

Modelling unemployment rate in the Czech Republic

ONDŘEJ ČÍŽEK

Department of Econometrics
University of Economics in Prague
W. Churchill Sq. 4, 130 67, Prague 3
CZECH REPUBLIC
cizeko@vse.cz

Abstract: Weak aggregate output demand and high unemployment are both basic features of the economy in the Czech Republic these days. For this reason, the model formulated in this paper is based on the following transition mechanism: “High unemployment leads to low consumption demand. Production is also low as no one can afford to buy the goods, which sustains high unemployment.” The paper contributes to the existing literature by incorporating this Keynesian principle of weak aggregate demand into the basic Diamond-Mortensen-Pissarides (DMP) model in a simple and novel way. Despite its simplicity, the proposed model captures the essence of the current economic crisis. The parameters of the model are econometrically estimated. The estimated model turns out to have multiple equilibria, which is interpreted from an economic point of view.

Key-Words: nonlinear modelling, unemployment, vacancies, multiple equilibria

1. Introduction

The unemployment rate in the Czech Republic has been switching between lower and a higher level in past two decades, which is illustrated in figure 1. Traditional view is that these fluctuations represent cyclical movements. In this paper, I explore a question whether or not this dynamics could be modelled by an alternative approach as transitions from a lower to a higher equilibrium unemployment rate.

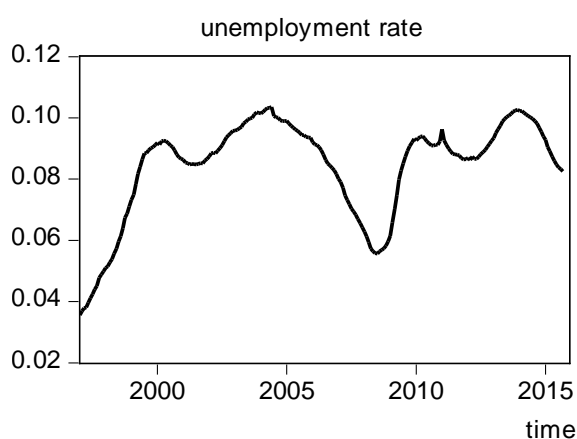


Fig. 1: Unemployment rate in the Czech Republic.

To this end, a nonlinear model of labour market in the Czech Republic is formulated in this paper. Nonlinearity arises by making a probability of finding a job endogenous. This probability is

modelled as a function decreasing in unemployment. It will be shown that this feedback mechanism causes the multiplicity of equilibrium unemployment rates. The intuition behind this result is that there is low demand for labour in times of high unemployment. Therefore, it is hard to find a job, which sustains unemployment at high levels.

This formulation principle is supported by the Nobel Prize winner in economics, Joseph Stiglitz [12], who states that Europe’s problem today is a lack of aggregate demand. This article contributes to the existing literature by incorporating the Keynesian principle of weak aggregate demand into the basic Diamond-Mortensen-Pissarides (DMP) model of the labour market. The DMP model is built upon theoretical foundations laid down by the Nobel Prize winners in economics Diamond [3], Mortensen [6] and Pissarides [7].¹

The idea that interaction between unemployment and aggregate demand for output might cause a multiplicity of equilibriums is not new in the literature. The most famous model was formulated by Diamond [3], in which a multiplicity of equilibriums is induced by the assumption of increasing returns in matching trading partners. The structure of this model is quite different from the structure of the model formulated in this paper. Despite this fact, multiple equilibrium points exist in *both* models due to the link between unemployment and aggregate output demand. For this reason,

¹ This work is summarized by Pissarides [8].

a comparison of these two models will be briefly described as well. In another example, Kaplan, Menzio [5] used the DMP modelling framework to show that the feedback between employment and product markets might generate multiple equilibria.²

The structure of the paper is as follows. First of all, the model is formulated in chapter 2. In the next chapter 3, it is compared with the very well known Diamond [3] model. Econometric estimation is described in chapter 4. After that, the estimated model is analysed in chapter 5, especially from a point of view of multiplicity of equilibria. In paragraph 6, economic consequences are discussed and the final chapter 7 concludes.

