Modeling the sovereign spreads via threshold cointegration: evidence from Brazil

BEATRIZ VAZ DE MELO MENDES
Federal University at Rio de Janeiro
COPPEAD–IM
BRAZIL
beatriz@im.ufrj.br

CLAUDIO CARDOSO FLORES
BCB & Federal University at Rio de Janeiro
COPPEAD
BRAZIL
claudio.cardoso@bcb.gov.br

Abstract: This paper analyses the dynamics of the Brazilian sovereign spread and its statistically significant connection with some selected variables by applying linear and threshold cointegration methods. We statistically test the effects of the recent subprime crisis finding strong equilibrium relations, stable enough to exhibit cointegrated behavior before and after the subprime crisis. The assumption of a threshold, linked to a certain level of market risk aversion, refines the methodology and brings flexibility to the modeling strategy. The final VECM model indicates the External debt and the volatility index VIX as the key drivers of the Brazilian sovereign spread. The findings suggest that, with the onset of the crisis, an automatic mechanism of adjustment between the spread and the VIX came into play while the market experimented an increasing risk aversion feeling. It becomes clear from the results that current beliefs on the economic fundamentals are not sufficient to deal with the market turbulence under a global crisis. We propose a methodology for assessing the robustness of the optimal threshold solution.

Key–Words: Threshold Cointegration, VECM models, Sovereign Risk, Emerging Markets

1 Introduction

Sovereign debt securities compose an important asset class in international financial markets, fulfilling global investors diversification objectives, as well as being a source of funding for several economies. Spreads calculated over these bonds are defined as the difference between the interest rate paid by a given economy when issuing a foreign currency denominated international sovereign debt, and the interest rate paid by a risk-free security issued by a country considered as a benchmark, usually the U.S. In this sense, this metric is commonly understood by stake holders in the issuing market as a proxy for the relative cost of raising funds and it would be able to compensate investors for the higher risks incurred, either credit, market, or liquidity risks.

Among the existing methodologies used to better understand sovereign risk, probably the most important and well-known are those used by rating agencies. Hybrid in their nature, they mix quantitative and qualitative analysis considering an extensive list of macroeconomic, financial, accounting, fiscal, demographic, social and political variables. Several academic papers, however, have adopted an empirical approach, looking for relationships between the spreads and variables taken as proxies for the abovementioned risks.

Many empirical studies have shown that the sovereign risk ratings issued by rating agencies may actually be explained by a small set of variables such as income *per capita*, GDP growth, inflation, external debt, level of economic development and default history. However, global financial variables may also indirectly explain such risk ratings by adequately explaining some relevant macroeconomic fundamentals, see Cantor and Packer (1996), Al-Sakkaa and Gwilym (2010), Martinez, Terceno and Teruel (2013), Al-Sakkaa, Williams and Gwilym (2013), among others.

Using panel data, Haugh, Ollivaud and Turner (2009) analyze the spreads from selected European countries, concluding that the increased risk aversion - characteristic of the recent financial crisis - besides being an important factor for itself, also acted as an amplifier for potential problems in fiscal fundamentals considered by the authors as very relevant for explaining sovereign risk.

In emerging countries, Hilscher and Nosbusch (2010) find that some macroeconomic fundamentals, in particular terms of trade and their volatilities, are important determinants for the sovereign debt spreads. On the other hand, Yeyati and Rozada (2008), through the use of panel cointegration, find that much of the emerging countries spreads variability can be explained by global factors such as risk appetite, liquidity and contagion. In addition, they show that such relationships are robust with respect to both time and the inclusion of country-specific factors, and would be also reliable for forecast purposes. In another interesting paper, Sun et al. (2011) select domestic and international variables and, through linear cointegration, they find some significant relationships between the spreads of 12 emerging countries and variables such as GDP, real effective exchange rate, risk aversion, liquidity and commodities prices.

Actually, there has been a lot of discussion, far from reaching a conclusion, on which would be the main driving forces for sovereign spreads. An extensive literature may be found on this topic, with some works advocating the use of country-specific, global variables and liquidity premiums, among others. Some examples are: Weigel and Gemmill (2006), Garcia-Herrero (2006), Longstaff et al. (2011) and Csonto and Ivaschenko (2013).

Considering that some of the above-mentioned papers highlight the well known fact that emerging markets sovereign spreads can be also affected by global financial markets events instead of being solely dependent on country-specific episodes, specially when a financial crisis occurs in the time span, this paper analyses the dynamics of the Brazilian sovereign spread and their statistically significant connection with these two sets of driving forces. Moreover, we assess the role played by the last financial crisis as a possible inductor of transient changes on the prevalence between spread's determinants.

