

# Volatility Forecast Combination of Brazilian Selic interest rate and exchange rate by means of Principal Component Analysis

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*Abstract:* - Using volatility models for macroeconomic variables can provide more efficient results than models which estimate the average of the process. In this context, the purpose of this research was to evaluate the efficiency of individual models and combination models in forecasting the Brazilian SELIC interest rate and exchange rate between January, 1974 and June, 2012 and from January, 1980 to May, 2012, respectively. The analysis of the series confirmed the presence of volatility in those periods, where Brazil's economic scenario, marked by both internal and external crises, was decisive for such variance. For this purpose, joint modeling was used for the average (ARIMA) and variance (ARCH, GARCH, EGARCH, TARARCH) of the process. The results showed that, in general, the performance measures considered (MAPE, MSE, and U-THEIL) are better for forecast combinations. In addition, forecast combinations by PCA using different kinds of weighting were not conclusive for the kinds of weighting used. This shows that when forecast combination by the PCA method is performed, the best alternative is to use more than one type of PCA in order to obtain the best results.

*Key-Words:* - time series, volatility models, selic interest rate, exchange rate, forecast combination, pca method.

## 1 Introduction

In a country's economic scenario, the behavior of macroeconomic variables, such as interest rate and exchange rate, can influence other variables and change the economy's performance. The interest rate, for example, plays a significant role in the economy by influencing economic agents, and exchange rate fluctuations directly influence country's imports and exports, thus affecting the balance of trade.

The assessment and measurement of variance within macroeconomic variables is not a simple task, because these variables are closely associated with other variables that depend not only on the Brazilian scenario, but also on the international scenario. However, many studies are currently being conducted to find models where estimates can reflect reality more accurately. These models started

being developed in the 1980 to measure, analyze and forecast the volatility intrinsic to the financial and/or macroeconomic series to reduce the risks of previous investment while trying to increase the expected return, as opposed to previous econometric models that focused on the first condition only.

Thus, measuring the variance of economic series allows decision-makers to obtain better results, reducing risks inherent in financial transactions. Although volatility models are widely applicable to economic series, their use is not restricted; several economic variables use this kind of modeling because they have volatile properties in the series.

The use of models that capture the volatility of the series are important because there are time series whose behavior typically features volatility clustering, i.e., periods in which the variable shows great fluctuations for a long period of time followed

by moments of relative calm. This justifies the use of such models, which enables research on the process that generates the series, producing consistent results because of the stability of the series over time. The macroeconomic series used in this research are the SELIC interest rate and the exchange rate. These series were chosen because they have a strong influence on economic activity and change companies willingness to invest.

The economic literature provides many volatility forecasting models with the aim of forecasting variance more accurately. The most prominent volatility models are known as the ARCH family models. According to Engle [7], Bollerslev [1], Nelson [15], Zakoian [26] and Bueno [2], volatility models have been developed over time, creating terms and parameters that did not exist in their predecessors. Thus, volatility modeling gradually improved and acquired properties of volatile series that had not been captured by early models. In this way, each volatility model has preserved its own properties and contains the parameters that can determine conditional variance (volatility).

According to Rausser and Oliveira [19], Libby and Blashfield [14], Makridakis and Winkler [16] and Werner [25], it is known that forecast combination produces better results than individual models when the level of a time series is forecasted. Thus, the research hypothesis of this study is to verify whether or not forecast combination is also better when applied to volatility models. It also investigates whether or not the use of principal component analysis (PCA) as a way of weighting the different models produces efficient results in the composition of the weights of the combination, as well as in dimensionality reduction.

The objective of this study was to make the forecast combination of the exchange rate and the SELIC interest rate by using the models ARCH, GARCH, EGARCH, and TARARCH, using the weights obtained by principal component analysis as the weighting factor, and verifying which method is the best to perform forecast combination, compared with others methods of combination.

This paper is organized as follows: Section 2 describes volatility models; Section 3 presents performance criteria; Section 4 on describes the methods for forecast combination used in this study; Section 5 contains the analysis and discussion; and Section 6 makes the final considerations, which are followed by the references.

## 2 Volatility Models

Many mathematical models are used for researching the volatility of financial series, and such models assume that conditional heteroscedasticity exists, that is, non constant variance in the residuals from linear models or from non autocorrelated series.