2. Model

2.1 Unemployment Dynamics

Unemployment is modelled in a continuous time. As data are available only at discrete dates, Shimer's [11] methodology is used to express the dynamics in discrete time

$$U_{t+1} = \frac{s_t}{s_t + f_t} \cdot (1 - e^{-(s_t + f_t)}) \cdot L_t + e^{-(s_t + f_t)} \cdot U_t, \quad (1)$$

where U_t is number of unemployed, L_t represent the labour force, s_t is separation rate and f_t is job-finding rate.

The measurement of transition rates f_t , s_t is also based on Shimer's [11] methodology. According to his evidence, there are substantial fluctuations in job finding probability during business cycle frequencies, while separation probability is nearly acyclic. This suggests that, in order to understand fluctuations in unemployment, one must understand the fluctuations in job-finding probability. The formulation of the model presented in this paper is based upon this result. The emphasis will, therefore, be given to the model of job-finding probability.

2.2 Matching Function

Standard Cobb-Douglas matching function is used to describes the formation of new relationships (matches) from the number of unemployed workers U_t and (unfilled) job vacancies V_t

$$M_t \equiv M(V_t, U_t) = A \cdot U_t^\alpha \cdot V_t^{1-\alpha} \quad (2)$$

where $A > 0$, $\alpha \in (0, 1)$.

The assumption of constant returns to scale is in line with most empirical work (see survey performed by Pissarides, Petrongolo [9]). Interpretation of this assumption is that the effectiveness of the matching process does not depend on the size of the labour market.

It will also be assumed that all unemployed workers U_t have an equal sampling probability. The job-finding probability F_t is then given by

$$F_t = \frac{M_t}{U_t} = A \cdot \theta_t^{1-\alpha}, \quad (3)$$

where $\theta_t \equiv \frac{V_t}{U_t}$ denotes the market tightness.

The corresponding job finding rate is

$$f_t = -\ln(1 - F_t). \quad (4)$$

The job-filling probability is given by

$$Q_t \equiv \frac{M_t}{V_t} = A \cdot \theta_t^{-\alpha} = \frac{F_t}{\theta_t}. \quad (5)$$

2.3 Job creation in the basic DMP model

Concepts described in previous two subchapters 2.1 and 2.2 are frequently used in the standard DMP model. The DMP model is closed by a free entry assumption. Under this condition, vacancies are created at a flow cost of C per period until they yield a zero profit. This can be described by the following condition

$$C = Q_t \cdot J_t. \quad (6)$$

where J_t is the present discounted value of a filled-job in a representative firm. The expression $Q_t \cdot J_t$ thus represents the expected profit from posting a vacancy, which is equated to the marginal cost C of posting a vacancy.

Assuming Cobb-Douglas matching function, the job filling probability Q_t can be substituted from (5) into (6), which yields

$$V_t = U_t \cdot \left(\frac{A \cdot J_t}{C} \right)^{\frac{1}{\alpha}}. \quad (7)$$

Given the number of unemployed, firms post more vacancies if the value of a filled job J_t is

² Kaplan, Menzio [5] also briefly discuss literature on multiple equilibria in the DMP modelling framework and summarizes various factors that may lead to the existence of multiple equilibria.

higher. However, the situation is complicated, as J_t , also depends on V_t and U_t .

The value of a job J_t is determined by the asset-pricing equation. The basic idea is as follows:³ Once a representative worker find a job, they produce y^s of a single homogenous good each period, for which they are paid a wage w . The company profit in a given period is therefore $y^s - w$. Production is supposed to last in future periods until the job is destroyed, which is modelled by a job destruction rate s . Future profits are discounted by interest rate r . The application of the asset pricing theory in this basic setup leads to

$$J = \frac{y^s - w}{s + r}. \quad (8)$$

This equation captures the basic idea for modelling job vacancies in the DMP framework. It claims that the value of a job is higher (and thus firms post more vacancies) when the marginal product of labour is high relative to the wage. The marginal product of labour, nonetheless, depends on unemployment U and the wage depends on unemployment U as well as vacancies V .

