An application of Real Option Analysis for the assessment of operative flexibility in the urban redevelopment

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Abstract: - The high variability of market prices and the uncertainty that, even in restrained timeframes, is characterizing the general economic situation, have led real estate operators to a prudent attitude, who tend to postpone or at least stagger the start of the initiatives on hold of more stable conditions. In this context it is appropriate to use evaluation tools enable to enhance the investment capacity to be adapted to possible changes of the conditions initially hypothesized. In the present research Real Options Analysis (ROA) is applied to the evaluation of an investment in urban redevelopment of a former industrial complex. The result obtained shows the efficacy of the instrument. Assuming that the entrepreneur considers affordable the implementation of the initiative if the outcome of the discounted cash flow analysis is at least equal to a threshold value calculated as a percentage of revenues, the application of ROA returns an extended NPV that meets this constraint, whereas the use of traditional NPV suggest to abandon the project idea. The binomial approach used also allows to accurately monitor the project's development, correlating it to the evolution of the market. The work must be attributed in equal parts to the authors.

Key-Words: - real options, binomial model, urban redevelopment, risk analysis, financial feasibility.

1 Introduction

The current historical moment is characterized by a unique combination of economic phenomena and national and international policy choices that have negatively affected the real estate sector. In Italy, as in Bulgaria, Greece, Ireland, the Netherlands, Portugal, Czech Republic, Romania and Spain, an oversupply, a contraction of sales and the price instability [13] are evident. Negative trends are expected in the short term in France [33].

Several factors, partly exogenous and partly endogenous to investment property, are contributing to determine this situation. Among the first, these must be mentioned: the drastic reduction, operated by the banks, of the disbursement of credit to households and businesses that do not have strong guarantees; the growth of interest on loans; the burden of taxation on real estate, and more generally, the tax burden; the loss of purchasing power of the currency, arising from a wage freeze, a rising inflation and the rise in the prices of consumer goods; the rampant unemployment.

In these macroeconomic and general circumstances, those related to the operations in the territory must be added, which occur especially in urban transformation initiatives or in the redevelopment of industrial brownfield sites in disuse [25]. The reference, in the first place, is the weight on the total cost of the initiative of the fixed expenses, linked to the presence in the production process of fixed factors of production. The amount of these costs always weighs with the same consistency on the budget of the operation, whatever the amount of product realized. In urban redevelopment fixed costs are related to the acquisition of land, its environmental remediation and restoration, the urbanization and the infrastructure for mobility, the recovery of existing buildings, the establishment of spaces and equipment of collective interest. These ones are consistent works whose amounts, for the model of negotiated planning that is being affirming, tend to be moved to load the private operator. The consistence of these amounts is a delicate moment of the initiative: if the amount is high, it may cancel or reduce the financial feasibility of the operation; the disproportionate incidence of fixed costs compared to variables costs can generate the "instability" of the budget of the operation to changing market conditions [30].

Secondly, the type of redevelopment should be considered, linked to the size of the initiative and the level of degradation of the area, which can affect the times of transformation, for the duration of the reclamation of areas, in relation to the intended uses, the geological nature of the soil, the characteristics of the private investor which realizes the transformation.

Thirdly, the financial structure of the initiative should be taken into account, valued in terms of debt and equity.

The phenomena outlined have an impact on supply and demand of real estate, inducing a cautious attitude in the operators, who prefer to postpone or at least stagger the launch of the initiative, waiting for more favorable market conditions, due to the variability of market prices and the uncertainty about their progress in restrained timeframes [11, 27, 34].

In the traditional valuation approaches, the estimative basics which require the detection of historical data in the spatial horizon of the analysis and/or the construction of time series of appreciable size collide with the high volatility of the possible evolution of scenarios [17]. Therefore it is appropriate to use "dynamic" assessment tools, which allow to exploit the ability of the investment to be adapted to potential changes of the conditions initially considered [38].