A novelty brought in here is the use of the powerful framework provided by the non-linear threshold cointegration methodology, useful when structural breaks and asymmetries are present. The investigation allows for a deeper knowledge of the relationships among the time series selected, which is, certainly, useful for understanding similar co-movements in many other emerging markets. Findings may be of useful guidance when facing future turbulent periods.

A related work is Sun et al. (2011), but we differ in: 1) the use of the threshold cointegration approach to analyze the relationships among an emerging market sovereign spread, macroeconomic and financial series; 2) the assessment of the effects of the last subprime crisis on the long run equilibrium relationship between the spread and its determinants, with very interesting findings. Among others, we find a risk aversion frontier which, when reached, changes completely the cointegrating relations, for just global factors become relevant. As such, one can interpret the asymmetries and non-linearities in the adjustment process as functions of a growing risk aversion. To the best of our knowledge, there is no previous work applying this methodology to sovereign spreads.

The remainder of this paper is organized as follows. Section 2 reviews the main concepts on linear and threshold cointegration, gathering results from several seminal works. The theoretical background is then used in Section 3, where we empirically examine and discuss the evolution of the Brazilian sovereign spread in the last decade. Section 4 summarizes the results and concludes the paper.

2 Cointegration

In general, a nonstationary behavior of a time series precludes the direct use of traditional econometric models, and some transformation is needed in order to fulfill the usually required stationarity assumption. The statistical models used in this work deal with a particular type of nonstationarity, the unit-root nonstationarity, arising from the persistence of past innovations. In such cases, stationarity can be obtained by d times differencing the original series, reason why they are known as integrated of order d, or $I(d)$.

2.1 Linear Cointegration

In the multivariate setting, a system with n $I(1)$ time series may possess less than n unit roots. In this case they are *cointegrated*. In other words, if there is a linear combination of n $I(d)$ time series which is $I(d{-}b), b > 0$, then the n time series are cointegrated. The formal definition of cointegration can be found in several seminal works such as Granger (1981, 1986), Engle and Granger (1987), Stock (1987), Phillips and Ouliaris (1990), and Johansen (1988). In what follows we set $d = b = 1$, since it adequately describes the observed behavior of the time series used in Section 3.

The most important characteristic of cointegrated times series is the existence of a long run equlibrium. In fact, the $I(0)$ behavior of their cointegrating relations can be seen in the mean reversion property presented by the residuals from a linear regression involving the $I(1)$ series. This means that, in the presence of cointegration, deviations from some atractor are allowed only in the short but never in the long run. In the long run there will be forces guaranteeing convergence towards equilibrium.

Let X_t be a n-dimensional $I(1)$ time series, $\mathrm{X}_t = (X_{1t}, ..., X_{nt})'$. If there is an $(n \times 1)$ vector β such that $\beta^{'}\mathrm{X}_t$ is $I(0)$, then X_t is cointegrated and β is a cointegrating vector. Since for any scalar c the linear combination $c\beta^{'}\mathrm{X}_t$ is also $I(0)$, to uniquely identify β some *normalization* procedure is needed, and one usually sets $\beta = (1, -\beta_2, \cdots, -\beta_n)'$.

Let $\beta^* = (\beta_2, \cdots, \beta_n)'$ and considers the partitions $(1, -\beta^{*'})^{'}$ and $(X_{1t}, \mathrm{X}_{2t})$ of β and X_t, respectively. The linear combination $\beta^{'}\mathrm{X}_t$ is known as the *long-run equilibrium* relationship, and it may be expressed as

$$X_{1t} = \beta^{*'}\mathrm{X}_{2t} + u_t \tag{1}$$

where $u_t \sim I(0)$. The error term u_t is known as the *disequilibrium error* or *cointegrating residual*.

The first step in the Engle-Granger cointegrating test is the ordinary least squares (OLS) estimation of (1). The OLS estimates are super-consistent (Stock, 1987), but possess small finite sample bias. Improved estimates (to be used in Section 3) may be obtained using the dynamic OLS method (DOLS, see Banerjee et al. (1986,1993)), where leads and lags of $\Delta\mathrm{X}_{2t}$ are also considered in (1). In the second step, an unit-root test is applied to the residual series $\{\hat{u}_t\}$. Rejection of the unit-root null hypothesis indicates cointegration.