For later comparison with my own model, I will briefly summarize the effects of unemployment on the number of vacancies posted by firms in the DMP model. The dependence of the wage and marginal product of labour on unemployment U and on vacancies V will become apparent.

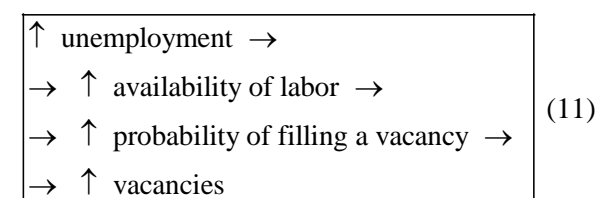
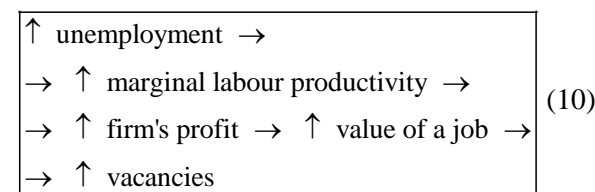
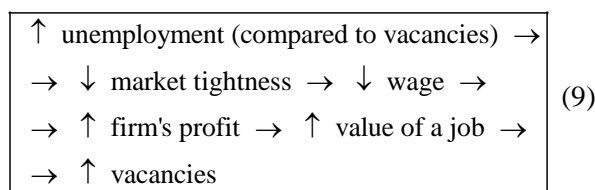
Firstly, a high unemployment U (compared to the number of vacancies V) tends to decrease wages w , because workers are in a weaker bargaining position. Lower wages means higher profits and the increased value of a job J , which encourages firms to create more vacancies V .

Secondly, an increase in the number of unemployed workers is supposed to lead to higher marginal labour productivity because of the diminishing returns in production. As a consequence, profits from employing additional workers are increased, which leads to more vacancies posted by the firms.

Finally, an increase in unemployment leads to the higher availability of labour. For this reason, firms have a higher probability of filling a vacancy Q . Therefore, there is also a higher expected profit

from posting a vacancy $Q \cdot J$. The result again is more vacancies V posted by the firms.

For convenience, these transmission mechanisms can be summarized as follows:



These transmission mechanisms describe the optimal behaviour of firms as well as workers, which is derived from microeconomic optimizations in the DMP framework. In a diagrammatic analysis, this optimal behaviour is traced by the vacancy supply (VS) curve. In an unemployment-vacancies plane, this curve slopes upwards because the rise in unemployment leads to more vacancies. VS curve is an analogy to the traditional labour demand curve.

2.4 Job creation in the presented model

The basic DMP framework summarized in the previous subchapter will be modified in this paper by introducing the Keynesian principle of weak output demand. The value of a job J_t is not evaluated on the basis of the current and expected future production y^s as in relation (8). The current and expected firm's future profits will instead be determined by the current and expected future sales. This modelling presumption is based on the fact that, in reality, firms typically do not have problems producing output.

The problem is to find customers, who will buy the products, which applies particularly in times of economic crisis. The production of a firm is constrained by demand and, for this reason, the relation (8) is modified as follows:

³ For notational simplicity, the time subscript will be omitted for a moment.

$$J_t = \frac{p(Y_t^d, Y_{t+1}^d, Y_{t+2}^d, \dots) \cdot y^s - w}{s + r} \quad (12)$$

where $p(\bullet) \in (0,1)$ represents a probability that an output produced will be sold, Y_t^d is the (real) aggregate demand in the period t , Y_{t+k}^d is the aggregate demand for the period $t+k$, which is expected in the current period t .

For simplicity, I assume that y^s , w , s and r from the equation (12) are constant over time. The assumption of a constant y^s reflects the Keynesian view that the supply-side of an economy does not play a crucial role in explaining the current economic crisis. Similarly, the separation rate s does not play an important role in explaining the rise in unemployment in the Czech Republic during the current economic crisis (Shimer [8]). The interest rate r has been low and stable since the beginning of the crisis. For these reasons, the separation rate and the interest rate are both regarded as constants.

