Second Order Slip Flow of Cu-Water Nanofluid Over a Stretching Sheet With Heat Transfer

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Abstract: - The boundary layer flow of Cu-water based nanofluid with heat transfer over a stretching sheet is numerically studied. Second order velocity slip flow model is considered instead of no-slip at the boundary. The governing partial differential equations are transformed into ordinary one using similarity transformation, before being solved numerically. Numerical solutions of these equations are obtained using finite element method (FEM). The variations of the velocity and temperature distribution as well as the skin friction and the heat transfer coefficients for some values of the governing parameters, namely, the nanoparticle volume fraction and slip parameters are shown graphically and discussed. Comparison with published results for pure fluid flow case is presented and it is found to be in excellent agreement.

Key-Words: - Nanofluid, Stretching surface, heat transfer, FEM, velocity slip condition

1 Introduction

The boundary layer flow over a stretching sheet plays an important role in aerodynamic, extrusion of plastic sheet, metal-spinning, manufacture of plastic and rubber sheets, paper production etc and thus, remains at the leading edge of technology development. In the industrial operation, metal or more commonly an alloy, is heated until it is molten, whereupon it is poured into a mould or dies which contains a cavity, of required shape. The hot metal issue from the die is subsequently stretched to achieve the desired product. When the super heated melt issue comes out from the die, it loses its heat and contract as it cools, this is referred as liquid state contraction. With further cooling and loss of latent heat of fusion, the atoms of the metal lose energy and become closely bound together in a regular structure. The quality of the final product greatly depends on the rate of cooling and the process of stretching.

In view of such applications, Crane [1] initiated the analytical study of boundary layer flow due to a stretching sheet. He assumed the velocity of the sheet to vary linearly as the distance from the slit and obtained an analytical solution. After this pioneering work, the flow over a stretching surface has drawn considerable attention and a good amount of literature in different field has been generated on this problem [2-10]. In these studies the no slip of the fluid velocity relative to the solid boundary was considered.

It is a well-known fact that, a viscous fluid normally sticks to the boundary. But, there are many fluids, e.g. particulate fluids, rarefied gas etc., where there may be a slip between the fluid and the boundary [11-12]. Wang [13] reported that the partial slip between the fluid and the moving surface may occur in particulate fluid situations such as emulsions, suspensions, foams and polymer solutions. Fang et al. [14] gave a closed form solution for slip MHD viscous flow over a stretching sheet. Wang [15] investigated the effect of surface slip and suction on viscous flow over a stretching sheet. Sajid et al. [16] analyzed the stretching flow with general slip condition. Sahoo [17] investigated the flow and heat transfer solution for third grade fluid with partial slip boundary condition. Bhattacharyya et al. [18] analyzed the boundary layer force convection flow and heat transfer past a porous plate embedded in the porous medium with first order velocity and temperature slip effect. Das [19] examined the influence of partial slip, thermal radiation, chemical reaction and temperature-dependent fluid properties on heat and mass transfer in hydro-magnetic micropolar fluid flow over an inclined permeable plate with constant heat flux and non-uniform heat source/sink. Das [20] examined the effects of partial slip, thermal buoyancy and heat generation/absorption on the flow and heat transfer of nanofluids over a permeable stretching surface. Noghrehabadi et al. [21] analyzed the effect of partial slip on flow and heat transfer of nanofluids past a stretching sheet. Zheng et al. [22] analysed the effect of velocity slip with temperature jump on MHD flow and heat transfer over a porous shrinking sheet. However, in all of these papers, only the first order Maxwell slip condition was considered. Recently, Wu [23] proposed a new second order slip velocity model. Fang et al. [24] analyzed the effect of second order slip on viscous fluid flow over a shrinking sheet. Fang and Aziz [25] studied the flow of a viscous fluid with a second order slip over a stretching sheet without considering the heat transfer aspect. Nandeppanavar et al. [26] studied the second order slip flow and heat transfer over a stretching sheet.

To the best of the authors knowledge, no information available on the effect of second order slip on the flow and heat transfer of a nanofluid past a stretching sheet. Therefore, in the present paper, we investigate the effect of second order slip on the flow and heat transfer over a stretching sheet immersed in a Cu-water nanofluid. The nanofluid model proposed by Tiwari and Das [27], which analyze the behavior of nanofluids taking into account the solid volume fraction is employed. The mathematical model of the problem is highly non-linear whose analytical solution is very hard to find out, so the only choice left is approximate numerical solution. Therefore, in this study, finite element method is used as a tool for the numerical simulation. Numerical results of the local skin friction coefficient and the local Nusselt number as well as the velocity and temperature profiles are presented for different values of the physical parameters.