### 2.1 ARCH Model

In the literature, models are commonly used to capture intrinsic volatility in financial time series because they allow capturing the behavior of variance in the series, which reveals the risk of making decisions towards these variables as well as the return of such variables, usually the price of assets, interest rates, exchange rates, among others. The - ARCH (p) – model, proposed by Engle in 1982 [7], seeks to estimate volatility by using the variance of previous errors. According to Souza, Souza, and Menezes [23], this kind of model emerges because the series depends on the residuals of the estimation of the process average. Thus, variance is observed to emerge from the volatility that exists between the periods of the residuals.

$$\varepsilon_t = \sqrt{\sigma_t^2} a_t \quad (1)$$

Where  $a_t$  errors are independent and identically distributed with mean 0 and variance 1.

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \cdot \varepsilon_{t-i}^2 = \omega + \alpha(B)\varepsilon_t^2 \quad (2)$$

Where p corresponds to the model order;  $\alpha_i$  is the autoregressive component of the quadratic residuals of the model (ARCH parameter);  $\varepsilon_t^2$  correspond to non-serially autocorrelated residuals.

This model must satisfy the assumption that the conditional variance must be positive; thus, the parameter must satisfy the following assumptions:  $\omega > 0$  and  $\alpha_i > 0$  and  $\sum_{i=1}^p \alpha_i < 1$  so that the assumption of stationarity is satisfied [3].

### 2.2 GARCH Model

Bollerslev [1] has proposed the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, where the author included the past variance of the series into the ARCH model. This way, the model captures the effects of the quadratic errors and of the variance itself at the past instants. Thus, it is possible to obtain a more

economic model, free from the estimation problems of the ARCH model, as shown in equation 4:

$$\varepsilon_t = \sqrt{\sigma_t^2} a_t \tag{3}$$

Which can be rewritten as:

$$\varepsilon_t = \omega + \sum_{i=1}^q \alpha_i \cdot \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \cdot \sigma_{t-j}^2 \tag{4}$$

Where  $\omega > 0$ , and  $\beta \geq 0$  ensuring that  $a_t \geq 0$ . Being able to be rewritten as:

$$\sigma_t^2 = \omega + \alpha(B)\varepsilon_t^2 + \beta(B)\sigma_t^2 \tag{5}$$

### 2.3 EGARCH Model

Nelson [15] has proposed an extension of the GARCH model, the Exponential GARCH (EGARCH), which captures the asymmetric effects of random shocks suffered by the series. These shocks tend to influence the variance of the series, and there is a difference of impact of these shocks compared to their impact signal. However, negative shocks are usually known to cause greater impact on volatility [24]. For this reason, this model can verify if positive or negative shocks influence volatility with the same weight.

$$\varepsilon_t = \sqrt{\sigma_t^2} a_t \tag{6}$$

Where Nelson [15] uses the logarithm (ln) of variance  $\sigma_t^2$  described in equation 6, thus modifying, the formulation of the model, besides including the term that captures the asymmetry of the series.

$$\ln \sigma_t^2 = \omega + \sum_{j=1}^p \beta_j \ln \sigma_{t-j}^2 + \sum_{i=1}^q \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E \left( \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) \right| + \sum_{k=1}^r \gamma_k \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \tag{7}$$

Where  $\varepsilon_t$  i.i.d.  $\sim \mathcal{N}(0,1)$ .

The parameter  $\gamma$  adjusts the asymmetry of the effects, evidencing that negative and positive shocks produce a different impact on the volatility of the series. If the effect of leverage is observed, with negative shocks (bad news) causing a greater impact on volatility than positive ones (good news), the parameter must be:  $\gamma < 0$ .

### 2.4 TARCH Model

The asymmetries in volatility can be captured by the model developed by [26]. This model is another variant of the GARCH model, referred to as TARCH by some researchers. In the financial markets, there are periods of price downfalls which are often followed by periods of intense volatility, while in periods of price increases, volatility is not so intense. This effect is designated as leverage. The conditional variance of the TARCH (Threshold Autoregressive Conditional Heteroscedasticity) model (p, q) can be defined by:

$$\varepsilon_t = \sqrt{\sigma_t^2} \varepsilon_t \tag{8}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \dots + \sum_{j=1}^s \alpha_j \sigma_{t-j}^2 + \gamma \varepsilon_{t-i}^2 d_{t-i} \tag{9}$$

Where  $d_t = \begin{cases} 1, & \text{se } \varepsilon_t < 0 \text{ ("Bad news")} \\ 0, & \text{se } \varepsilon_t > 0 \text{ ("Good news")} \end{cases}$

If  $\gamma = 0$ , there is no asymmetry in the conditional variance. Negative market forecasts ( $\varepsilon_t < 0$ ), such as an abrupt dollar downfall or political instability, have an impact of  $(\alpha + \gamma)$  while positive information ( $\varepsilon_t > 0$ ), for example, intense demand for an asset, has an impact  $\alpha$ . To confirm the leverage effect for this model,  $\gamma > 0$ .