There are two basic modelling issues in the DMP framework. The first is how workers and firms come together and the second is how they determine wages. It should be stressed here that the issue of determining a wage will not be modelled in this paper. I make an assumption of constant wages, which is definitely an important simplification. Nonetheless, it can be justified from an empirical as well as theoretical point of view.

There is a good deal of empirical literature dealing with sticky wages, which cannot be surveyed here. Bewley's [1] book tackles the issue of wage rigidity from a novel methodological perspective and evidence is found to support various wage-stickiness theories.

Within the DMP modelling framework, a number of authors (e.g. Shimer [10], Hall [4]) emphasized that the basic DMP model has difficulty accounting for the volatile behaviour of labour market activity through the business cycle, at least for standard calibration of parameters. Shimer [10] showed that replacing the Nash bargaining solution with a fixed wage dramatically increases the variability of unemployment and vacancies.

The probability $p(Y_t^d, Y_{t+1}^d, Y_{t+2}^d, \dots)$ is the only variable that is not considered constant over time in the relation (12). This probability could be viewed as a function, which is increasing in aggregate demand. For this reason, the value of a job is viewed as an increasing function of total output demand

$$J_t = J(Y_t^d, Y_{t+1}^d, Y_{t+2}^d, \dots) \quad (13)$$

The current and future expected domestic demand $Y_t^d, Y_{t+1}^d, Y_{t+2}^d, \dots$ is determined by current as well as past levels of total income. Since total aggregate income is negatively correlated with the current as well as past unemployment $U_t, U_{t-1}, U_{t-2}, \dots$, there should be a negative relationship between $Y_t^d, Y_{t+1}^d, Y_{t+2}^d, \dots$ and the current as well as lagged values of unemployment $U_t, U_{t-1}, U_{t-2}, \dots$. The value of a job can be viewed as a decreasing function of unemployment

$$J_t = J(U_t, U_{t-1}, U_{t-2}, \dots) \quad (14)$$

For simplicity, it will be assumed that only current unemployment U_t is relevant regressor

$$J_t = J(U_t), \quad \frac{\partial J}{\partial U_t} < 0 \quad (15)$$

Current income plays an important role in consumption decisions of households. This Keynesian argument was empirically confirmed by Campbell, Mankiw [2]. Nonetheless, this simplification does not play a crucial role in the presented model. Various versions of life-cycle hypotheses could thus be incorporated into the modelling framework used in this paper.

The transmission mechanism from unemployment U_t to the value of a job J_t can be summarized as follows:

$$\begin{array}{l} \uparrow \text{unemployment} \rightarrow \downarrow \text{total income} \rightarrow \\ \rightarrow \downarrow \text{domestic demand for products} \rightarrow \\ \rightarrow \downarrow \text{firms' profits} \rightarrow \downarrow \text{value of a job} \end{array} \quad (16)$$

It is important to stress that the relation (15) is in striking contrast to the model of vacancies in the DMP model. Recalling the mechanisms (9)-(11), it is apparent that a rise in unemployment has exactly the opposite effect on the value of a job in the DMP model.

The effect of unemployment on vacancies is described by the function $V(U_t)$, which is obtained by substituting function (15) into the equation (7)

$$V(U_t) = U_t \cdot \left(\frac{A \cdot J(U_t)}{C} \right)^{\frac{1}{\alpha}} \quad (17)$$

of the number of unemployed ($\partial J / \partial U_t < 0$). Using the symbol e to denote the number of employed workers, as in the Diamond model, the condition $\partial J / \partial U_t < 0$ could be equivalently expressed as

$$\frac{dJ}{de} > 0, \tag{25}$$

from which the analogy to (23) becomes more apparent.

As Diamond [3] explained, the result (23) might cause the existence of multiple equilibrium unemployment rates. My model is similar in this respect as the backward causality described by (20) is the reason for the existence of multiple equilibriums as well. This topic will be discussed in detail later in the chapter 5.

4. Econometric Estimation

Econometric estimation was performed using monthly data from the Czech Republic. All these data were collected from the official website of the Ministry of Labour and Social Affairs of the Czech Republic: <http://portal.mpsv.cz/sz/stat/nz/mes>.