The cointegrating relations may be written under the semantics of an Error Correction Model (ECM), or its matrix representation VECM (Engle and Granger (1987) and Johansen (1988)). A VECM$(p-1)$ model for n I(1) time series X_t follows from a cointegrated vector autoregressive model of order p, VAR(p), with h cointegrating relations:

$$\Delta\mathrm{X}_t = \Phi_0 + \Phi_1\mathrm{X}_{t-1} + \sum_{j=1}^{p-1}\Gamma_j\Delta\mathrm{X}_{t-j} + \Lambda_t \tag{2}$$

where the innovations Λ_t are Gaussian white noise, Φ_0 is a vector of possible deterministic components, and $\Phi_1 = \alpha\beta^{'}$, where α and $\beta^{'}$ are, respectively, $(n \times h)$ and $(h \times n)$ matrices. That is, under the assumption that there are h cointegrating relationships, only h linear combinations of X_{t-1} (the h elements of $\beta^{'}\mathrm{X}_{t-1} \sim \mathrm{I}(0)$) would appear in equation (2). The factorization $\Phi_1 = \alpha\beta^{'}$ is not unique. To obtain unique values for α and β some restrictions on the model are needed. It should be noted that the classic cointegration technique implies that the correction α of past disequilibriums will always exist in a linear and continuous (symmetric) way.

The number of cointegrating relations h is usually not known, and should be estimated. The Johansen procedure provides two test statistics for deciding on h, the trace and the maximum eigen value statistics, besides the normalized β. The remaining parameters in model (2) may be then estimated by maximum likelihood, see details in Johansen (1988).

2.2 Threshold Cointegration

An underlying assumption in the Engle-Granger approach is the reversion to the equilibrium occurring at the same constant speed for all time points. However, asymmetries may exist in the speed of adjustment α towards the equilibrium. For example, they may differ according to the cointegrating residual is either above or below some threshold. The seminal research on threshold cointegration and some recent studies include, among others, Balke and Fomby (1997), Enders and Granger (1998), Enders and Falk (1998), Enders and Siklos (2001), Hansen and Seo (2002), Tsai et al. (2012).

The Enders and Skilos (2001) framework may be embedded within the context of a SETARMA (Self-Exciting Threshold Autoregressive Moving Average) model, a well known times series model able to handle nonlinearities. Let $\{Y_t\}$ be a stochastic process, with $t \in T$, where T is a time span. Y_t follows a SETARMA $(l; p_1, \cdots, p_l; q_1, \cdots, q_l)$ process if

$$Y_t = \sum_{i=1}^{l}\left(\phi_0^{(i)} + \sum_{j=1}^{p_i}\phi_j^{(i)}Y_{t-j} + \sigma_i(a_t - \sum_{j=1}^{q_i}\varphi_j^{(i)}a_{t-j})\right)\mathcal{H}_{\{Y_{t-d}\in\Re_i\}} \tag{3}$$

where $l \in N^*$ is the number of regimes; for $i = 1, 2, \cdots, l$, $\sigma_i a_t \sim WN(0, \sigma_i^2)$, $\mathcal{H}_{\{Y_{t-d}\in\Re_i\}}$ is the Heaviside indicator function, and $\Re_i = [r_{i-1}, r_i)$ is such that $-\infty = r_0 < r_1 < \cdots < r_l = +\infty$; Y_{t-d} is the threshold variable; $d \in N^*$ is a possible delay; and $\phi_j^{(i)}$, $\varphi_j^{(i)}$, p_i and q_i are coefficients and orders of the autoregressive and moving average components, respectively, in each regime.

In the analysis of Section 3 we fit a SETAR model to the residuals $\hat{u}_t$ obtained in the first step of the Engle-Granger test. That is, we obtain the least squares estimates of

$$\Delta\hat{u}_t = \sum_{i=1}^{l}\left(\phi_0^{(i)} + \phi_1^{(i)}\hat{u}_{t-1} + \sum_{j=1}^{p_i}\phi_{j+1}^{(i)}\Delta\hat{u}_{t-j} + \sigma_i a_t\right)\mathcal{H}_{\{\hat{u}_{t-d}\in\Re_i\}}\,. \tag{4}$$

For fixed d and p_i, the optimal threshold is obtained through a grid search, by ordering the series of lagged residuals $\{\hat{u}_{t-d}\}$ and considering each one as a possible solution defining the regimes. As suggested in Enders and Siklos (2001), in the case $l = 2$ the suitability of the model may be assessed through two test statistics, the $tmax = max(t(\phi_1^{(1)}), t(\phi_1^{(2)}))$, for testing the null hypothesis of *no cointegration*, and the F statistic for testing the joint null hypothesis $H_0 : \phi_1^{(1)} = \phi_1^{(2)} = 0$. Having found the threshold solution as the one providing the smaller residual sum of squares (RSS), a VECM similar to (2) will be fitted.