The Real Options Analysis (ROA), where changes in scenario assumptions are predictable, allows decision makers to adopt pricing strategies and to evaluate the effects that result from the investment choices (options), different from the initial ones [12].

The ROA is a technique for evaluating investments that can be used with success to manage the uncertainty related to possible changes of scenery. Compared with the "static" approach, that considers the present value of cash flows expected to the most likely future scenario, the ROA, when it is possible to transform the uncertainty of the project into risk, allows to carry out the risk analysis for the different options.

Ultimately, having math flexible architecture to changes that occur during the development of the project, the ROA extrinsic the investment capacity to adapt to new events.

In the literature several applications of the ROA in the field of real estate development have been treated. Some Authors [2, 5, 32, 34, 38] have investigated flexibility in real estate development, considering the option to delay development, time to build option, the option to alter land density, the option to switch land use and others. In land development, Capozza and Sick [6] examined the value of the option to redevelop a property. Quigg [28] examined the empirical prediction of real option-pricing models using 3,000 urban land transaction data of developed property and unimproved land parcel within the city of Seattle; the Author concluded that the value of vacant urban land should reflect not only its value based on its best immediate use, but also its option value should development be delayed [20]. Grenadier [16] aims to choose the optimal tenant mix relative to a shopping center. For this purpose the standard features and a distinction between the different types of store, the qualitative preferences (high level of interior trim or not) and the cost of flexibility have been considered. Childs et al. [7] use a numeric model to analyze the effect of sequential investment on property value by evaluating a sequence of American calls (options to wait) without dividends. In the presence of relatively low costs to conversion, flexibility with respect to mixed uses and redevelopment contributes significantly to the value of the built property or undeveloped land [21]. Ward and French [37] applied an option pricing model to examine the right to restrain upwards-only rent reviews on the attractiveness of property as an investment and found that the impact of the right is non-trivial. Rocha et al. [29] consider instead various opportunities for real estate development in the Brazilian reality, such as the acquisition of information, the wait or the abandonment of the investment. Considering fixed the building volume of the area because of planning restrictions, it is shown that the optimal decision, instead of the alternative to develop the whole lot or to divide the initiative in two sequential stages, depends on the time in which the expected incomes exceed the critical point of threshold. Geltner [14] have included real option theory as a core part of land development decision-making.

2 Aim of the study

With specific reference to urban redevelopment, this work aims to highlight the potential of Real Options Analysis (ROA) as a tool to support the choices of investment in real estate [4, 22]. In fact, although the urban regeneration identifies an area where long the attention of the main European countries is focusing, the application of ROA in real estate development so far is relatively limited [31].

The opportunity to employ a "dynamic" process of evaluation is particularly incisive in the current economic situation, characterized by high volatility in property values and by the resulting uncertainty related to the calculation of the Net Present Value associated with the application of traditional Discounted Cash Flow Analysis (DCFA) [19]. The enhancement of the project's ability to adapt to any possible disruption of the scenario initially hypothesized is the basis for the launch of real estate operations that require a substantial financial commitment [29]. Therefore, the ROA for the verification of the financial feasibility of an investment property that provides for the urban redevelopment of a former industrial complex and the construction of a park for mixed use (residential, commercial and underground garages) is here applied. The ability to delay the start of work and to divide the execution of the works into functional lots constitute an added value that can be caught in the development of the financial analysis only through the use of ROA.

The analytical formulations of ROA are manifold. In this work the binomial paradigm is used, which develops the changes of the initial value of the investment through probabilistic multiplicative states, that represent the evolution of the initial situation to a favorable scenario or to an unfavorable scenario [9].