Job-finding probability F_t is measured by the Shimer's [11] method. To apply this methodology, the series for the short-term unemployed persons⁴ was used.

Market tightness was calculated as $\theta_t = V_t / U_t$, where time series for V_t and U_t was directly obtained from the above mentioned website.

Firstly, the stochastic version of the regression (3) was estimated

$$F_t = A \cdot \theta_t^{1-\alpha} \cdot e^{\varepsilon_t}, \tag{26}$$

where ε_t is i.i.d. random error.

The estimation was performed by ordinary least squares (OLS) after log-linearization for the data ranging from 1997 M1 to 2015 M8. The results are as follows:⁵

$$\ln(\hat{F}_t) = \frac{-1.69}{(0.03)} + (1 - 0.78) \cdot \ln(\theta_t), R^2 = 0.59. \tag{27}$$

The estimate $\hat{\alpha} = 0.78$ is in line with the results of other empirical studies, which are summarized by Pissarides, Petrongolo [9] and according to which this parameter ranges from 0.2 to 0.8.

⁴ The number of workers, whose unemployment has not exceeded one month.

⁵ Standard errors of the estimated coefficients are indicated in parentheses below the parameters.

Secondly, the regression (20) was estimated in the following form

$$\theta_t^{0.78} = a - b \cdot u_t + \eta_t, \tag{28}$$

where η_t is i.i.d. random error and 0.78 is the value of the estimated coefficient $\hat{\alpha}$.

The parameter θ is not estimated econometrically. The lowest value of market tightness was attained was approximately equal to 0.06, which can be seen from figure 2. We definitively conclude that $\theta \leq 0.06$. However, it is impossible to say anything more concerning the value of the θ parameter on the basis of the historical data. For this reason, the value of the θ parameter is not obtained using econometric techniques. Instead, various plausible values of this parameter will be taken into account and consequences to the model properties will be analysed.

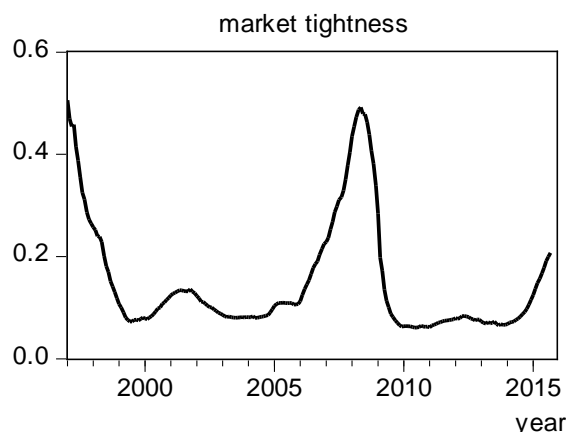


Fig. 2: Market tightness in the Czech Republic.

The regression (28) was estimated by standard OLS for data ranging from 1997 M1 to 2015 M8 as follows:

$$\hat{\theta}_t^{0.78} = \frac{0.77}{(0.02)} - \frac{6.59}{(0.26)} \cdot u_t, R^2 = 0.74. \tag{29}$$

5. Equilibrium unemployment rates

Under the assumption of constant labour force $L_t = L$ and separation rate $s_t = s$, the equation (1) is slightly modified as follows:

$$u_{t+1} = \frac{s}{s + f_t} \cdot (1 - e^{-(s+f_t)}) + e^{-(s+f_t)} \cdot u_t,$$

where $u_t = U_t / L$ is unemployment rate.

This equation implies that a stationary unemployment rate $u_t = u$ satisfies

$$0 = \left(\frac{s}{s + f(u)} - u \right) \cdot (1 - e^{-(s+f(u))}), \quad (30)$$

where $s = 0.0105$ is the separation rate in 2015 M8 (the last date in the dataset) and $f(u)$ indicates that a stationary value of f is a function of a stationary unemployment rate u .

