. Comparison to the Buyevich's results on the drops in a flow around a hot plate

In contrast with the results of Buyevich et al. [7, 8], the drop's oscillations are high-frequent here because the drop is placed on a hot plate while in the case considered in [7, 8] the drops are moving in the flow and have the momentum (the drops move to the plate and due to this they oscillate with big amplitudes comparing to the case studied here where the drops are oscillating only due to their intensive evaporation).

4.4. Peculiarities of the drop's oscillations over hot plate

Thus, as shown in figures, the radius of the drop is decreasing monotonically in time up to the complete drop disappearance due to evaporation, while the distance of the drop over surface of the heated plate and the drop's velocity are the high-frequent oscillating functions of time.

Precise model of droplet impacts on hot surfaces and computer code for its numerical simulation were developed in [10, 11]. The model is good for investigation of the physical behaviours of the drop on hot surface but it is complicated for engineers and does not allow computing the complete evaporation time for a drop. To use this model one needs to get a computer code from the developers or program it oneself.

4.5. The observations of the small drop's oscillations over hot plate

Some interesting observations were made for small drops with the radius less than 2.5 mm at plate temperatures above 300°C (i. e., above the Leidenfrost temperature) in [5] using a pure water.

The drops tended to oscillate rather markedly in the vertical direction. They moved approximately I-2 mm up and down, with a frequency that was clearly visible.

However, when a small amount of surfactants (2-10 ppm) was added to the water, no such oscillations were observed, and the drops remained perfectly calm. These observations indicate that surfactants might have a stabilising effect on small drops in the film boiling region.

Despite of a number of theoretical and experimental papers on the Leidenfrost phenomenon, the authors did not find yet the data to validate the model completely, therefore this question remains open for further investigation.

4.6. The effect of surfactants on a dynamics of the Leidenfrost drop

The most important and complex problem in the Leidenfrost drop's dynamics is a surface tension coefficient for the liquid with surfactants. This predetermines the drop bottom surface area in a contact with a hot plate; therefore uncertainties with the surface tension coefficient are of paramount interest for the phenomenon studied. For the engineering calculations, the Kutateladze correlation [12] can be adopted:

$$\frac{q_{cr}}{h_{fv}\rho_v}\left(\frac{\rho_f}{g\sigma}\right)^{1/4} = \mathrm{K}\left(\frac{\rho_f}{\rho_v}\right)^{1/2}\left(1-\frac{\rho_v}{\rho_f}\right)^{1/4}, \quad (26)$$

where the Kutateladze constant for was approximately estimated at K=0.16 by many experimental data [12].

5 Conclusion

The high-frequent oscillating character of the drop's evaporation on the heated surface was revealed and modelled.

The results obtained showed reasonable correlation with the experimental data [5] by the time of a complete drop evaporation.

But a number of uncertainties still exist in the problem, which requires more detail investigation. For example, there are unclear questions concerning the influence of surfactants, interfacial instability of the drop's surface, etc.

These and other peculiarities of the Leidenfrost phenomenon are a subject for the further investigations, which are of interest for many industrial and technical applications.

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