In many cases n is too large and/or some of the VECM equations have no economic rationale. For example, here we are only interested in the long and short term dynamics of the Brazilian sovereign spread. In such situations, a further refinement of the VECM model, the partial VECM, is more appropriate. The model splits X_t in two sets of dependent and exogenous variables, yielding more accurate estimates, see Harbo et al. (1998).

Formally, consider the partition of the vector X_t into X_{ct}, treated as dependent, and X_{et}, containing all other variables. It is possible to get an efficient estimation of this model whenever the variables in X_{et} are at least weakly exogenous with respect to the parameters of cointegration. See Johansen (1992) for the necessary and sufficient conditions for exogeneity. In this case, the partial system, derived from the VECM described in equation (2) can be written as:

$$\Delta \mathrm{X}_{ct} = \Phi_0 - \alpha\beta'\mathrm{X}_{t-1} + \Psi\Delta\mathrm{X}_{et} + \sum_{j=1}^{p-1}\Gamma_j\Delta\mathrm{X}_{t-j} + \Lambda_t \tag{5}$$

which can be extended by considering the SETAR model with multiple regimes.

3 Analysis of the Brazilian Sovereign Spread

The time series analyzed cover the period from 1997 to 2013. They are: the Brazilian sovereign spread, represented by the Emerging Market Bond Index Plus - Brazil (EMBI+BR); the Commodities Price Index (CRB), a proxy for measuring the performance of the Brazilian economy in commodities international trading; the volatility index (VIX) as a proxy for measuring the global market risk; and the 10 years US Interest Rate, selected to represent international liquidity as well as to assess investors' willingness to invest in emerging markets. These time series compose the set of global variables considered in this paper and were collected both on a daily basis (3980 observations) and on a monthly basis (194 observations). The country-specific variables were collected on a monthly basis and are: International reserves (%GDP), External debt (%GDP), Domestic debt (%GDP) and Terms of Trade. Due to their own specificities, all could be used by investors and stakeholders to form an opinion on the Brazilian health. In order to better understand the effects of the last subprime crisis we split the data in two periods defined by the date of July, 1st, 2007.

Figure 1 shows the evolution through time of the EMBI+BR in levels and first differences, along with the sample autocorrelation functions of both series. Figure 2 shows the varying time dynamics of the External debt, Domestic debt, and VIX series, in levels (a joint plot with the EMBI+BR) and their changes. The series present the expected behavior, seeming to be unit root non-stationary in levels and stationary in the first differences. The Ljung-Box test confirms the visual inspection, and the Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests provide strong evidence supporting the I(1) null hypothesis for the series in levels for all periods, while rejecting the null at any reasonable level for their first differences and any period. All care was taken when specifying the deterministic terms for the PP test as well as the correct autoregressive order for the ADF test.

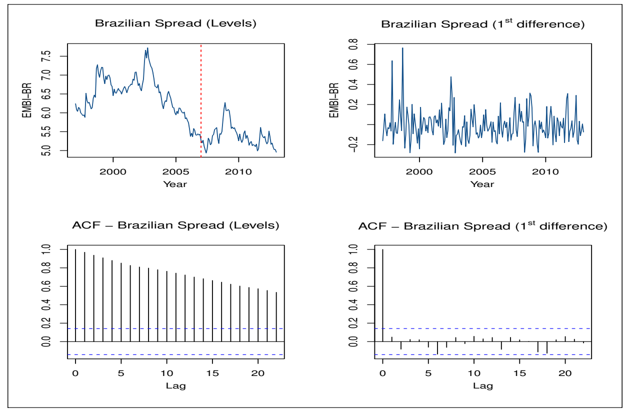

Figure 1: *The Brazilian spread in levels and changes, along with their corresponding autocorrelation functions. The dotted vertical line is drawn at the date of July, 1st, 2007.*

