Modelling and Forecasting of the Annual Inflation Rate in the Unstable Economic Conditions

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Abstract: - The monthly time series of the annual inflation rate is defined as the growth rate of the monthly time series of the consumer price index with respect to the corresponding month of the previous year. The annual inflation rate might not always be the appropriate measure of inflation, mainly due to the fact that it does not provide up-to-date information on the level of inflation. The harmonic analysis shows that the annual inflation rate deforms and delays the information with respect to the monthly inflation rate. This conclusion can be extremely important in the forecasting of the inflation rate, as well as in the process of economic decision making. The non-traditional method for the construction of the annual inflation rate forecasts is proposed. The advantage is that it is able to catch breaks and other instabilities in the future development of the time series.

Key-Words: - inflation rate, harmonic analysis, linear filtration, forecasting.

1 Introduction

Inflation is a very important macroeconomic indicator, which measures the change in the general level of prices of goods and services consumed by households. It plays a crucial role in monetary policy, specifically in the targeting of inflation through the setting of interest rates. It is used for the calculation of real interest rates, the increase of the real value of assets as well as the valorization of wages, pensions and social benefits.

Due to the widespread use of inflation and its significant role in the economy, it is of the utmost importance to find a good way to measure of inflation, as well as a method for inflation forecasting (Stock, Watson, 1999, Ciccarelli, Mojon, 2010, Diron, Mojon, 2008, and other). In this paper we argue that the widely used measure of inflation, specifically the annual inflation rate (introduced below), might not always be the appropriate way to measure it, mainly due to the fact that it does not provide up-to-date information on the level of inflation. It is shown that the information in the annual inflation rate is delayed in comparison with the monthly inflation rate and the annualized inflation rate. Therefore, the information in the monthly inflation rate can be used for the annual inflation rate forecasting, which is effective especially in the period of the unstable economic conditions.

The papers consists of four parts. In the first part the monthly and annual inflation rates are defined and the development of these rates based on the European harmonized consumer price index are described. The second part deals with the problem of delayed information in the annual inflation rate in comparison with the monthly inflation rate. In the third part there is explained the principle of the newly proposed forecasting method for the annual inflation rate. The fourth part of the paper contains the empirical verification of the new forecasting method.

2. Monthly and annual inflation rate

Inflation is informally defined as the change in the consumer price index during the period of either one month or one year – this definition leads to either the monthly, or the annual, inflation rate. The monthly time series of the monthly inflation rate can be defined as

$$IR_{m,t} \equiv \frac{CPI_t}{CPI_{t-1}}. \qquad (1)$$

This definition implies that the time series of the monthly inflation rate is the growth rate (of the monthly time series of the consumer price index) with respect to the previous month. Similarly, the monthly time series of the annual inflation rate can be defined as

$$IR_{a,t} \equiv \frac{CPI_t}{CPI_{t-12}}. \qquad (2)$$

This definition means that the time series of the annual inflation rate is the growth rate (of the monthly time series of the consumer price index) with respect to the corresponding month of the previous year.[1]

Figure 1 presents the development of the natural logarithm of the harmonized consumer price index from January 1998 up to February 2012 – $HCPI_t$ – which is the indicator of inflation and price stability for the European Central Bank. In Figure 2 there is the logarithm of the monthly inflation rate, $IR_{m,t}$, from January 1998 up to February 2012, which is based on the harmonized consumer price index. Figure 3 shows the development of the logarithm of the annual inflation rate – $IR_{a,t}$ – from January 1998 up to February 2012, also it is computed from the harmonized consumer price index.

The logarithm of the monthly inflation rate is characterised by a relatively strong seasonal pattern in contrast with the logarithm of the annual inflation rate. In this time series it is clearly seen unstable behavior from 2007.