In order to describe the function $f(u)$ in more detail, let's start with the equilibrium value of market tightness, which is obtained from (29)

$$\theta(u) = (0.77 - 6.59 \cdot u)^{1/0.78}. \quad (31)$$

From now on, I will return to the specification (20), which yields a stationary value of market tightness in the following form

$$\theta(u) = \max \{0.77 - 6.59 \cdot u, \underline{\theta}\}^{1/0.78}. \quad (32)$$

Different values of the lower bound $\underline{\theta} \geq 0$ will be discussed later in this section.

The probability of finding a job in a stationary state is obtained from (27)

$$F(u) = e^{-1.69} \cdot \theta(u)^{1-0.78} \quad (33)$$

and the corresponding job-finding rate is

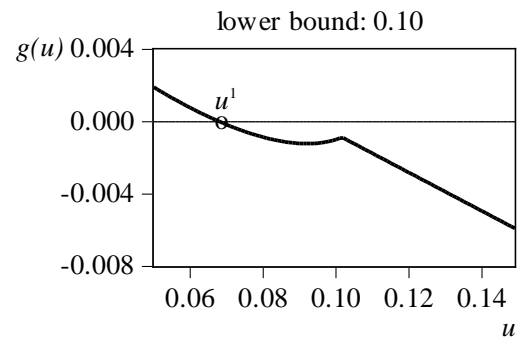
$$f(u) \equiv -\ln(1 - F(u)), \quad (34)$$

which defines the function $f(\bullet)$ in equation (30).

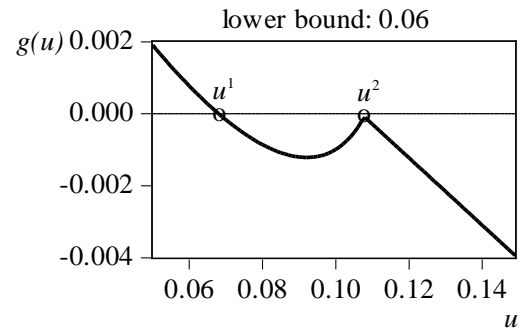
Equation (30) is nonlinear, hence multiple solutions may exist. The expression on the right hand side of (30) is a function of the variable u , which will be denoted $g(u)$

$$g(u) \equiv \left(\frac{s}{s + f(u)} - u \right) \cdot (1 - e^{-(s+f(u))}). \quad (35)$$

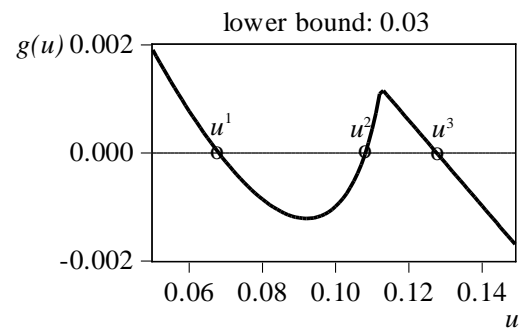
The function $g(u)$ calculated for various values of the lower boundary of market tightness $\underline{\theta}$ is depicted at the following figure



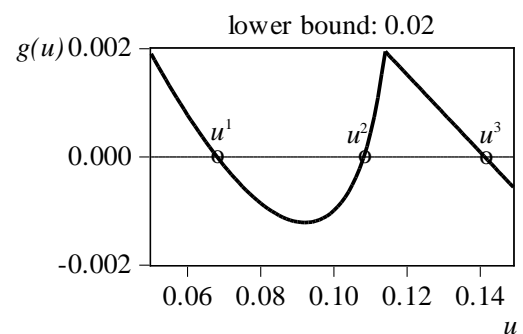
(a)



(b)



(c)



(d)

Fig. 3: Equilibrium unemployment rates for different values of the lower boundary of market tightness.

In the graph (a), we can see that there is only one solution to the equation $g(u) = 0$. This means that the equilibrium unemployment rate is unique in this case and equals approximately to $u^1 = 0.07$. This equilibrium is stable, which follows immediately from the fact that the value $g(u_t)$ can be interpreted as $u_{t+1} - u_t$.