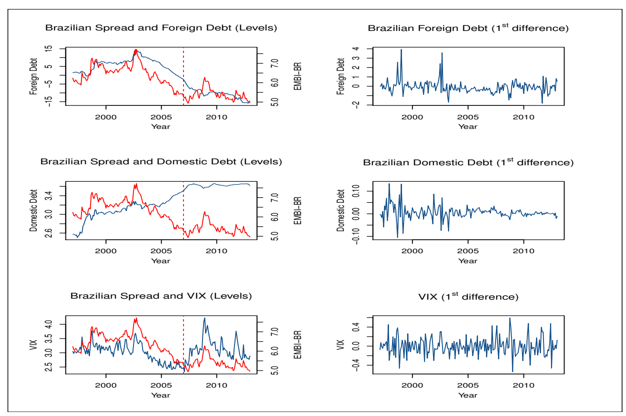

Figure 2: *The Brazilian External debt, the Domestic debt, and the VIX, in levels and changes. The dotted vertical line is drawn at the date of July, 1st, 2007.*

The Engle and Granger (1987) test is carried on the residuals from the regressions built for all 7 pairs of I(1) series involving the Brazilian spread, and for all 3 periods (entire, pre- and post-crisis). For the pre-crisis period we reject the null only for the External debt, meaning that just this variable was cointegrated with the Brazilian spread. This situation completely changes with the advent of the subprime crisis, being only the volatility index VIX cointegrated with the spread after 2007. However, the Johansen test indicates the Domestic debt as the series cointegrated with the spread before crisis. Since the assumptions behind the two tests are different, we continue considering all three time series as possible candidates for explaining the dynamics of the sovereign spread.

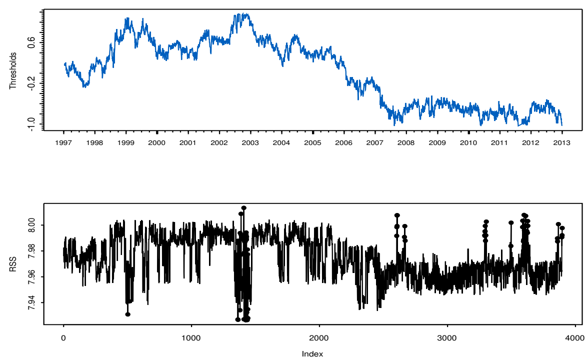

Figure 3: *Plots of the threshold values (upper row) and corresponding RSS values (lower row) along time for the parametrization* $\{l = 2; p_1 = p_2 = 4; d = 1; \phi_0^{(i)} \neq 0\}$, $i = 1, 2$. *The balls in the second row plot indicate the residuals which could not be a feasible solution since their ordered position in the sample are smaller than* 15%.

Assuming some market efficiency premises, Sovereign spreads fluctuations should compensate an investor for the assumed risks. As such, one could say that before the crisis, the Brazilian spread dynamics could be somehow closer to the credit risk, but, with the advent of the crisis, the market risk would have gained more importance. This suggests that a risk aversion level would exist such that, once reached, could have strengthened the link between the volatility index and the spreads' evolution. Thus, we start the analysis by proposing a simple model which considers a threshold cointegration relation between the daily Brazilian spreads and the VIX.

Initially, the series of daily residuals $\{\hat{u}_t\}$ is obtained from the linear regression fit between the spread and the VIX. We consider model (4) with 2 regimes, $l = 2$, $p_1 = p_2 \in \{1, 2, 4\}$, with and without the deterministic terms $\phi_0^{(1)}$ and $\phi_0^{(2)}$, and set $d \in \{1, 2, 3, 4\}$, resulting in 24 different parametrizations. For each d, the ordered lagged residuals are taken as candidates for the threshold value. However, as Enders and Siklos (2001) suggest, to guarantee enough data for each regression, the $k\%$ smaller and larger residuals, $k = 15$, are not considered. The global final threshold solution would be the one producing the smaller RSS among all 24 solutions. To illustrate, Figure 3 shows the threshold values (upper row plot) and corresponding RSS values (lower row plot) along time for the case $\{l = 2; p_1 = p_2 = 4; d = 1; \phi_0^{(i)} \neq 0\}$, $i = 1, 2$. The final solution $\hat{r}_1 = -0.3476$ arises from this model specification.

The balls in the second row plot of Figure 3 indicate the residuals which could not be a feasible solution since their ordered position in the sample are smaller than 15%, actually smaller than $k = 2\%$. It is interesting to see though, that some are actually the smaller RSS values, occurring at important economic periods for Brazil, with internal crisis, uncertainty, and so on, which generated atypical residuals. They may also indicate that 3 or 4 regimes could be more appropriate.

It is worth to note that 22 threshold solutions belong to the interval $[-0.7944, -0.2573]$, and that their corresponding RSS values are very much close. Interpreting the set of all 22 solutions as estimates of the true value resulting from different model specifications, we draw in Figure 4 the *confidence* region $L = [L_1, L_2] = [-0.7944, -0.2573]$ for the true threshold.