The work is structured as follows. In Section 3 the case study, i.e. the urban redevelopment of a former industrial complex located in the central area of a municipality in the Province of Salerno (Italy), is illustrated. The dimensional parameters of the initiative and the prices and the costs of building products are outlined, and financial analysis is carried out using a "traditional" DCFA. In Section 4 the ROA is applied to the case study. The theoretical and practical aspects of risk analysis, of strategic analysis and quantitative analysis are developed, that identify the three phases in which the implementation of the ROA is divided. Finally, the results of the calculations are discussed and the conclusions of the work are taken.

3 The case study

The case study concerns a redevelopment of a former industrial complex located in the central area of a town in the Province of Salerno (Italy).

The area, which covers 17,000 m^2 of surface, is in fair conditions of usability and need only few interventions of reclamation. On the area insist 71,942 m^3 of abandoned buildings, mostly in poor conditions.

The specificity of the types of construction and the decay of the buildings do not recommend the conversion to new uses of the existent structures, but direct to their replacement.

The redevelopment is expected by the current planning instrument. On the basis of planning rules, in particular, 40,030.65 m^3 of new buildings can be realized, that correspond to 9,304.77 m^2 of residential gross floor area and 2,358.00 m^2 of commercial gross floor area.

Seven of the eleven buildings in the initiative have a height of 12.50 m (for four floors); the ground floor is intended to commercial use, the upper floors are house residences. Three buildings have three floors with a total height of 9.45 m, and are intended to residential use. The last building, also in residential use, have two floors, with a total height of 6.50 m.

Under eight of the eleven buildings, underground garages are planned, for a total of 75 units, whereas the other three buildings are served by parking spaces on the ground floor.

The project will be completed by the external work and the construction of a link road between two existing municipal roads located north and south of the complex.

The financial feasibility of the initiative has been verified with the traditional DCFA. The data for the construction of the business plan have been obtained by integrating the amounts reported in the official lists of the territory with the information gathered through a survey of construction companies and operators in the local real estate. The main financial data are presented in Table 1.

COSTS	
Land purchase	150.00 €/m^2
Demolition of existent buildings	17.00 €/m^3
Constructions of new buildings	900.00 €/m^2
Construction of underground boxes	15,000.00 €/box
External work	290.00 €/m^2
REVENUES	
Residential market value	2,200 €/m^2
Commercial market value	2,300 €/m^2
Box market value	30,000 €/box

Table 1 – Market data for the implementation of the financial analysis

As basis of the evaluation the following assumptions have been considered: the time of the valuation is the second half of 2013; the analysis period, including the construction phase, is four years, divided into eight semesters; the current prices system has been assumed re-evaluated annually on expected inflation; sales have been assumed to be uniformly distributed over the eight semesters of the analysis period in the proportion of 12.50% per semester; the annual discount rate is 6.50%, which corresponds to a half-year rate equal to 3.20%.

The calculations return a Net Present Value (NPV) of the project equal to € 5,998,935.29.

This amount , which corresponds to 25.36% of the total revenues of the entire initiative (*Revenues*), amounting to € 23,655,107.61, identifies the total profit of the operation (U_p).

However, given the current economic conditions and the significant uncertainty involved in this operation, the market survey has revealed that the profitability threshold currently set by the operators in the zone to launch similar investments ($\overline{U}$), computed in terms of the ratio between the NPV and the total revenues, is at least 27%.

Therefore the results obtained by the canonical DCFA induce to abandon the project.

4 Application of the ROA to the case study

The usefulness of real options emerges just in cases where the NPV is close to zero or, as in this case, it reaches values close to the threshold of acceptability of an investment [24].

In these circumstances, the identification and the translation into monetary terms of real options, i.e. the strategic opportunities that are activated with the implementation of the project and that can be captured with the actions that the entrepreneur can undertake in response to changes of scenario, can lead to recognize the convenience of initiatives that according to traditional logic would be discarded [35].

In fact, the entrepreneur, in relation to its capacity, can respond to changes in the market through behaviors that can benefit from the positive scenario or viceversa that may limit the consequences generated by an unfavorable evolution of the variables that affect the feasibility of the initiative [3].