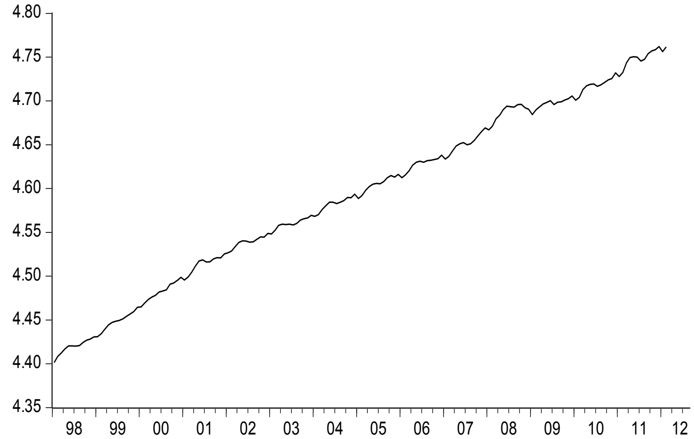

Fig. 1. The log $HCPI_t$ from January 1998 till February 2012

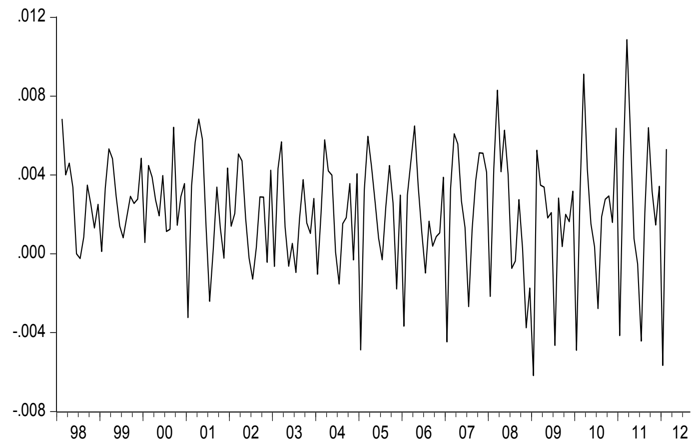

Fig. 2. The log $IR_{m,t}$ from January 1998 till February 2012

Fig. 3: The log $IR_{a,t}$ from January 1998 till February 2012

Data source: ECB (2013)

[1] The annual inflation rate is frequently defined also as $(CPI_t\text{-}CPI_{t-12})/CPI_{t-12} = IR_{a,t}\text{-}1 \approx \log IR_{a,t}$ and the monthly inflation rate as $(CPI_t\text{-}CPI_{t-1})/CPI_{t-1} = IR_{m,t} - 1 \approx \log IR_{m,t}$ (Arlt, J. (1998) and Arlt, J. & Arltová, M. (2013))

3. The effect of the annual inflation rate time delay

A crucial point to notice is that

$$\log IR_{a,t} = \log IR_{m,t} + \log IR_{m,t-1} + \log IR_{m,t-2} + \ldots + \log IR_{m,t-11}, \quad (3)$$

i. e., when coming from the log monthly inflation rate to the log annual inflation rate, we take a moving sum of 12 numbers, which are spread uniformly from time $t - 11$ to time t, and the result is assigned to time t. In other words: the aggregate information contained in the range of times from $t - 11$ to t is assigned to the endpoint of this range. But in the fact it measures the inflation at its yearly level which effectively corresponds to time $t - 5.5$ (i.e. the center of the range $t - 11$ to t). This intuitively implies that the log annual inflation rate must be delayed behind the log monthly inflation rate, and the annualized log monthly inflation rate ($12^* \log IR_{m,t} = \log IR_{anualized,t}$) which truly corresponds to time t.