2 Problem formulation

Consider the two-dimensional flow over a flat sheet with heat transfer in a water based nanofluid containing Cu nanoparticles. We assume that the sheet coincides with the plane $y = 0$ and the flow is confined to $y > 0$. Two equal and opposite forces are applied along the x- axis so that the wall is stretched keeping the origin fixed. It is assumed that the sheet is stretched with velocity $u_w = cx$, where $c > 0$ is the stretching rate. It is also assumed that the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium. The thermo-physical properties of the water and Cu are given in table 1. Assuming that the nanofluid is viscous and incompressible, and using the nanofluid model as proposed by Tiwari and Das [27], the governing boundary layer equations of mass, momentum and thermal energy for nanofluids can be written as (see Tibari and Das [27]),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{nf}\left(u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}\right) = \mu_{nf}\,\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$(\rho c_p)_{nf}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_{nf}\,\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where y is the coordinate measured in the direction normal to the sheet, u and v are the velocity components along the $x-$axes and $y-$axes, T is the nanofluid temperature, ρ_{nf} is the effective density of the nanofluid, μ_{nf} is the effective dynamic viscosity of nanofluid, k_{nf} is the thermal conductivity and $(\rho c_p)_{nf}$ is the heat capacity of the nanofluid, which are given by

$$\begin{gathered}\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,\, \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \\ (\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s \\ \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}\end{gathered} \tag{4}$$

Table 1. Thermo-physical properties of water and Cu

ϕ	ρ $(kg\, m^{-3})$	c_p $(J\, Kg^{-1} K^{-1})$	k $(W m^{-1} K^{-1})$
Pure water (H_2O)	997.1	4179	0.6130
Copper (Cu)	8933	385.0	401.0

where ϕ is the solid volume fraction of the nanofluid, ρ_f is the density of the base fluid, ρ_s is the density of the nanoparticle, μ_f is the dynamic viscosity of the base fluid, $(\rho c_p)_f$ is the heat capacity of the base fluid, $(\rho c_p)_s$ is the heat capacity of the nanoparticle, k_f is the thermal conductivity of the base fluid and k_s is the thermal conductivity of the solid nanoparticle.

Eqs. (1) - (3) are subjected to the following boundary conditions:

$$u = u_w + U_{slip}, \quad v = 0, \quad T = T_w = T_\infty + C\left(\frac{x}{l}\right)^2 \text{ at } y = 0 \tag{5}$$

$$u = 0, \quad T = T_\infty \quad \text{as} \quad y \to \infty$$

where U_{slip} is the slip velocity at the wall. The Wu's slip velocity model (valid for arbitrary Kundsen numbers, K_n) is used in this paper and is given as follows [23]

$$\begin{aligned} U_{slip} &= \frac{2}{3}\left(\frac{3-\alpha l^3}{\alpha} - \frac{3}{2}\frac{1-l^2}{K_n}\right)\lambda\frac{\partial u}{\partial y} \\ &\quad - \frac{1}{4}\left(l^4 + \frac{2}{K_n^2}\left(1-l^2\right)\right)\lambda^2\frac{\partial^2 u}{\partial y^2} \\ &= A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2} \end{aligned} \tag{6}$$

Following Nandeppanavar et al. [26], we introduce the following similarity transformation:

$$u = cx\, f'(\eta), \quad v = -(c\nu_f)^{1/2}\, f(\eta), \quad \eta = y\sqrt{c/\nu_f}, \quad \theta(\eta) = (T - T_\infty)/(T_w - T_\infty) \tag{7}$$

where primes denote differentiation with respect to η. Using transformation (7), Eq. (1) is automatically satisfied, while Eqs. (2) and (3) respectively reduce to the following nonlinear ordinary differential equations:

$$\frac{\mu_{nf}}{\mu_f} f''' + \left((1-\phi) + \phi\frac{\rho_s}{\rho_f}\right)(ff'' - f'^2) = 0 \tag{8}$$

$$\frac{k_{nf}}{k_f}\theta'' + \Pr\left((1-\phi) + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\left(f\,\theta' - 2\,f'\theta\right) = 0 \tag{9}$$

subject to the boundary conditions

$$\begin{aligned} &f(0) = 0,\ f'(0) = 1 + \gamma\, f''(0) + \delta\, f'''(0),\ \theta(0) = 1 \\ &f'(\infty) \to 0, \quad \theta(\infty) \to 0 \end{aligned} \tag{10}$$

where $\gamma = A\sqrt{c/\nu_f}\ (>0)$ is the first order velocity slip parameter, $\delta = B(c/\nu_f)\ (<0)$ is the second order velocity slip parameter and $\Pr = \mu_f / k_f$ is the Prandtl number.

Physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu, which are defined as

$$C_f = \frac{\tau_w}{\rho_f\, u_w^2}, \quad Nu = \frac{q_w}{k_f\,(T_w - T_\infty)} \tag{11}$$

where τ_w is the surface shear stress and q_w is the surface heat flux, which are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

Using the similarity variables (5), we obtain

$$\mathrm{Re}_x^{1/2} C_f = \frac{1}{(1-\phi)^{2.5}} f''(0), \mathrm{Re}_x^{-1/2} Nu = -\frac{k_{nf}}{k_f}\theta'(0) \tag{13}$$

where $\mathrm{Re}_x = u_w\, x / \nu_f$ is the local Reynolds number.

3 Method of Solution

The set of ordinary differential equations (8)-(9) are highly non-linear, and cannot be solved analytically. Therefore, the finite element method [28-30] is implemented to solve this system numerically. For computational purposes, the dimensionless spatial coordinate is discretized by uniform elements of step size $h = 0.01$. Care has been taken in choosing η_∞ for a given set of parameters because for a fixed value of η_∞ (where η_∞ corresponds to $\eta \to \infty$) for all calculations may produce inaccurate results.

The Gauss quadrature formula has been used to calculate the integrals. Owing to the nonlinearity of the system of equations an iterative scheme has been used to solve it. An initial guess is taken at each node point. The system of equations is then linearized by incorporating the functions, which are assumed to be known values of the functions f and θ. After applying the given boundary conditions, the remaining system of equations has been solved using Gauss-elimination method. This gives us new values of unknowns. This process continues till the absolute differences of two successive iterate value of unknowns is less than the accuracy of 0.0001.

4 Code verification

In order to verify the accuracy of the applied numerical scheme, comparisons of the present results corresponding to the values of heat transfer coefficient for $\gamma = 0$, $\delta = 0$ and $\phi = 0$ for prescribed surface temperature case are made with the available results of Grubka and Bobba [3] and Chen [4], as presented in Table 2. The results are found in excellent agreement, and thus gives confidence that the numerical results in our case are accurate.

Table 2. Comparison with previous studies for $\{-\theta'(0)\}$ $\gamma = \delta = \phi = 0.0$

Pr	Grubka & Babba [3]	Chen [4]	Present
0.72	1.0885	1.08853	1.08891
1.00	1.3333	1.33334	1.33330
3.00	2.5097	2.50972	2.50960
7.00	----	3.97150	3.97120
10.0	4.7969	4.79689	4.79640
100	15.712	15.7118	15.70127

Table 3. Reduced skin friction coefficient and reduced Nusselt number for various value of ϕ

ϕ	Skin friction	Nusselt Number
0.0	-0.0712	1.5310
0.1	-0.0831	1.5618
0.2	-0.1129	1.7245

Table 4. Reduced skin friction coefficient for various value of γ and δ

γ	$\delta = -0.5$	$\delta = -1.0$	$\delta = -2.0$	$\delta = -3.0$
0.5	-0.2718	-0.0875	-0.0375	-0.0221
1.0	-0.2145	-0.0831	-0.0366	-0.0218
2.0	-0.1582	-0.0756	-0.0350	-0.0212
3.0	-0.1303	-0.0695	-0.0335	-0.0207

Table 5. Reduced Nusselt number for various value of γ and δ

γ	$\delta = -0.5$	$\delta = -1.0$	$\delta = -2.0$	$\delta = -3.0$
0.5	2.3158	1.5892	1.1954	0.9992
1.0	2.1411	1.5618	1.1861	0.9945
2.0	1.9351	1.5133	1.1681	0.9853
3.0	1.8144	1.4713	1.1516	0.9763

5 Results and discussion

A systematic study of selected control parameters governing the flow regime i.e. nanoparticle concentration (ϕ), first order slip parameter (γ) and second order slip parameter (δ) has been conducted and the results are depicted in Figs. 1-10 and Tables 3-5. In the present computations the following default parameter values have been prescribed: $\phi = 0.1$, $\gamma = 1.0$, $\delta = -1.0$, $\Pr = 6.2$ (water).

