

# Multilateral Bargaining in the Vertically Differentiated Market

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*Abstract* : In this paper, we examine the bargaining strategy of a distributor who sells vertically differentiated, i.e. high and low brand products. In this multilateral bargaining the negotiations are interdependent due to bargaining externality. We derive and analyze the equilibrium solutions for both simultaneous and sequential bargaining games between the distributor and the manufacturers of the high and low brand products. The results shows that the optimal bargaining strategy for the distributor heavily depends on the relative quality and price level of the low brand product comparing to those of the high brand product. It is also shown that, for more bargaining profit, the distributor has strong incentive to prefer a low brand product which has lower quality level per unit price.

*Keywords*: Multilateral bargaining, Vertically differentiated Market, Simultaneous bargaining, Sequential bargaining, Nash equilibrium solution

## 1. Introduction

It is common for a distributor to arrange vertically differentiated products for more market coverage and profit. Such vertical product arrangement involves multilateral negotiations between the distributor and the manufacturers of the vertically differentiated products. Most of the previous literature on the vertically differentiated market can be classified into pricing [4] and determination of the quality level of the low brand products [5][8][9][11]. Although a great deal of research had been made about the vertically differentiated market with respect to these categories, studies investigating the bargaining process between the distributor and the manufacturers of the vertically differentiated products are rare, suggesting a need for further research that addresses this issue. This study seeks to fill such need.

This study addressed the multilateral bargaining strategy of a distributor who deals with vertically differentiated products. The significant feature of such multilateral bargaining is that the bargaining outcomes between the distributor and the manufacturers are interdependent. In other words, the bargaining outcome between the distributor and a

manufacturer can heavily influenced by the bargaining outcome between the distributor and the other competing manufacturer. In this study, we analyzed the effect of the above mentioned externality on the bargaining strategy of a distributor in a vertically differentiated market context.

The results show the optimal bargaining strategy for the distributor heavily depends on the relative quality and price level of the low brand product comparing to those of the high brand product. It is also shown that the distributor has strong incentive to prefer a low brand product which has lower quality level per unit price

The remainder of this paper is as follows. In section 2, a theoretical bargaining model along with the assumptions for the market used in this study. In section 3, the Nash equilibria for both simultaneous and sequential bargaining games are derived and analyzed. Section 4 discusses the optimal bargaining strategy for the distributor, and section 5 concludes this study with some further research area.

## 2. Bargaining Model

This study investigates the supply price bargaining strategy of a distributor(D) with the high brand product manufacturer(H) and the low brand product manufacturer(L). We assume, without loss of generality, that the production costs of both high and low brand products are zero. We also assume that retail pricing decisions precede the bargaining for supply prices. This assumption is consistent with the market situations where distributors compete and secure consumer orders with committed prices before negotiating with suppliers, i.e. manufacturers [6]. The bargaining model of supply price  $w = (w_h, w_l)$  between a distributor and the manufacturer  $i$  ( $i = h$  or  $l$ ) is given in the equation (1) [10].

$$\begin{aligned} & \text{Max}_{w_i} [\pi_D(w) - \pi_{D_i}^0(w)]^\alpha \times [\pi_i(w) - \pi_i^0(w)]^{1-\alpha} \\ \text{s.t. } & \pi_D(w) \geq \pi_{D_i}^0(w) \text{ and } \pi_i(w) \geq \pi_i^0(w). \quad (1) \end{aligned}$$

In equation (1),  $\pi_D(w)$  and  $\pi_i(w)$  represent the expected profits of the distributor and the manufacturer  $i$  ( $i = h$  or  $l$ ), respectively;  $\pi_{D_i}^0(w) = \pi_D(w_i = \otimes, w_j)$  and  $\pi_i^0(w) = \pi_i(w_i = \otimes, w_j)$  represent the expected profits of the distributor and the manufacturer  $i$  when the negotiation with the manufacturer  $i$  has been broken off (denoted as  $w_i = \otimes$ );  $\alpha$  denote the market power of the distributor over the manufacturers. In this study we assume that  $\pi_i^0(w) = 0$  under the market assumption that the manufactures sell their products only through the corresponding distributor.

To analyze the bargaining game in equation (1), we need to know the sales volume of each product for each scenario of bargaining result. The bargaining between the distributor and the manufacturer may result in either success or failure. If the bargaining of one manufacturer fails, some customers of the corresponding product may switch to the rival product. Thus the demand of each product is affected not only by the result of its own bargaining but by the result of the rival's bargaining. Let us determine the sales volume of each product based on the concept of surplus value of the customer. The surplus value of a customer for a particular product means the difference between price and perceived value. Thus a customer would buy a product only if the surplus value for the product is positive. If we denote the surplus values of a customer for the high and low brand products as  $sv_h$  and  $sv_l$ , then the total demand can be classified as the following segments.