The opportunity to defer the launch of the work, due to the uncertainty related to the performance of the real estate market, as well as the ability to divide the project into three functional lots to be carried out in sequence, are the added value of the intervention that can be quantified with the use of ROA.

In the literature, different approaches are illustrated for the implementation of the Real Options Analysis. Whatever the procedure used, it is possible to distinguish three logical-operative moments that lead to the explicitation of the value of the options:

- Risk analysis;
- Strategic analysis;
- Quantitative analysis.

Although the path is organized in a "cascade" mode, so that the results of each phase form the starting point of the next, the connections between the various steps do not allow a clear separation and give rise to a linear iterative process, which can sometimes require the return on the factors already analyzed, in order to deepen the study or broaden the spectrum of the investigation.

In the terminology of the ROA, the project of which is estimated the extended NPV - sum of the NPV obtained by the application of a traditional DCFA and the value of real options - constitutes the "underlying asset". The ROA developed here pertains to the so-called *discrete* models [10], in which the value of the underlying asset changes into specific points in time and can take only specific predetermined values. The simplicity of the analytical formulations and the schematic of the logical process make the procedure easy to understand and give an additional ease of use.

Therefore, the ROA of the 1° functional lot is developed, considering it as the "main" project (underlying asset), whose the extended Net Present Value is determined, whereas the other two lots constitute opportunities for future developments.

4.1 Risk analysis

The risk analysis allows the detection and the investigation of the uncertain variables of the initiative: the decisions related to the project depend on their evolution [18].

This phase is divided into two steps: the identification of the critical variables of the project and the valuation of the volatility.

The identification of the critical variables depends on the specific area in which the initiative pertains, the market at the time when the valuation is developed, the availability of data and information on which the analysis is set. Of these variables the connection with the underlying project is also necessary to analyze.

In the present case the selling price of residential building products has been identified as a critical variable.

The choice of this variable has depended on a variety of reasons. Meanwhile, the selling price, as well as being the synthesis of the forces operating on the market at the time of the exchange, incorporates the market expectations about the future. The decisions of the investor seem to be linked to the selling prices, as the launch of the initiative in question is attached to the condition that the profit, calculated as a percentage of expected revenues, exceeds the threshold of 27% of revenues.

Even if the project involves a mix of functions consisting of residential use, commercial use and parking, the selling price of the residences identifies the critical variable. This is for several reasons. First of all, it is the use with the greatest weight within the framework of the realizations; the major changes in terms of price are expected for the residential destination: in fact, the commercial use does not show large variations over time whereas the fiscal policy and the credit crunch to households have ended up hitting just residential real estate.

The next step involves the formalization of a stochastic model that simulates the evolution of the uncertain variable.

For the study of real options the class of Markovian stochastic processes is considered, which have the property that the value assumed by the uncertain variable at a given time contains all the information necessary to determine the likelihood of its future states. Among the Markovian processes, in particular, a process called *Brownian motion* is chosen. An uncertain variable can be described by a Brownian motion if it is characterized by two parameters, the *drift*, which identifies its instantaneous percentage variation expected, and the variance of the percentage change, named *volatility*.

In the literature, different approaches are proposed to estimate volatility. The availability of historical data of the critical variables previously identified is a necessary condition for the implementation of these techniques. When a significant sample of past values of the uncertain variables is not known, a sensitivity analysis could be used, i.e. the analysis of different values of volatility and therefore the definition of a range of variation for the value of the investment.

The procedure applied in this work for the estimation of the volatility of the selling prices uses the regression analysis, implemented on historical data of uncertain variables identified [23, 26]. Estimated the function that interpolates the data of the time series of selling prices, the volatility of the investment is calculated as the standard deviation of the residuals computed between the detected values and the values estimated.