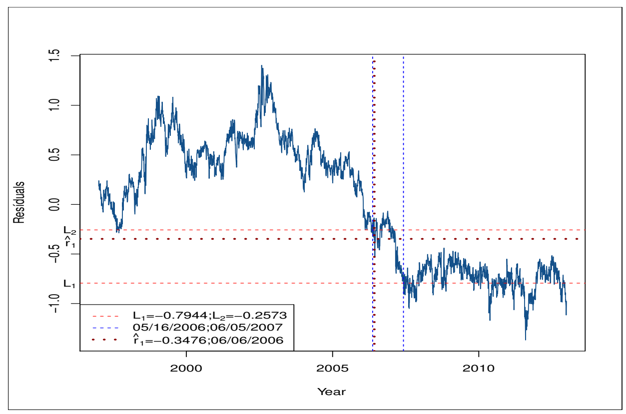

Figure 4: *Time series plot of the residuals from the regression between the daily Brazilian spread and the VIX. Horizontal dotted lines represent the interval* $[L_1, L_2]$ *containing the 22 threshold solutions and the global one.*

The dates of all solutions are concentrated in a period defined by May, 15, 2006 and June, 5, 2007, being the date of the global solution June, 6, 2006. This is, actually, a very particular configuration which shows that the threshold may be indeed a consequence of the financial crisis. All residuals smaller than $\hat{r_1} = -0.3476$ (in regime 1) belong to crisis period, whereas those greater than the threshold belong to the initial portion of the data, as illustrated in Figure 4.

Table 1 gathers the results from the classical linear fit and the threshold non-linear alternative (model (4)) for the winning parametrization resulting in $\hat{r}_1 = -0.3476$, where the smaller AIC indicates the better performance of the threshold model. The rejection of the null hypotheses of the statistical tests indicates that the long run adjustment speeds are different, that is, the market would be approximately 10 times faster when rising the spread value (regime 1) than when reducing it, in line with the increasing risk aversion experimented during the crisis. The estimates $\hat{\phi}_1^{(1)} < 1$ and $\hat{\phi}_1^{(2)} < 1$ satisfy the necessary and sufficient conditions for ergodicity, implying in consistent estimators, see Petrucelli and Woolford (1984) and Chan (1985). The short run dynamics are different in regimes 1 and 2, being $\phi_2^{(i)}$ aproximately 4 times larger in the crisis period. The test statistic *tmax* $= -1.7368$ is significant at the 10% level, and the F test rejects the null at the 1% level.

Table 1: *Results from the linear and the winning nonlinear fit based on model (4) with $\hat{r_1} = -0.3476$. Table shows the AIC values from both models along with the percentage of points in regimes 1 and 2, parameters estimates, their t-values, and statistical significance of the asymmetric long term cointegration tests.*

Number of observations		3975
% of points in regime 1		37.21%
% of points in regime 2		62.79%
AIC of threshold model		-13404.77
AIC of linear model		-13358.62
Parameter	Estimate	t-value
$\hat{\phi}_0^{(1)}$	-0.0342	$-5.3676^{(***)}$
$\hat{\phi}_1^{(1)}$	-0.0460	$-5.3634^{(***)}$
$\hat{\phi}_2^{(1)}$	-0.1917	$-7.8849^{(***)}$
$\hat{\phi}_3^{(1)}$	-0.0743	$-3.0326^{(***)}$
$\hat{\phi}_4^{(1)}$	-0.0735	$-3.0193^{(***)}$
$\hat{\phi}_0^{(2)}$	0.0016	1.1141
$\hat{\phi}_1^{(2)}$	-0.0044	$-1.7368^{(*)}$
$\hat{\phi}_2^{(2)}$	-0.0461	$-2.1801^{(**)}$
$\hat{\phi}_3^{(2)}$	-0.0634	$-2.9869^{(***)}$
$\hat{\phi}_4^{(2)}$	-0.0551	$-2.5915^{(***)}$
Statistical Test		F-value
$H_0 : \hat{\phi}_1^{(1)} = \hat{\phi}_1^{(2)} = 0$		$15.8952^{(***)}$
$H_0 : \hat{\phi}_1^{(1)} = \hat{\phi}_1^{(2)}$		$10.8421^{(***)}$

(***), (**), and (*) indicate, respect., stat. sig. at the 1%, 5%, 10% levels.