Nonetheless, the lower bound on market tightness $\underline{\theta} = 0.10$ is not realistic. We already know from figure 2 that market tightness θ_t can attain a value as low as 0.06. When the lower bound is decreased to $\underline{\theta} = 0.06$, there is also another equilibrium $u^2 = 0.11$. This equilibrium is semistable as unemployment rate u converges to it when $u > u^2$, but converges to u^1 when $u < u^2$.

If the lower bound is further decreased to $\underline{\theta} = 0.03$, there are three solutions to the equation $g(u) = 0$. Therefore, three equilibrium unemployment rates $u^1 = 0.07$, $u^2 = 0.11$ and $u^3 = 0.13$ exist in this case. The equilibria u^1 and u^3 are stable, while the equilibrium u^2 is now unstable.

Decreasing the lower bound to $\underline{\theta} = 0.02$ does not change the equilibria $u^1 = 0.07$, $u^2 = 0.11$. But the equilibrium u^3 would now be $u^3 = 0.14$. If the lower bound is decreased even further to the value of zero, then the equilibria $u^1 = 0.07$, $u^2 = 0.11$ still would not change, but the equilibrium u^3 would now be $u^3 = 1$.

6. Economic Discussion

Market tightness in the Czech Republic was 0.5 at the beginning of the economic crisis in 2008. There was a significant decrease in subsequent periods and was as low as 0.06 in 2010.

From these facts, it seems to me that the market tightness could quite easily fall even to a zero value. A multiple equilibrium model of the labour market is thus more suitable than a traditional model with unique equilibrium.

The existence of multiple equilibrium unemployment rates in my model can be explained by a less effective labour market during times of high unemployment. Firms open only a few vacancies during a recession (crisis) because demand for their output is low. For this reason, it is

hard for unemployed workers to find jobs, which maintains unemployment at high levels.

Comparing figure 1 with figure 3 reveals that observed unemployment rate does not switch between equilibria u^1 , u^3 . The reason is that the equilibrium u^3 must be higher than $u^2 = 0.11$. But from figure 1, we can see that the observed unemployment rate has not exceeded the level of 0.10. Therefore, the dynamics of the observed unemployment rate could be equally well described by a traditional model with only one equilibrium. Nonetheless, the formulated model warns us that multiple equilibria exist and that the observed unemployment rate u_t might begin to converge to the equilibrium u^3 whenever $u_t > u^2 = 0.11$.

Note from figure 1 that the unemployment rate in the Czech Republic was approximately equal to 0.10 in 2013, which was very near to the unstable equilibrium u^2 . The dynamics of the unemployment rate in 2014 and 2015 were thus unclear late in 2013. It is probably the case that a small negative shock could have caused the convergence to the equilibrium u^3 . On the other hand, a positive shock might have led to convergence at the point u^1 , which is what seems to have happened.

7. Conclusion

The important conclusion is the multiplicity of equilibrium unemployment rates for the econometrically estimated model. Specifically, there are two stable equilibrium points, provided that the number of unfilled vacancies (market tightness) is sufficiently low in times of high unemployment. The lower stable equilibrium unemployment rate attains a value of 0.07. The higher stable equilibrium emerges when market tightness is allowed to fall below a value of 0.06. The lower it is allowed to fall, the higher the second stable equilibrium. If the lower boundary for market tightness is 0.03 then a higher equilibrium would attain a value 0.13.

There is also an unstable equilibrium unemployment rate, which equals 0.11. The importance of this lies in the fact that the unemployment rate in the Czech Republic was 0.10 in 2013. The labour market in the Czech Republic was thus very close to unstable equilibrium. In such a situation, only a tiny negative shock could have caused the convergence to a higher ineffective stable equilibrium. Similarly, only a small positive shock might have led to the convergence to the

lower equilibrium point, which is what seems to have happened.

Despite the existence of multiple equilibria, the hypothesis that the observed unemployment rate has been switching between two equilibria in the last two decades turned out not to be correct. The reason is that the less effective equilibrium unemployment rate should be well above the value of 0.10, while the observed unemployment rate has not exceeded this value.

Acknowledgements

Financial support of VŠE IGA IG403014 and VŠE IP400040 is gratefully acknowledged by the author.

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