It was illustrated with the help of the spectral analysis in Arlt, Bašta (2010). The decomposition of log monthly inflation rate, $\log IR_{m,t}$, of length T into the sum of sines of different frequencies $f_k = k/T$, amplitudes $A_k \geq 0$ and phases φ_k has form

$$\log IR_{m,t} = \sum_{k=0}^{\lfloor T/2 \rfloor} A_k \sin(2\pi f_k t + \varphi_k), \qquad t = 0, \ldots, T-1, \quad (4)$$

$k = 0, \ldots, \lfloor T/2 \rfloor$, where $\lfloor T/2 \rfloor$ stands for the closest integer less or equal to $T/2$. From definition of the log annual inflation rate in (3) it follows that

$$\begin{aligned} \log IR_{a,t} &= \sum_{i=0}^{11} \log IR_{m,t-i} \cong \\ &\cong \sum_{i=0}^{11} \sum_{k=0}^{\lfloor T/2 \rfloor} A_k \sin(2\pi f_k (t-i) + \varphi_k) = \\ &= \sum_{k=0}^{\lfloor T/2 \rfloor} \sum_{i=0}^{11} A_k \sin(2\pi f_k (t-i) + \varphi_k). \end{aligned} \quad (5)$$

Equation (5) is an approximate equality because the values of the log monthly inflation rate for $t < 0$ are needed to calculate all values of the log annual inflation rate. After some trigonometric modifications Arlt, Bašta (2010) gained equation

$$\sum_{i=0}^{11} A_k \sin(2\pi f_k (t-i) + \varphi_k) = c_k A_k \sin(2\pi f_k t + \varphi_k + w_k), \quad (6)$$

from which it follows that the moving sum of the monthly inflation rates (3) changed the amplitude from A_k to $c_k A_k$ and phase from φ_k to $\varphi_k + w_k$, where c_k and w_k are factors depending on f_k only. Therefore, the log annual inflation rate can be expressed as

$$\log IR_{a,t} \cong \sum_{k=0}^{\lfloor T/2 \rfloor} c_k A_k \sin(2\pi f_k t + \varphi_k + w_k). \quad (7)$$

Then

$$\begin{aligned} c_k A_k \sin(2\pi f_k t + \varphi_k + w_k) &= \\ &= c_k A_k \sin(2\pi f_k (t + w_k / (2\pi f_k)) + \varphi_k) = \\ &= c_k A_k \sin(2\pi f_k (t - \Delta T_k) + \varphi_k). \end{aligned} \quad (8)$$

It is seen, that the delay of the annual inflation rate is

$$\Delta T_k = -\frac{w_k}{2\pi f_k}. \quad (9)$$

It was proved by Arlt, Bašta (2010) that the low frequencies ($f_k < 1/12$), which create the main systematic part of the time series dynamism, are all delayed by 5.5 month and the high frequencies ($f_k > 1/12$) are delayed by a different amount, which may lead to some form of deformations in the log annual inflation rate.

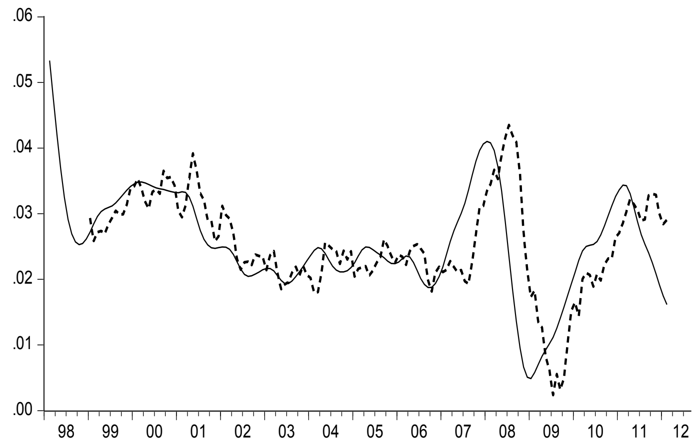

Fig. 4: The log $IR_{a,t}$ (dashed) and the smoothed log $IR_{annualized,t}$

The annual inflation rate, along with the smoothed and annualized monthly inflation rate in the log transformation, computed on the basis of $HCPI_t$ from January 1998 to February 2012, are presented in Fig. 4. The smoothing of the annualized log monthly inflation rate is achieved by the Hodrick-Prescott (HP) filter. It is a two-sided linear filter that computes the smoothed series $s_t = \log IR_{annualized,t}|_{smoothed}$ of series $y_t = \log IR_{annualized,t}$ by minimizing the variance of y_t around s_t, subject to a penalty of the second difference of s_t. The HP filter chooses s_t to minimize

$$\sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} ((s_{t+1} - s_t) - (s_t - s_{t-1}))^2. \quad (10)$$