Variations of the reduced skin friction coefficient and the reduced Nusselt number with respect to nanoparticle concentration(ϕ), first order slip parameter (γ) and second order slip parameter (δ) are shown in Tables 3-5. It is observed that the reduced skin friction coefficient is an increasing function of nanoparticle concentration(ϕ), whereas it a decreasing function of first order slip parameter (γ) and second order slip parameter (δ). The heat transfer rate is also an increasing function of nano particle concentration (ϕ) and a decreasing function of first order slip parameter (γ) and second order slip parameter(δ). The positive value of the reduced Nusselt number shows that the heat is transferring from the plate to the fluid i.e. cooling of the plate. Thus, it can be concluded that the nanofluid can be effectively used for the fast cooling of the plate, while slip effect slow down the heat transfer rate.

Figs. 1-2 depict the effect of nanoparticle concentration (ϕ) on the variation of velocity and temperature in the boundary layer. On observing these figures, we see that velocity decreases and temperature increases in the boundary layer region with the increase of nanoparticle concentration. It has been found that when the volume fraction of the nanoparticle increases from 0 to 0.2, the thickness of the thermal boundary layer increases (See Fig. 2). Since nanoparticle enhanced the thermal conductivity of the fluid, and higher values of thermal conductivity are accompanied by higher values of thermal diffusivity. The high value of thermal diffusivity causes a drop in the temperature gradients and accordingly increases the boundary layer thickness as demonstrated in Fig. 2. As temperature gradient decreases with the increase of nanoparticle fraction, so Nusselt number i.e. rate of heat transfer should also reduce with the increase of nanoparticle fraction from 0 to 0.2. But in case of nanofluid, the Nusselt number is a multiplication of temperature gradient and the thermal conductivity ratio (conductivity of the nanofluid to the conductivity of the base fluid). The reduction in temperature gradient due to the presence of nanoparticles is much smaller than thermal conductivity ratio, which accompanied the enhancement of Nusselt number by increasing the volume fraction of nanoparticles, as it can be seen from Table 3.

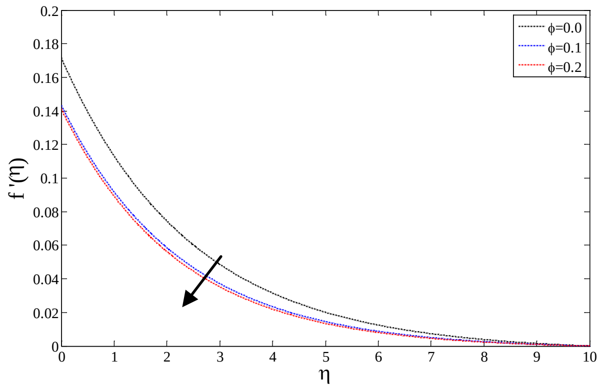

Fig. 1 Velocity distribution for various value of ϕ

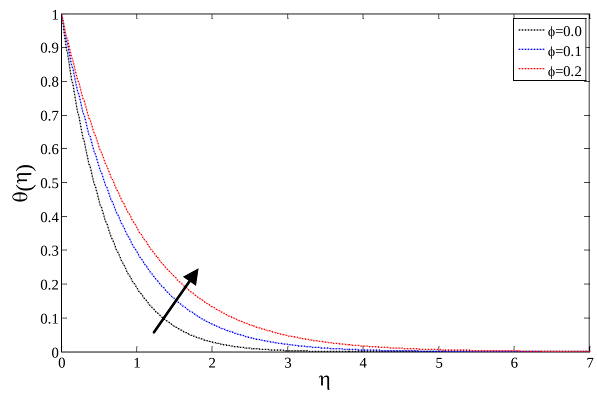

Fig. 2 Temperature distribution for various value of ϕ

Figs. 3-6 show the effect of first order slip parameter on velocity, temperature, shear stress and effective temperature gradient in flow field. It is seen that for increased first order slip the lateral velocity decreases near the surface but increases at large distance. Thus, first order slip of fluid on the stretching surface causes decrease in flow velocity. Fig. 4 shows that temperature of flow field increases with the increase of first order slip parameter. These results are in very good agreement with reported results in partial slip case by Mahmoud [19] and Noghrehabadi [21]. It is observed from Figs. 5-6 that the magnitude of the wall shear stress i.e. $(1/(1-\phi)^{2.5}) f''(0)$ decreases with the increase of first order slip parameter and the value of the effective temperature gradient at the wall i.e. $-(k_{nf}/k_f)\theta'(0)$ is also decreases with the increase of first order slip parameter, which is in agreement with the results presented in Table 4-5.