Having decided on the threshold value, unit root tests are applied on the residuals $\{\hat{u}_{it}\}$, $i = 1, 2$, obtained from the regressions between the sovereign spread and the VIX index considering each of the two regimes. We find strong evidence for rejecting the null hypothesis when using $\{\hat{u}_{1t}\}$, the residuals from post-crisis regime, but fail to reject the null for the second regime. This corroborates the idea that an automatic mechanism of adjustment between the spread and the VIX came into play once the threshold had been reached, resulting in $\mathrm{I}(0)$ residuals.

In other words, when pricing the sovereign spread in the pre-crisis period, due to market efficiency, it would exist a tendency to balance the credit and the market risks. However, in particular for an emerging market, this balance also depends upon the global picture, and, as off 2006, the risk aversion perception may have grown to a level where this balance leaned towards the market risk. Thus, the estimated threshold may be interpreted as a risk aversion frontier, representing the maximum disequilibrium between the spread and the market risk, and strong comovements between these series would be observed after that. Considering that many emerging markets spreads have showed similar behavior, this collection of risk aversion frontiers could have important applications. For example, one could investigate their positions on the time line. Such ordered dates would clearly have a one-to-one relationship with the set of corresponding economies and, as such, a new sovereign ratings rank could be proposed. It is also worth to note that, after the event of the crisis, the spread being cointegrated only with the volatility index suggests that economic interventions with direct manipulation of macro and fiscal fundamentals probably would be less effective, although remaining, of course, still relevants.

Accordingly, we propose a special version of the threshold VECM model for the entire period, based on the monthly series that showed to be at some extent cointegrated with the Brazilian Spread (S), namely the External debt (E), the Domestic debt (D), and the VIX (V). As estimate of the threshold time point characterizing the onset of the crisis, we take the previously found date of June/2006. For each series we define two data sets, one containing the observations from January/1997 to June/2006, the regime 2, and the other one containing the observations from July/2006 up to January/2013, regime 1. The unit root tests applied to the 8 series all accept the null at the 5% significance level. The following VECM model is then fitted:

$$\Delta S_t = \sum_{i=1}^{2} \Bigg({\phi_0}^{(i)} + \alpha^{(i)} \hat{u}_{t-1}^{(i)} + \gamma_1^{(i)} \gamma_2^{(i)} \gamma_3^{(i)} \gamma_4^{(i)} \Delta S_{(t-1)}^{(i)} \Delta D_{(t-1)}^{(i)} \Delta E_{(t-1)}^{(i)} \Delta V_{(t-1)}^{(i)} + \lambda_t^{(i)} \Bigg) \mathcal{H}_{(\Re_i)} \qquad (6)$$

where $\Re_i$ represents the regime i, $i = 1, 2$ (post- and pre-crisis), defined by the threshold date of June/2006, and where $\{\lambda_t^{(i)}\}$ are the white noise innovations.

Table 2 shows the results from the estimation of the final model (6). The VECM estimates and corresponding (t-values) are shown in the upper (lower) part of the table for regime 2 (1). In regime 2 only the Spread and the External debt are cointegrated, and in regime 1 there exists a cointegrating relation between the Spread and the VIX. The long run relations are statistically significant with different speeds of adjustment. The short run dynamics of ΔE and ΔV are also statistically significant at the 5% level. The residuals from this fit are stationary and show no significant autocorrelations in the first and second moments.

Table 2: *Results from the VECM model (6). Table provides the estimates and (t-values) of the cointegrating vector coefficients for both pre- and pos-crisis. The notations S, D, E, V refer, respectively, to the Brazilian spread, Domestic debt, External debt, and VIX.*

Before Crisis					
Cointegrating Vector					
	S	D	E	V	
	1	–	-0.071	–	
		–	(-4.19)	–	
VECM					
$\hat{\phi}_0^{(2)}$	$\hat{\alpha}^{(2)}$	$\hat{\gamma}_1^{(2)}$	$\hat{\gamma}_2^{(1)}$	$\hat{\gamma}_3^{(2)}$	$\hat{\gamma}_4^{(2)}$
0.640	-0.106	-0.097	–	0.1217	–
(2.53)	(-2.55)	(-1.17)	–	(6.24)	–
After Crisis					
Cointegrating Vector					
	S	D	E	V	
	1	–	–	-1.353	
		–	–	(-2.54)	
VECM					
$\hat{\phi}_0^{(1)}$	$\hat{\alpha}^{(1)}$	$\hat{\gamma}_1^{(1)}$	$\hat{\gamma}_2^{(1)}$	$\hat{\gamma}_3^{(1)}$	$\hat{\gamma}_4^{(1)}$
0.019	-0.022	-0.093	–	–	0.2628
(0.61)	(-2.04)	(-0.94)	–	–	(2.29)