Parameter λ controls the smoothness of series s_t. It is seen that the peaks and troughs, especially in the period of instability, are delayed in the log annual inflation rate behind the smoothed annualized log monthly inflation rate.

4. The forecasting of the annual inflation rate

The annual inflation rate is very frequently used in economic practice; more frequently than the monthly inflation rate and annualized monthly inflation rate. One of the arguments for the practical application of the inflation rate is its relative smoothed shape in comparison with the annualized monthly inflation rate, which usually contains a strong seasonal pattern. As we mentioned above, the principle problem of the annual inflation rate is the significant time delay in comparison with the monthly, and annualized monthly, inflation rate. In fact, the problem is only of a technical nature and it can be solved by moving of the annual inflation rate back by five or six months. In papers Arlt, Arltová (1999) and Arlt, Bašta (2008, 2010), it was justified this issue in detail. In this paper we will no longer draw attention to the analysis of the problem of delayed information in the annual inflation rate and its essential influence for example to the monetary policy of the central banks. Instead, we will use our knowledge for the proposal a new, nontraditional method of the annual inflation rate forecasting, with the horizons $h = 1, 2, 3, 4, 5, 6$.

In literature, there are many methods for inflation forecasting, their survey is given for example in Faust, Wright (2013). The following approaches are frequently used in economic practice: Box-Jenkins methodology based forecasts, Phillips-curve motivated forecasts, unobserved component models based forecasts, VAR and VECM based forecasts etc.

The basic approach is based on the modelling of CPI_t, for example some types of SARIMA, VAR or VEC models can be used for CPI_t forecasting. The forecasts of the annual inflation rate are then computed from the CPI_t forecasts. The CPI_t is usually the nonstationary time series with relatively strong seasonality. The non-seasonal difference of log CPI_t represents the log monthly inflation rate, log $IR_{m,t}$. The SARIMA model of this inflation rate directly follows the model SARIMA for the log CPI_t. The forecasts of the monthly inflation rates are then used for the construction of the annual inflation rates forecasts (see eq. 3). The seasonal difference of the log CPI_t represents the log annual inflation rate, log $IR_{a,t}$. It seems that the most suitable way how to predict the annual inflation rate is to find the model directly for it. But as argue Lütkepohl and Xu (2011), HEGY tests for seasonal unit roots (Hylleberg, Engle, Granger and Yoo (1990)) for CPI_t of the EU countries typically do not reject the seasonal unit roots at some frequencies but reject for others. So, the seasonal difference leads to over-differencing of log CPI_t in some frequencies which can cause serious problems in the identification of the suitable time series model.

Our new method for the annual inflation rate forecasting is based on different principle. It uses the fact, that the annual inflation rate is delayed behind the annualized monthly inflation rate by approximately six months and that the behavior of these two inflation rates are very similar (after the smoothing of the annualized inflation rate). The basic forecasting principle is simple. It can be expressed in the following formula

$$\log IR_{a,T}(h) = \log IR_{anualized,T-6+h\,|\,smoothed} \quad \text{for } h = 1, 2, 3, 4, 5, 6, \qquad (11)$$