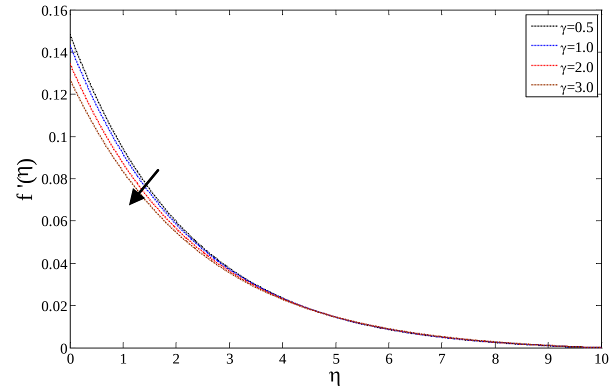

Fig. 3 Velocity distribution for various value of γ

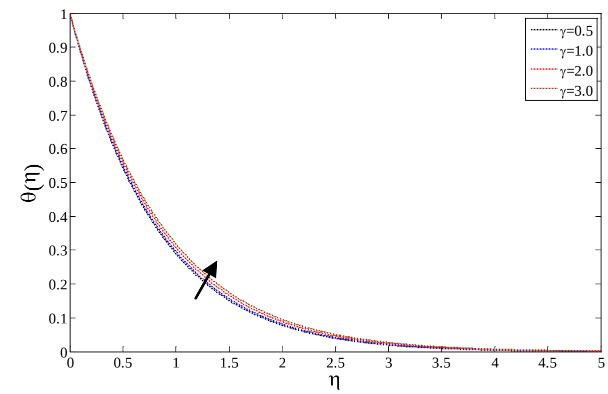

Fig. 4 Temperature distribution for various value of γ

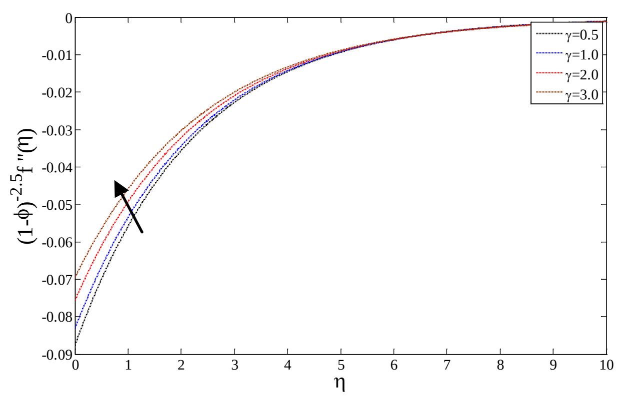

Fig. 5 Shear stress distribution for various value of γ

Figs. 7-10 depict the influence of second order slip parameter on the velocity, temperature, shear stress and effective temperature gradient profile in boundary layer region. From these figures, we observe that velocity decreases and temperature increases with the increase of second order slip parameter $|\delta|$. The magnitude of the wall shear stress and effective temperature gradient at the wall decrease with the increasing value of $|\delta|$.