The analytical evaluation of the volatility (σ) is performed using the statistical formula of the standard deviation:

$$\sigma = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (R_i - \overline{R})^2} \qquad (1)$$

where:

$\sigma =$ volatility of the uncertain variable X;
n = number of years for which historical data of the variable X are available;
$R_i =$ residual at year i expressed as a percentage, i.e. the percentage deviation of the historical data X_i with respect to the corresponding value of the interpolating function X_{ir}, deviation obtained with the equation $R_i = \frac{(X_i - X_{ir})}{X_{ir}}$;
$\overline{R}$ = average of residuals R_i.

Table 2 shows the components for the calculation of the residues R_i. In Table 3 the estimated statistical characteristics of the interpolating function are illustrated. "R-squared corrected", the "significance of the F Fischer" and the "significance of the Student's t" denote a good reliability of the linear regression model with intercept adopted. The use of the formula (1) returns a value of volatility (σ) equal to 22.03%.

4.2 Strategic analysis

Strategic analysis concerns the identification of areas of managerial adaptability, i.e. the strategic opportunities inherent in the project and which the investor could provide.

Of the three phases of ROA, strategic analysis is the *subjective* moment: it is at this stage that the ability of the investor - or of the technician who operates in support of its decision-making - to explain the added value of the project related to options that are immanent in it, is manifested.

This analysis defines the characters of the options provided, necessary to quantify their value in the later stage of quantitative analysis. In fact, the type and the algorithms that synthesize the condition of exercise is necessary to identify for each option.

In the literature, the options are grouped into various types [36]. In this case, the opportunities inherent in the main project are related to the possibility to *defer* the start of the works and to *expand* the size of the initial operation.

The option to defer reflects the opportunity to delay the implementation of a project, to take advantage of the likely positive developments or to avoid negative market trends that have been estimated to occur until the extension date for the launch of the investment: the value of the option is related to the possibility of obtaining additional information in the time of deferral (these opportunities exist if in the period of deferment may happen something that increases the NPV of the project).

This option is applicable whenever the implementation of the project can be delayed in time without compromising the technical and administrative feasibility.

The time indeed has a financial value that should be considered in the assessment of the project (lost revenues determined by the postponement of the initiative); but it also has an informative value, which is an important resource, especially in those areas that, for contingent or structural circumstances, are subject to rapid change. It is an option particularly strong in the areas where the final product is characterized by volatile market prices (in relation to changes in demand, changes in interest rate, impacts of fiscal policies), or in which the choices related to the project generate strong margins of irreversibility (typical case of real estate). It is as if the project competes in time "against himself".

The option to expand is a *call* option that may be issued on the intervention being evaluated on the basis of the opportunity to make additional investments that increase the cash flows of the initial project. It assesses the opportunity to expand the scale of the project.

A *compound* option is considered [15], that consists of the following opportunities in chronological order: a) the option to defer the launch of the works of the project in three years, b) the option to expand the implementation of the 2° functional lot, viable from the second year and by the end of the fifth semester of the analysis period considered, c) the option to expand the realization of the 3° functional lot, feasible from the second year. The volumes and gross floor surfaces (SLP) pertaining to each functional lot considered are shown in Table 4. It must be noted that the listed opportunities to expand and to defer are *sequential* options. In fact, every viable option changes the economic characteristics of the main project, thus defining a new underlying asset which is related to the possible activation of the subsequent options.