Total demand : Customers of  $0 < sv_h$  or  $0 < sv_l$   
Customers in this set will always purchase one of high brand or low brand products. Thus this set means the total demand for the vertically differentiated products, which is assumed to be 1 in this study.

Segment 1: Customers of  $0 > sv_h$  and  $0 < sv_l$   
Customers in this segment are very price sensitive, so they do not purchase the high brand product but purchase only the low brand product. It is shown that the size of this segment is proportional to the quality level of the low brand product per unit price (denoted as  $\theta$ ) [4]. Thus the size of this segment can be denoted as  $k_1\theta$ , where  $k_1$  represents a constant.

Segment 2: Customers of  $0 < sv_h < sv_l$   
If both the high and low brand products are sold by the distributor, the customers in this segment will purchase the low brand product. They will purchase the high brand product only if the low brand product is not sold. The size of this segment can also be denoted as  $k_2\theta$ , where  $k_2$  represents a constant [4].

Segment 3: Customers of  $0 < sv_l < sv_h$   
Customers in this group prefer the high brand product. They will purchase the low brand product only if the high brand product is not sold. The size of this segment can also be denoted as  $k_3\theta$ , where  $k_3$  represents a constant [4].

Segment 4: Customers of  $sv_l < 0$  and  $0 < sv_h$   
Customers in this group do not want any low quality product. They only purchase the high brand product. The size of this segment can be represented as  $1 - (k_1 + k_2 + k_3)\theta$ .

Based on the above discussions, the sales volume for each scenario of bargaining result can be summarized as follows, where it is assumed that  $k_1 = k_2 = k_3 = 1$  without loss of generality.

Scenario 1 : Both the high and low brand products are sold through bargaining settlement

Sales volume of the high brand product :  $1 - 2\theta$

Sales volume of the low brand product :  $2\theta$ .

Scenario 2 : Only the high brand product is sold through bargaining settlement

Sales volume of the high brand product :  $1 - \theta$

Scenario 3 : Only the low brand product is sold through bargaining settlement

Sales volume of the low brand product :  $3\theta$

It is easy to see that inequality  $0 \leq \theta \leq 1/3$  should be satisfied because the total demand was assumed to be 1.

### 3. Nash Equilibria

In this study we consider two types of bargaining game, namely, simultaneous and sequential. To derive the equilibrium solutions for the simultaneous and sequential bargaining, we first need the reaction functions of both the high and low brand product manufacturers. Suppose that the retail prices of the high and low brand products, namely  $P=(p_h, p_l)$ , are given ( $p_l < p_h$ ). Then the reaction functions of the low and the high brand product manufacturers can be derived based on the game model and market assumption in section 2, and are given in equation(2) and (3), respectively.

$$w_l = \begin{cases} (1 - \alpha)p_l, & \text{if } w_h = \otimes; \\ \frac{(1-\alpha)}{2}(2p_l - p_h + w_h), & \text{if } w_h > p_h - 2p_l; \\ \otimes, & \text{if otherwise.} \end{cases} \quad (2)$$

$$w_h = \begin{cases} (1 - \alpha)p_h, & \text{if } w_l = \otimes; \\ (1 - \alpha)[p_h - \frac{\theta}{1-2\theta}(p_l - w_l)], & \text{if otherwise.} \end{cases} \quad (3)$$

In equation (2), the condition  $p_h - 2p_l < w_h$  is derived from the bargaining settlement condition for the low brand product manufacturer, namely  $0 < w_l < p_l$ . In equation (3), however, we see no such settlement condition for the high brand product. This implies that if the distributor’s bargaining for the low brand product manufacturer reaches a settlement, then always does the bargaining for the high brand product.