The use of these models favors the behavior analysis of the time series, jointly estimating the average and the process variance, enabling distinct models to capture different behaviors, since the models are not excluding; they are able to complement each other.

### 3 Evaluation Criteria

This section describes the methodological steps as a way to test the research hypothesis, which is the use of principal component analysis (PCA) as a way to weight the forecast combination, and achieve the

research objective, which is to perform the forecast combination for the volatility of the series of interest rate and SELIC rate, in order to obtain the better results than the models previously used.

First, the series was estimated by the linear models of the ARIMA general class; later, a series of residuals is obtained, with such series presenting the characteristics of white noise. Volatility is then estimated by the models of the ARCH family. This way, the ARIMA-ARCH mixed model is obtained.

After estimating various models for the series, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) penalization criteria are used to help to choose the best model, using the maximized value of the Likelihood Function for the estimated model (L), the number of parameters (n) and the size of the sample (T) [23].

$$AIC = -2 \ln(L) + 2n \quad (10)$$

$$BIC = -2 \ln(L) + n \ln(T) \quad (11)$$

As described by Diebold and Lopez [6], there are other evaluation criteria that help to evaluate the performance of the forecasting models. These indicators evaluate precision capacity. Among the accuracy evaluation measures available in the literature, the expressions 12, 13, and 14 will be used; they respectively correspond to: mean absolute percentage error (MAPE), mean squared error (MSE) and U-Theil statistics.

$$MAPE = \frac{1}{n} \sum_{i=1}^n \frac{|(Z_i - \hat{Z}_i)|}{Z_i} \quad (12)$$

$$MSE = \frac{1}{n} \sum_{i=1}^n (Z_i - \hat{Z}_i)^2 \quad (13)$$

$$U - Theil = \frac{\sqrt{\frac{1}{n} \sum_{t=1}^n (\hat{Z}_i - Z_i)}}{\sqrt{\frac{1}{n} \sum_{t=1}^n (\hat{Z}_i)^2 + \frac{1}{n} \sum_{t=1}^n (Z_i)^2}} \quad (14)$$

Where n corresponds to the number of standard forecasts carried out,  $Z_i$  represents the real value at instant i, and  $\hat{Z}_i$  represents the value forecast at instant i.

#### 4 Forecast Combination Techniques

To carry out the forecast combination of the interest rate and exchange rate, the ARCH, GARCH, EGARCH and TARCH individual models were used to obtain the best estimates and consequently the best values forecasted by each model, following the MAPE, MSE, and U-Theil criteria. To compose the proposed combinations, principal component analysis (PCA), Simple Average (SA) and the Ordinary Least Squares (OLS) method will be used. The first forecast combination technique is PCA, first proposed by Pearson [18] and later by Hotelling [13]. Other researchers, such as Morrison [17], Seber [22], Reinsel [20], and Jackson [11];[12], showed that this method enables the reduction of the data set to be analyzed, for example, when the data are composed by a great number of interrelated variables.

When the set of original variables are transformed into a new set that keeps, at most, the variance of the data set, reduction of information takes place. This new set of transformed variables is called Principal Components (PC), which are independent and non-correlated, thus favoring analysis, especially when many variables have to be analyzed [21].

Each principal component is represented by equation 15:

$$CP_j = a_{1j}X_1 + a_{2j}X_2 + \dots + a_{pj}X_{p1} = a_jX \quad (15)$$

Where the vector of constants that must keep the normality condition.

Forecast combination by the PCA method will be carried out by choosing the best individual models out of the four different models - ARCH, GARCH, EGARCH, TARCH. PCA will be performed with the forecasts within the samples of each model previously mentioned. This analysis will allow the construction of weights for each model in the principal components in addition to the weighting of eigenvalues.

The principal component analysis allows the use of more than one weighting factor, which can provide the weights given by the factor loading or factor loadings based on the correlation of the variables - which in this study are models of the ARCH family, or the Variable Contributions weight or contribution of the variables also based on the correlations of the models in each PC. Besides these considerations, weighting will also be used to explain each component as performed by Casarin et al [4]. In addition, verification can take place through the correlation matrix between the PCs and the estimates of the models to find out which model has

greater representation in each component, thus forming the combination based on the strength of expression of each model.