Year	***Detected price*** [€/m^2] ***(a)***	***Estimated price*** [€/m^2] ***(b)***	***Residual (c) = (a)-(b)***	***Percentual residual R_i = (c)/(b)***
1971	45	29.32	15.68	0.53
1972	47	33.12	13.88	0.42
1973	48	36.91	11.09	0.30
1974	48	40.71	7.29	0.18
1975	49	44.51	4.49	0.10
1976	49	48.30	0.70	0.01
1977	46	52.10	-6.10	-0.12
1978	60	55.90	4.10	0.07
1979	69	59.69	9.31	0.16
1980	74	63.49	10.51	0.17
1981	76	67.28	8.72	0.13
1982	76	71.08	4.92	0.07
1983	72	74.88	-2.88	-0.04
1984	93	78.67	14.33	0.18
1985	107	82.47	24.53	0.30
1986	108	86.26	21.74	0.25
1987	108	90.06	17.94	0.20
1988	100	93.86	6.14	0.07
1989	93	97.65	-4.65	-0.05
1990	91	101.45	-10.45	-0.10
1991	90	105.24	-15.24	-0.14
1992	94	109.04	-15.04	-0.14
1993	97	112.84	-15.84	-0.14
1994	94	116.63	-22.63	-0.19
1995	90	120.43	-30.43	-0.25
1996	97	124.23	-27.23	-0.22
1997	104	128.02	-24.02	-0.19
1998	102	131.82	-29.82	-0.23
1999	99	135.61	-36.61	-0.27
2000	99	139.41	-40.41	-0.29
2001	101	143.21	-42.21	-0.29
2002	116	147.00	-31.00	-0.21
2003	130	150.80	-20.80	-0.14
2004	129	154.59	-25.59	-0.17
2005	127	158.39	-31.39	-0.20
2006	149	162.19	-13.19	-0.08
2007	190	165.98	24.02	0.14
2008	205	169.78	35.22	0.21
2009	226	173.58	52.42	0.30
2010	239	177.37	61.63	0.35
2011	235	181.17	53.83	0.30
2012	228	184.96	43.04	0.23

Table 2 - Components for the calculation of the residues

Statistical regression	
R-multiple	0,87
R-squared	0,76
R- squared correct	**0,75**
Standard error	26,55
Osservations	42

	gdl	*SQ*	*MQ*	*F*	*Significance F*
Regression	1	88920,47	88920,47	126,14	**6,11407E-14**
Residual	40	28196,67	704,92		
Total	41	117117,14			

	Coefficients	*Standard error*	*Stat t*	*Significance value*
Intercept	25,52	8,34	3,06	**0,003**
Variable X 1	3,80	0,34	11,23	**6,11407E-14**

Table 3 - Statistical characteristics of the interpolating function

		Volume [m^3]	SLP [m^2]	SLP [n]	Market value [€/m^2]	Market value [€/n]
Main project 1° lot	Residential	19,003.37	4,075.41	-	2,200	-
	Commercial		1,358.00	-	2,300	-
	Underground parking	-	-	50	-	30,000
2° lot	Residential	13,975.75	3,000.00	-	2,200	-
	Commercial		1,000.00	-	2,300	-
	Underground parking	-	-	-	-	-
3° lot	Residential	7,051.53	2,229.36	-	2,200	-
	Commercial		-	-	-	-
	Underground parking	-	-	25	-	30,000

Table 4 - Sizing and economic parameters for the functional lots

4.3 Quantitative analysis

In the quantitative analysis the value of real options is determined. The analytical formulation adopted borrows the logic of *dynamic programming* for solving optimization problems of decisions. This technique involves the construction of a *binomial* model, according to which the value of the underlying asset is a stochastic variable that changes over time evolving through only two probabilistic multiplicative states, defined by the coefficients $u>1$ and $d<1$. These coefficients are, respectively, the evolution of the initial state to a favorable scenario (u) or to an unfavorable scenario (d). The method of solution develops the possible values of the underlying asset over the life of the option and then discounts at the present time the value of optimal decisions in the future.

The dynamic programming is based on the principle of *Bellman*: defined an initial strategy, the optimal strategy for the next period is the one that would be chosen if the whole analysis began at that time. The optimal strategy is identified by an iteration backwards method, and by discounting future values and cash flows and embedding them in the current decision.

In the quantitative analysis of the binomial model, there are three steps:

1) valuation of the NPV of the underlying project;
2) modeling of the stochastic process through the scenarios tree;
3) construction of the decision tree and determination of the extended NPV.