#### 3.1. Simultaneous bargaining

The equilibrium solutions for the simultaneous

bargaining can be derived by solving the reaction functions in equations (2) and (3) as a linear system, which are given in equations (4), (5) and (6), respectively. In the following equations, the superscript S stands for ‘simultaneous’.

$$w_h^S = \frac{(1-\alpha)[(2-(5-\alpha)\theta)p_h - 2\alpha\theta p_l]}{2-(5-\alpha(2-\alpha))\theta} \quad (4)$$

$$w_l^S = \frac{(1-\alpha)[- \alpha(1-2\theta)p_h + (2-(5-\alpha)\theta)p_l]}{2-(5-\alpha(2-\alpha))\theta}, \quad (5)$$

$$\pi_D^S = (1 - 2\theta)(p_h - w_h^S) + 2\theta(p_l - w_l^S). \quad (6)$$

To justify the above solutions, as mentioned earlier, the bargaining settlement condition for the low brand product manufacturer should be satisfied. The settlement condition can be derived by plugging the above equations into the condition  $p_h - 2p_l < w_h$  (or  $0 < w_l < p_l$ ). The resulting settlement condition is given in the equation (7) and depicted in Fig. 1.

$$\frac{\alpha(1-2\theta)}{2-(5-\alpha)\theta} < \frac{p_l}{p_h}. \quad (7)$$

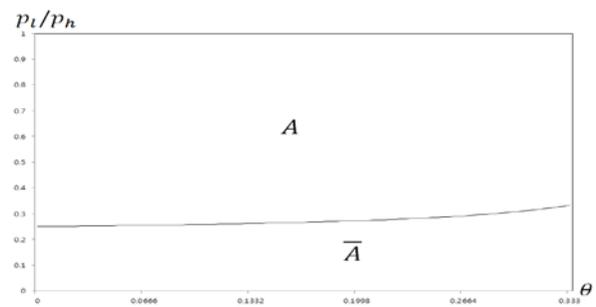


Fig. 1. Settlement Condition for the Low Brand Product (Simultaneous Bargaining,  $\alpha=0.5$ )

In Fig.1, A is the area where the settlement condition in equation (7) is satisfied, whereas  $\bar{A}$  is the area where the condition is not satisfied. In other words, A is the area where the distributor sells both the high and low brand products through simultaneous bargaining, whereas the distributor sells only the high brand product in area  $\bar{A}$ . We see the division into A or  $\bar{A}$  is more dependent on  $p_l / p_h$  than  $\theta$ .

#### 3.1. Sequential bargaining : Bargaining first with the low brand product manufacturer

The procedure to derive equilibrium solutions for the sequential bargaining can be found in [2][7]. The resulting equilibrium solutions for this sequential bargaining first with the low brand product manufacturer are given in equations (8), (9) and (10). In the following equations, the superscript LF stands for ‘Low brand First’.

$$w_l^{LF} = \frac{(1-\alpha)[- \alpha p_h + (3-\alpha)p_l]}{3-2\alpha + \alpha^2}, \tag{8}$$

$$w_h^{LF} = \frac{(1-\alpha)[(3-6\theta + \alpha(\alpha-2+3\theta-\alpha\theta))p_h - 2\alpha\theta p_l]}{(1-2\theta)[3-(2-2\alpha)\alpha]}, \tag{9}$$

$$\pi_D^{LF} = (1-2\theta)(p_h - w_h^{LF}) + 2\theta(p_l - w_l^{LF}). \tag{10}$$

To justify the above solutions, the bargaining settlement condition for the low brand product manufacturer should be satisfied. The settlement condition can be derived by plugging the above equations into the condition  $p_h - 2p_l < w_h$  (or  $0 < w_l < p_l$ ). The resulting settlement condition is given in the equation (11) and depicted in Fig. 2.

$$\frac{\alpha}{3-\alpha} < \frac{p_l}{p_h}. \tag{11}$$

In Fig.2,  $B$  is the area where the settlement condition in equation (11) is satisfied, and  $\bar{B}$  is the area where the condition is not satisfied. In other words,  $B$  is the area where the distributor sells both the high and low brand products through sequential bargaining first with the low brand product manufacturer, whereas the distributor sells only the high brand product in area  $\bar{B}$ . It is interesting to see the division into  $B$  or  $\bar{B}$  only depends on  $p_l/p_h$ .

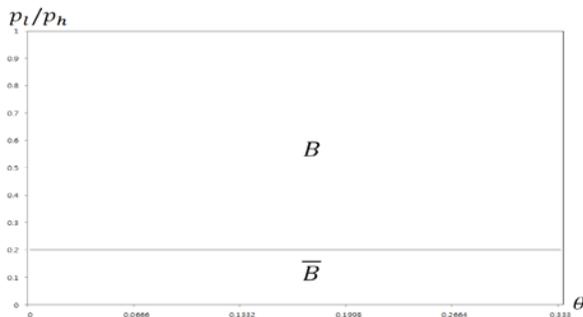


Fig.2. Settlement Condition for the Low Brand Product (Bargaining first with the Low Brand Manufacturer,  $\alpha=0.5$ )

### 3.2. Sequential bargaining : Bargaining first with the high brand product manufacturer

Different from the sequential bargaining first with the low brand product manufacturer, the following two possible options exist for the sequential bargaining first with the high brand product manufacturer. First, the distributor and the high brand product manufacturer can set a relatively high wholesale price which satisfies the settlement condition ( $p_h - 2p_l < w_h$ ). In this case, the low brand product can enter the market by bargaining settlement. Second, the distributor and the high brand product manufacturer can kick out the low brand product by setting a relatively low wholesale price which does not satisfy the settlement condition of the low brand product.