The formulation of the CPs principal components is described in equation 16.

$$CP_j = a_{1j} * X_1 + a_{2j} * X_2 + a_{3j} * X_3 + a_{4j} * X_4 \quad (16)$$

Where is the  $a_{ij}$  the factor loadings or the contribution of the models, and  $X_i$  is the variables that form the linear combination. Besides, it should be noted that the components to be used in the present research will be the significant ones, that is, the ones whose eigenvalues are bigger than one ( $\lambda_j > 1$ ).

The forecast combination by PCA will be given by three different weightings to verify if any of the weightings provides a better forecast result according to the performance indicators. The weights of the first combination by PCA will consider the values given by the matrix of factor loadings shown in equation 17. The second PCA is given by weighting the contribution of models in the components represented by equation 18. The third and final PCA combination will be given considering the weight of the explanation of each component according to equation 19.

$$PCA_1 = r_{11} * X_1 + r_{21} * X_2 + r_{31} * X_3 + r_{41} * X_4 \quad (17)$$

$$PCA_2 = a_{12} * X_1 + a_{22} * X_2 + a_{32} * X_3 + a_{42} * X_4 \quad (18)$$

$$PCA_3 = \lambda_1 * (X_1 + X_2 + X_3 + X_4) + \dots + \lambda_p * (X_1 + X_2 + X_3 + X_4) \quad (19)$$

Considering that  $X_1, X_2, \dots, X_n$  are the variables, represented by the ARCH family models;  $r_{ij}$  correspond to the factorial load or the correlation between the PCs and the forecasts of the models used;  $a_{ij}$  correspond to the contribution of each one of the models in the formation of the PC, that is, the weight that each model has on the formation of the PC, and  $\lambda_p$  represents the eigenvalues referring to the main component and that represents the variance corresponding to the explanation of each component. In this study, eigenvalues bigger than one were selected.

The second combination technique to be used was proposed by Gupta and Wilton in [9], and it is based

on the arithmetic mean of the forecast models as shown below by expression 20.

$$F_c = \frac{ARCH + GARCH + EGARCH + TARCH}{4} \quad (20)$$

Where  $F_c$  is the forecast combination,  $F_1, F_2, \dots, F_n$  correspond to the forecasts of the individual models and  $n$  is the number of individual models used in the combination. According to Makridakis and Winkler [16], using a combination of arithmetic average is better than using the forecast of a bad model. This method is one of the most frequently used in the literature for the sake of convenience and because of the good results it generates compared to individual analyses. The third technique is the estimation by Ordinary Least Squares (OLS) method, having as regressors the forecasts of individual models and as a dependent variable, the differenced series that gave rise to each model, where it is known that each regressor will have a weight factor to be estimated by considering the time horizon used in the forecast.

$$F_c = \alpha_0 + \beta_1 ARCH + \beta_2 GARCH + \beta_3 EGARCH + \beta_4 TARCH \quad (21)$$

Where  $\alpha_0$  corresponds to the linear coefficient of the regression;  $\beta_n$  is the angular coefficient, the one which will give the weight to each model (regressor);

Using the forecast combination by the techniques mentioned above aims to produce better results than those achieved by using individual models. In addition, it can also show which combination method provides better forecast performance, considering the series under study.

Soon after volatility was modeled with ARCH models, the principal component analysis will be used for the combination of forecasts formulating three different kinds of combination by PCA, considering the weight factors: factor loading, the contribution of models and eigenvalues. These will be evaluated jointly to verify which kind of weighting works best. Furthermore, the combinations by PCA will be compared to combinations by OLS and Arithmetic Mean, and also the individual models. These comparisons will be made by the following performance criteria: MAPE, MSE and U-THEIL. The combined forecasts are expected to have better results than individual forecasts. In addition, it is believed that the combination by PCA will provide better results than the others for all criteria because of the differenced method to obtain the weights.

## 5 Results and Discussion

In this research, two macroeconomic series were used: the SELIC interest rate series from January, 1974 to June, 2012 and the exchange rate series from January, 1980 to May, 2012. They are both relevant in the Brazilian macroeconomic scenario; they influence other economic variables as well as decisions for investments by Brazilian companies.