4.3.1 Valuation of the NPV of the underlying project

The use of a traditional DCFA returns the Net Present Value of the project. In this step the evaluator has to discern the revenues and costs of the intervention, the analysis period, the discount rate and other parameters that influence the outcome of the evaluation.

In order to consider only the voices characterized by components of uncertainty for further analytical processing, the NPV with the exception of the investment costs has been determined, as these items are routinely characterized by deterministic nature. The value obtained (*VA*) is a stochastic variable that does not yet take into account the flexibility inherent in the investment.

The Net Present Value of the underlying project is determined by considering the same parameters used in case the DCFA is applied to the entire project solution and quantifying revenues and costs related to the different size of the intervention. The analysis period is still four years, divided into six-month intervals ($dt = 0.5$) and it is assumed that the sales are spread evenly over the eight semesters. The result is a NPV of the main project (first functional lot) equal to € 2,682,295.

The value (*VA*) of the underlying project is formed by the accumulation at the time of the assessment of revenues generated from the sales of building products.

The subsequent processing requires the development of a DCFA for the two options to expand identified (2° and 3° functional lots). Table 5 summarizes the results in terms of present value of the revenues and present value of the costs of the three functional lots evaluated individually.

	Present value of revenues [€]	*Present value of costs [€]*
Main project	11,424,130	8,741,835
2° functional lot	7,481,970	5,556,610
3° functional lot	4,753,650	3,362,370

Table 5 - Revenues and costs of the functional lots

4.3.2 Modeling the stochastic process through the scenarios tree

In the binomial model, the mathematical expressions for the calculation of the two possible states of evolution (u and d) of the value of the underlying project are:

$$\begin{cases} u = e^{\sigma \cdot \sqrt{dt}} \\ d = e^{-\sigma \cdot \sqrt{dt}} \end{cases} \quad (2)$$

where:

e = Euler number;

σ = riskiness of the underlying project revenues (volatility);

dt = time interval between two successive periods.

Depending on the values of *u* and *d*, *VA* of the underlying project changes in each time interval with a "jump" evolution, represented by a typical tree structure (*scenarios tree*). Considering an analysis period of five time intervals, the value of the underlying project changes according to the scenarios tree shown in Figure 1.

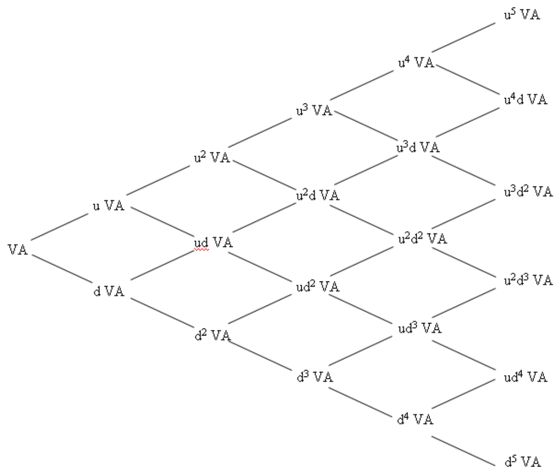

Figure 1 – Evolution of the event tree

The formulas of (2) are used to define the evolution of *VA* of the 1° functional lot (main project). The parameters required for the construction of the scenarios tree are summarized in Table 6.

σ	22.03%
u	1.169
d	0.856
dt	0.5

Table 6 - Parameters for the construction of the scenarios tree

4.3.3 Construction of the decision tree and determination of extended NPV

Starting from the scenarios tree, the binomial model enhances the options inherent in the underlying project by constructing a tree diagram (decision tree) for each option identified.

A specific maximization function is associated with each class of options. The maximization function works by comparing, in the exercise period (*m*) of the option and for every possible scenario of that period, the value of the activation of the option with the alternative "zero" (non-exercise of the option), and selecting the major of the two values. The maximization algorithm, applied in all scenarios *n* of the period in which the option is feasible, allows to determine a vector of the majors, with size $n \cdot 1$.