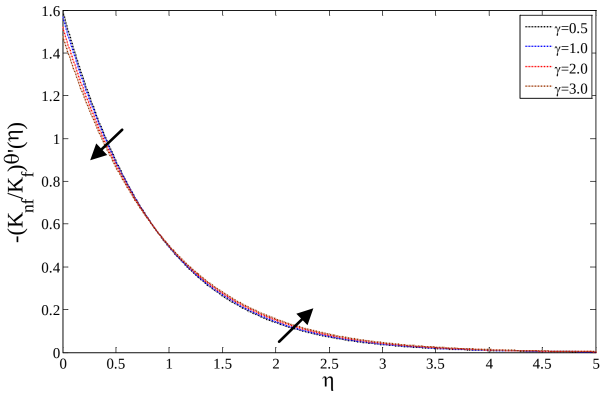

Fig. 6 Temperature gradient distribution for various value of γ

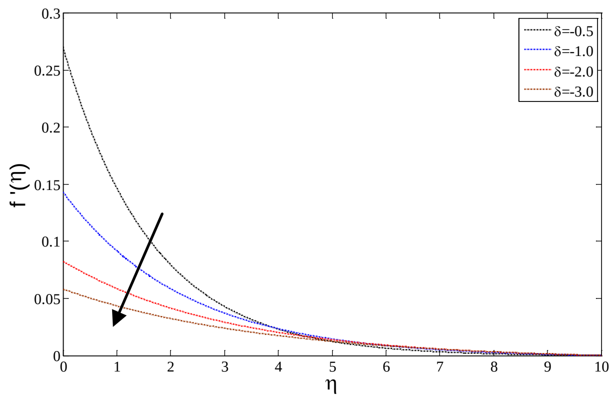

Fig. 7 Velocity distribution for various value of δ

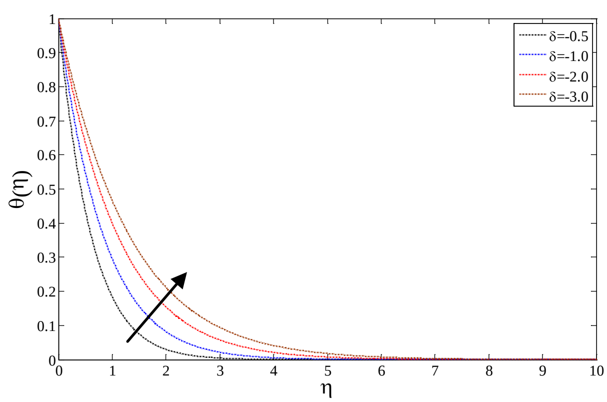

Fig. 8 Temperature distribution for various value of δ

6 Conclusions

In this paper, the problem of two-dimensional flow of a viscous and incompressible Cu-water nanofluid over a stretching flat sheet with second order slip conditions is studied. The governing partial differential equations for mass, momentum and energy are transformed into ordinary differential equations using a similarity transformation. These equations were solved numerically using finite element method. We found that first order and second order slip parameters reduce the skin friction as well as the rate of heat transfer. The results also indicate that with the increase of nanoparticle volume fraction, skin friction coefficient as well as heat transfer rate increases.

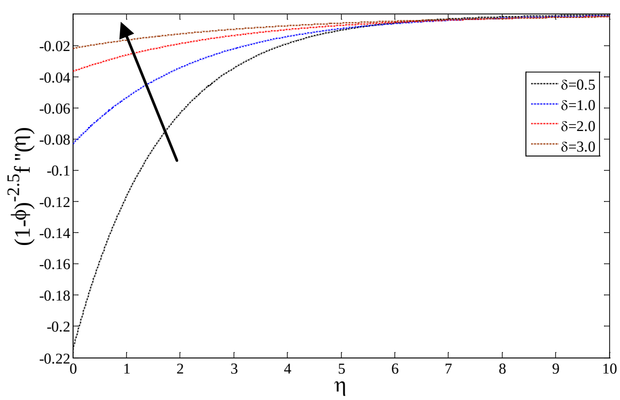

Fig. 9 Shear stress distribution for various value of δ

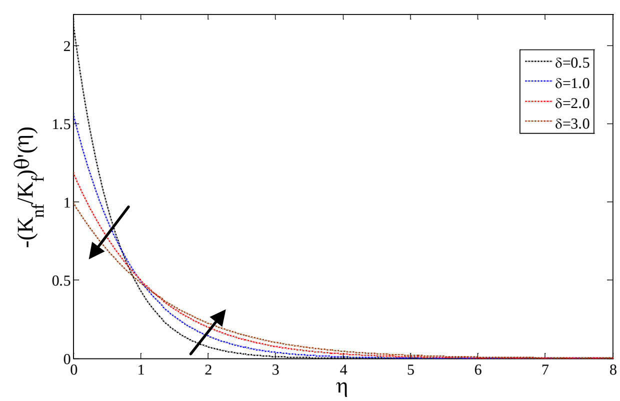

Fig. 10 Temperature gradient distribution for various value of δ

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