### 5.1 Analysis and Modeling of the Time Series

The first step of this research was to perform a visual analysis of the series over time. Figure 1 shows that, apparently, the series are not stationary at level. Also, autocorrelation was observed between observations by means of CAF and PCAF, where the coefficients slowly decrease as the number of lags increases, indicating that there is a strong dependence on the data. After this step, the following tests were performed: Augmented Dickey-Fuller (ADF), and the test proposed by KWIATKOWSKI, PHILLIPS, SCHMIDT, SHIN (KPSS), where it was confirmed that the series are not stationary at level. Then, the first difference was verified in the series, and tests were subsequently redone showing that both series, at first difference, became stationary.

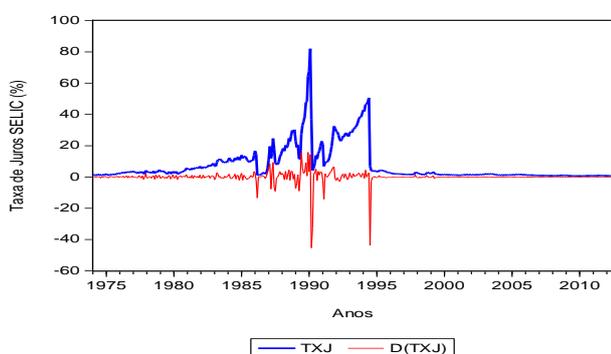


Fig. 1 – Behavior of the SELIC interest rate at level and with a simple difference.

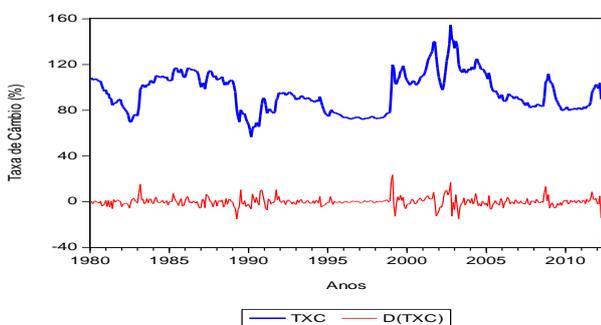


Fig. 2 - Behavior of the exchange rate level and with a simple difference.

After modeling of the series, several models were estimated in the search for the most suitable data set, and the selection was made by using the AIC and BIC criteria. Subsequently, the residuals of these models were analyzed and characterized as white noise. Still on the residual analysis, through the Jarque-Bera test, it was found that there are clues to heavy tails ( $kurtosis > 3$ ) for both modeling. In addition, the residuals do not have a normal distribution with  $p\text{-value} < 0.0001$ . The ARCH-LM test showed that there is heteroscedasticity in the residuals of the quadratic model ( $p\text{-value} < 0.0001$ ). After this first stage of the modeling of the process average, the joint modeling of the series was started, with addition of the modeling of variance by the models of the ARCH family. Thus, Table 1 shows the competing models of joint modeling of the average and volatility. The competing models shown in Table 1 have their significant parameters with 95% confidence. In addition, all models described have white noise residuals. The models that capture the asymmetry in the volatility shocks are EGARCH and THARCH, these models have the parameter  $\gamma$  nonzero, and confirming the asymmetry of information shocks us.

According to Ferreira, Menezes and Mendes [8], the leverage effect exists when  $\gamma < 0$  for the models EGARCH and  $\gamma > 0$  for the model THARCH. For the models in Table 1, the existence of the leverage effect was observed only for the model THARCH (0,1) for the exchange rate. The other models were asymmetric, but there was no effect of leverage.

### 5.2 Forecast Combinations and Individual Forecasts

The individual models and the combination forecast methods for the two series used in this study were performed with the following software: Statistica 9.1, Eviews 7.1. As shown in the previous section, the models considered for the forecasts were the volatility models ARCH, GARCH, EGARCH, and TARCH. For forecast combination techniques, PCA was used in three ways: PCA<sub>1</sub>, weighting the forecasts by the factorial loads of the selected component, PCA<sub>2</sub>, weighting the forecasts by the contribution of the models in the PCs; and PCA<sub>3</sub>, weighting the forecasts by the explanation percentile of the CPs, the Ordinary Least Squares (OLS) Method and the Simple Average (SA).

The PCA technique enables a linear combination of the ARCH models, of each model within their respective components, and the eigenvalues, which are the contributions that will provide the eigenvalues of each component. Therefore, these









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