where $\log IR_{a,T}(h)$ is the forecast of the log annual inflation rate at time T for h months ahead and $\log IR_{anualized,T-6+h\,|\,smoothed}$ is the smoothed annualized log monthly inflation rate. The advantage of this approach is that the forecasts of the annual inflation rates are based on the annualized monthly inflation rates computed directly from the real data. It follows that it is able to catch breaks and other instabilities in the ”future” six months development of the annual inflation rate, and can thus be efficient, especially in the unstable time periods. The drawback of this method is the fact, that the annualized monthly inflation rate is strongly seasonal and must be smoothed. As it was mentioned above, the HP filter can be used. The smoothing of the ends of the time series by two-sided filter (as is HP filter) is usually problematic and does not reflect the reality. The solution is to forecast ”ex ante” the annualized monthly inflation rate, which gives the HP filter enough values to smooth more properly the end of the analyzed real time series. Then the smoothed forecasted values are removed. The similar solution is used in the X13ARIMA seasonal adjustment method. The quality of smoothing and the annual inflation forecasts depends on the accuracy of the monthly inflation rate forecasts.

5. The practical verification of the proposed method

The empirical verification of the proposed forecasting method is based on the recursive forecasting of the *HCPI* annual inflation rate. The forecasts with a horizon of 6 months start from the prediction threshold of September 2006, and repeat themselves with the prediction threshold of each subsequent month. The Hodrick-Prescott filter is used for smoothing the last six values of the actual annualized monthly inflation rate. Before that, however, it is necessary to calculate the annualized monthly inflation rate forecast for 12 months ahead, on the basis of the SARIMA models (until the prediction threshold April 2009 it is SARIMA$(0,0,4)(1,0,0)_{c,}$ and after that it is SARIMA(1,0,0)(1,0,1)). The accurate forecasts improve the quality of the smoothing of the time series ends considerably. The forecasts accuracy “ex post” is measured by the Mean Squared Errors, and by the Theil Inequality Coefficient. Its values lie between zero and one; zero indicates a perfect fit.

Fig. 5 shows some of the forecasts together with the real log annual inflation rate time series. A period with significant breaks was deliberately chosen. It is clearly seen that the forecasts are able to capture the instability, as well as the fundamental changes in the behavior of the "future" development of the annual inflation rate.

Table 1 contains the Mean Squared Errors as well as the Theil Inequality Coefficients of recursive forecasts for 6 months. The values of the second measure vary, but they are very close to zero. It is interesting to compare the size of Theil Inequality Coefficients with the real values of the annual inflation rate, which has been captured in Fig. 6. With decreasing values of the predicted time series, the Theil Inequality Coefficients have the tendency to grow, which means that the accuracy of the predictions decreases.

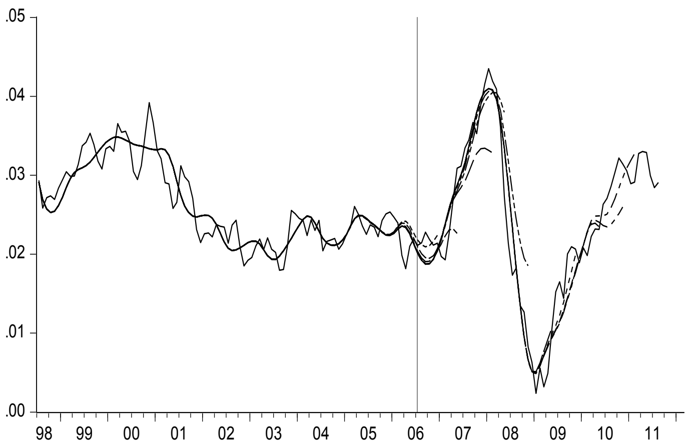

Fig. 5. The $\log IR_{a,t}$, the smoothed $\log IR_{annualized,t}$, the $\log IR_{a,T}(h)$ with the horizon of 6 months