#### 3.2.1. Allowing the low brand product to enter the market by setting a relatively high $w_h$

The equilibrium solutions for this type of sequential bargaining are given in equations (12), (13) and (14). In the following equations, the superscript HF stands for ‘High brand First’.

$$w_l^{HF} = \frac{(1-\alpha)[(1-\theta-\alpha\theta)p_h - \alpha\theta p_l]}{1-\theta[1+\alpha(2-\alpha)]}, \tag{12}$$

$$w_h^{HF} = \frac{(1-\alpha)[(2\alpha\theta-\alpha)p_h + (2-(2+(5-3\alpha)\alpha)\theta)p_l]}{2-2\theta[1+\alpha(2-\alpha)]}, \tag{13}$$

$$\pi_D^{HF} = (1-2\theta)(p_h - w_h^{HF}) + 2\theta(p_l - w_l^{HF}). \tag{14}$$

To justify the above equilibrium solutions, precondition for this type of bargaining, namely  $p_h - 2p_l < w_h^{HF}$ , should be satisfied, which is given in equation (15). In Fig.3 C denotes the area where the condition in equation (15) is satisfied.

$$\frac{\alpha(1-2\theta)}{2-(2-\alpha)(1+3\theta)\theta} < \frac{p_l}{p_h} \tag{15}$$

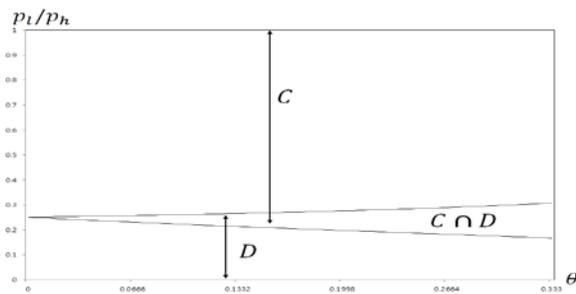
#### 3.2.2. Throwing the low brand product out of the market by setting a relatively low $w_h$

The equilibrium solutions for this type of sequential bargaining are given in equations (16) and (17). In the following equations, the subscript ‘ho’ stands for ‘High brand Only’.

$$w_{ho}^{HF} = \frac{(1-\alpha)[(1-\theta)p_h - 3\alpha\theta p_l]}{1-\theta}, \quad (16)$$

$$\pi_{Dho}^{HF} = (1 - \theta) \quad (17)$$

To justify the above equilibrium solutions, precondition for this type of bargaining, namely  $w_{ho}^{HF} < p_h - 2p_l$ , should be satisfied, which is given in equation (17). In Fig.3  $D$  denotes the area where the condition in equation (17) is satisfied.



<Figure 3> Feasibility Conditions for the Equilibrium Solutions (Bargaining first with the High Brand Manufacturer,  $\alpha=0.5$ )

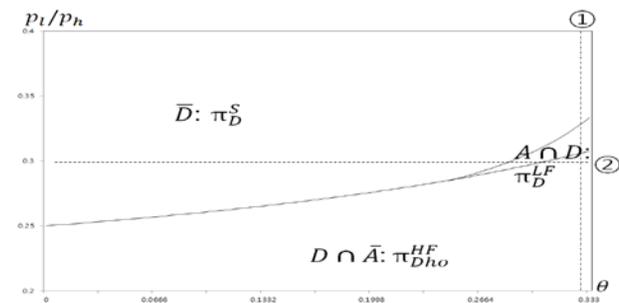
In Fig.3, the low brand product may or may not enter the market in the area  $C \cap D$ .