3.1 Sensitivity analysis

When applying Chan's (1993) methodology, we followed the suggestion of Enders and Siklos (2001) on not considering as possible thresholds the smallest and largest k% ordered residuals, $k = 15$, in order to run feasible regressions. This makes the best threshold solution a function of k, $\hat{r}_1(k)$. However, there is no guarantee that such a choice of k would work well for all data sets at hand. Other proportions may work better and, depending on data configuration, it may be more reasonable to assume that the proportion of smallest residuals, say, k_S, could be different from the proportion of largest ones, k_L, and a more careful investigation on k would certainly lead to a better solution.

Figure 5 illustrates this difficulty in the case of the winning model involving the daily series of EMBI+BR and VIX. The figure shows the plot of the ordered residuals in the horizontal axis, versus the corresponding RSS values in the vertical axis, along with vertical lines indicating the varying choices for the percentuals k_S and k_L, 1%, 2%, 5%, 15%, and 30% at the left and right tails.

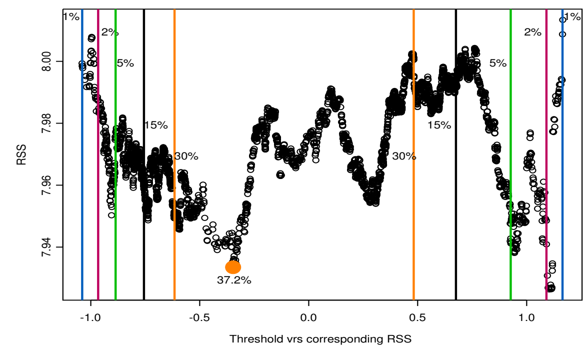

Figure 5: *Plot of the ordered thresholds versus corresponding RSS values, along with varying choices for the percentuals k_S and k_L. The vertical lines are drawn at the 1%, 2%, 5%, 15%, and 30% at the left and right tails.*

It is clear that a k of approximately 50% would imply in two regressions with the largest possible number of observations, resulting in more efficient estimates. Thus, it would be interesting to ask which would be the breakdown proportion, that is, the largest k_S (or k_L) providing the same solution $\hat{r}_1 = -0.3476$. We observe the global solution is in fact very robust, and would not change for all k_S smaller than 37%. On the right tail, only unreasonable proportions of k_L smaller than 2% would lead to a different solution. In summary, the choice of k must be data driven, and could be asymmetric, depending on the tails of the residuals distribution.

4 Conclusion

The existence of cointegrating relations among unit-root non-stationary series is, per-se, an interesting topic, extremely useful whenever the usually required stationarity assumption does not hold. This paper reviews the most important results on cointegration and investigates the dynamics of the Brazilian sovereign spread and their statistically significant connection with some selected variables, aiming also to assess the effect of the last subprime crisis.

The analysis employs a large sample from 1997 through 2013 which includes the subprime crisis. The cointegration approach shows how the selected sets of economic and financial series can explain the sovereign spread movements, and how the global financial environment can influence their explanatory power. When searching for the Brazilian sovereign spread determinants, we found strong equilibrium relations, stable enough to exhibit cointegrated behavior before and after the subprime crisis, according to the risk perception of the market. Although just one emerging market is investigated, some lessons may be learned about the effect of the global crisis and findings may apply to others emerging markets due to their similarities.

The assumption of the existence of a threshold linked to a certain level of risk aversion, refines the methodology and brings flexibility to the modeling strategy. It becomes clear from the results that current beliefs on the economic fundamentals are not sufficient to deal with the market turbulence under a global crisis.

While some economic and financial indicators play an important role in explaining the sovereign spread under "normal" conditions, just a global volatility index is able to explain the spread's dynamics during and after a crisis. Of course our results are restricted to the series selected, however extensive experimentation made showed that no further statistical findings are achieved by including other variables.

Another contribution of the paper is the analysis of the robustness of the solution. The results show that the choice of the proportion of ordered residuals considered as possible thresholds must be data driven.

Finally, it would be interesting to investigate the duration of the post-crisis model, that is, whenever the post-crisis model would become obsolet and new (country specific) fundamentals would be appropriate for explaining the sovereign spreads. We believe that our final cointegration model supports a common belief that whenever periods of global stress are combined with a local uncertanty on country-specific economy, usually global factors gain even more importance and become the drivers of the sovereign spread.

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