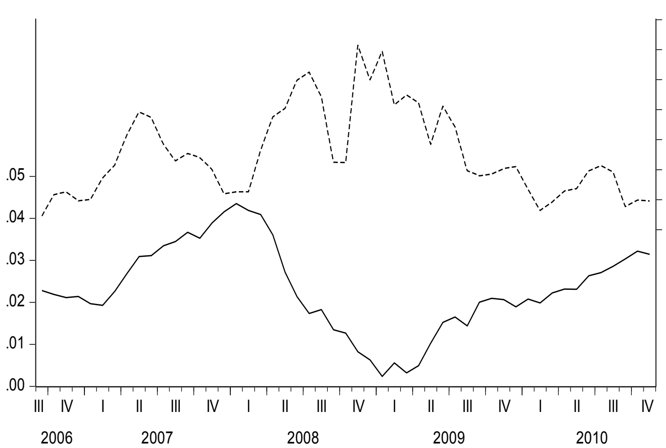

Fig. 6. The Theil Inequality Coefficients (dashed), the $\log IR_{a,t}$

Table 1. The MSE and the Theil Inequality Coefficients of the recursive forecasts

Date	MSE	Theil	Date	MSE	Theil
2006-09	0.0010	0.0229	2008-11	0.0071	0.3073
2006-10	0.0024	0.0581	2008-12	0.0040	0.2497
2006-11	0.0026	0.0630	2009-01	0.0041	0.2971
2006-12	0.0020	0.0480	2009-02	0.0028	0.2080
2007-01	0.0022	0.0502	2009-03	0.0043	0.2243
2007-02	0.0040	0.0863	2009-04	0.0039	0.2113
2007-03	0.0052	0.1078	2009-05	0.0036	0.1422
2007-04	0.0079	0.1580	2009-06	0.0050	0.2059
2007-05	0.0101	0.1960	2009-07	0.0049	0.1713
2007-06	0.0103	0.1867	2009-08	0.0033	0.0984
2007-07	0.0084	0.1424	2009-09	0.0032	0.0898
2007-08	0.0072	0.1146	2009-10	0.0034	0.0924
2007-09	0.0083	0.1271	2009-11	0.0039	0.1017
2007-10	0.0083	0.1199	2009-12	0.0047	0.1052
2007-11	0.0073	0.1001	2010-01	0.0030	0.0675
2007-12	0.0046	0.0599	2010-02	0.0014	0.0321
2008-01	0.0048	0.0632	2010-03	0.0021	0.0463
2008-02	0.0050	0.0632	2010-04	0.0030	0.0644
2008-03	0.0104	0.1323	2010-05	0.0033	0.0685
2008-04	0.0140	0.1880	2010-06	0.0048	0.0982
2008-05	0.0136	0.2021	2010-07	0.0055	0.1067
2008-06	0.0150	0.2492	2010-08	0.0052	0.0958
2008-07	0.0134	0.2627	2010-09	0.0022	0.0385
2008-08	0.0089	0.2218	2010-10	0.0029	0.0493
2008-09	0.0033	0.1124	2010-11	0.0029	0.0478
2008-10	0.0026	0.1117			

6. Conclusion

In this paper we argue that the principal property of the annual inflation rate is its approximate six months time delay in comparison with the annualized monthly inflation rate. We use it to suggest the new, nontraditional approach to annual inflation rate forecasting. This method is verified for the case of the *HCPI* annual inflation rate. The recursive "ex post" forecasts are computed for the very unstable period. The accuracy of the "ex post" forecasts is measured by the Mean Squared Error, and the Theil Inequality Coefficient. It has been found that even in the periods of great instability, the proposed method is very efficient and able to create relatively accurate forecasts, catching even the considerable breaks in the future development of the forecasted time series.

As it was mentioned above, the accuracy of the annual inflation rate depends on the quality of the monthly inflation rate forecasts. Currently, we investigate the quality of different forecasting methods, mainly the method based on the SARIMA models. For this purpose, the automatic model selection procedure was created and the simulation experiment was implemented. The accuracy evaluation of the forecasts "ex post" plays very important role in the automatic forecasting model selection. In the short time, the suggested forecasting method will be analyzed by another simulation study. We plan to perform empirical study in which we will analyze the accuracy of the forecasts of the annual inflation rates of most of the EU countries.

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