## 4. Optimal Bargaining Strategy of the Distributor

### 4.1. Locally Optimal Strategies

In the previous section, we derived the expected profit of the distributor for each bargaining type ( $\pi_D^S$  in equation (6),  $\pi_D^{LF}$  in equation (10),  $\pi_D^{HF}$  in equation (14),  $\pi_{Dho}^{HF}$  in equation (17)). To derive the local optimal bargaining strategy of the distributor, we need to compare the expected profits by each of the area in Fig.1 ~ Fig. 3. By numerical analysis the following results were derived. First,  $\pi_D^S$  turned out to be the highest profit in the area  $\bar{D}$ , which implies selling both the high and low brand

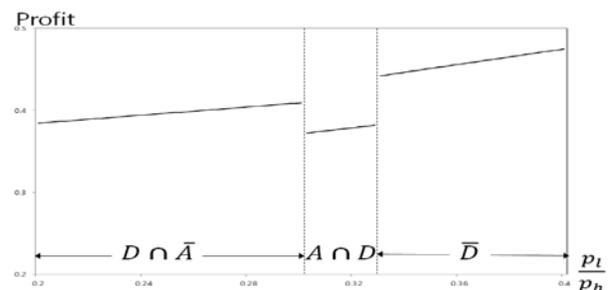
product resulting from simultaneous bargaining is optimal for the distributor. Second,  $\pi_{Dho}^{HF}$  turned out to be the highest profit in the area  $\cap \bar{A}$ , which implies selling only the high brand product resulting from the sequential bargaining first with the high brand product manufacturer is optimal for the distributor. Lastly,  $\pi_D^{LF}$  turned out to be the highest profit in the area  $A \cap D$ , which implies selling both the high and low brand product resulting from the sequential bargaining first with the low brand product manufacturer is optimal for the distributor. The locally optimal bargaining strategies are depicted in Fig. 4.



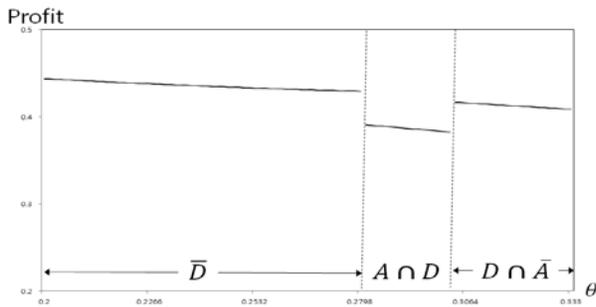
<Figure 4> Changes of the Optimal Bargaining

### 4.1. Globally Optimal Strategy

Among the three locally optimal strategies which would be the most favorable strategy to the distributor? To answer this question, the changes of profits as the function of  $p_l/p_h$  (along the dotted line ① in Fig. 4) and  $\theta$  (along the dotted line ② in Fig. 4) are analyzed in Fig. 5 and Fig. 6, respectively.



<Figure 5> Effect of  $p_l/p_h$  on the Distributor’s Profit



<Figure 6> Effect of  $\theta$  on the Distributor's Profit

In Fig. 5 and Fig. 6, we see the profit of the distributor is proportional to  $p_l/p_h$ , and inversely proportional to  $\theta$ . It is also shown that area  $\bar{D}$  is most favorable to the distributor, whereas it is advantageous to the distributor to avoid the area  $A \cap D$  as far as possible.

## 5. Concluding Remarks

This study addressed the multilateral bargaining strategy of a distributor who deals with vertically differentiated products. To do this, we derived and analyzed the Nash equilibrium solutions for both simultaneous and sequential bargaining games as functions of the relative quality and price level of the low brand product comparing to those of the high brand product. The implications derived from the analysis are as follows.

First, it is found that the following three types of bargaining are locally optimal : ① the simultaneous bargaining which results in selling both the high and the low brand products, ② the sequential bargaining first with the high brand product manufacturer which results in selling only the high brand product, ③ the sequential bargaining first with the low brand product manufacturer which results in selling both the high and the low brand products.

Second, it is shown that the profit of the distributor is proportional to the relative price level of the low brand product comparing to that of the high brand product ( $p_l/p_h$ ), and inversely proportional the

quality level of the low brand product per unit price ( $\theta$ ). This implies that, for more bargaining profit, the distributor has strong incentive to prefer a low brand product which has high price and lower quality level per unit price.

Third, among the three locally optimal strategies, the simultaneous bargaining which results in selling both the high and the low brand products was found to be the most favorable strategy to the distributor. Besides, it is advantageous to the distributor to avoid the sequential bargaining first with the low brand product manufacturer which results in selling both the high and the low brand products as far as possible.

The above results may be regarded as qualitative in that the model used in this study incorporates some assumptions for deriving solutions in a closed form. We expect that it would be possible to get more practical guidelines for multilateral bargaining by conducting some empirical or simulation studies for resolving those assumptions.

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