Unlike the scenarios tree, which is built from left to right, i.e. from the initial moment until the final instant of the time period of evaluation, the evolution of the decision tree is defined backward, moving from the period on which the option is exercised (*m*) up to the year zero. The operation is carried out by weighting the elements of the vector of the majors determined, through coefficients called *risk neutral probability*, determined by (3):

$$p = \frac{e^{(r_f - \delta)dt} - d}{u - d} \quad (3)$$

where:
e = Euler number;
r_f = risk free discount rate;
δ = loss in value of the underlying project[1].

The result is then brought to the current moment through a risk-free discount rate and because of the time interval *dt* between two successive periods.

In the case of the activation of *sequential* options during the analysis period, the operative logic of the algorithm requires the verification of the global effect of the options from the furthest one chronologically, then gradually implementing the procedure for the other options closer to the time of the assessment. The mathematical expressions that lead to the value of the compound option are built in such a way that the value of the *i*-th option "has memory" of the value of the chronologically later option [9].

In the present case, it is necessary to determine in advance the expansion coefficients for the second and the third functional lots (respectively, a_2 and a_3) and the strike prices of the three options (I_{a2} and I_{a3} for the options to expand and I_d for the option to defer), in order to implement the related optional algorithms.

Developed the DCFA for the two options to expand, the corresponding coefficients are considered to be equal to the percentage increase of the revenues of the main project determined by the exercise of each option to expand considered. The strike prices are given, for the two options to

[1] The loss of value resulting from cash flows or returns of convenience occurred between two points of possible decision are a characteristic of real assets, which must be taken into account in an evaluation model of real options. For option pricing models it is very convenient to treat the cash flows or returns of convenience as a constant percentage of the value of the underlying asset [1]. The parameter δ is representative of a constant proportionality between the cash flows of each reporting period and the present value of the underlying asset.

expand, by the investment costs of the second and the third functional lots; for the option to defer, by the investment cost of the main project.

In Table 7 the main parameters that contribute to the estimation of risk neutral probability using the formula (3) and to the definition of the algorithms of the binomial model are explained.

σ	22.03%
u	1.169
d	0.856
r_f	3%
δ	12.50%
p	0.313
dt	0.5
a_2	0.65
a_3	0.42
I_{a2}	5,556,610
I_{a3}	3,362,370
I_d	8,741,835

Table 7 - Parameters of the algorithms of the binomial model

The mathematical formalization of the maximization functions of the sequential options identified (option to defer and two options to expand) is shown in Table 8. In Table 9 the meaning of the terms that appear in the equations is explained, using the apex i = (*a3*, *a2*, *d*) to indicate the *i*-th option among those in analysis (to expand 3° lot, to expand 2° lot, to defer).

The analysis returns a value of the compound option considered (V_{option}) amounted to € 3,998,052.

The extended NPV of the main project is equal to the sum of the NPV of the underlying asset and the total value of the sequential options identified:

$$extended\ NPV = NPV + V_{option} = 2{,}682{,}295 + 3{,}998{,}052 = 6{,}680{,}347\ € \quad (4)$$

The return for the entrepreneur (U_p), equal to the ratio between the value of transformation and total revenues of the initiative is the following:

$$U_p = \frac{extended\ NPV}{Revenues} = \frac{6{,}680{,}347}{23{,}655{,}107.61} = 28.24\% > \overline{U} = 27\% \quad (5)$$

Therefore, the entrepreneur's profit is greater than the threshold of financial acceptability fixed. The difference in value between the application of ROA and the development of a traditional DCFA - difference of about € 700,000 - allows to appreciate the financial feasibility of the project analyzed.

The binomial approach employed, compared to other procedures of ROA characterized by higher mathematical formalisms, provides the operator with an additional benefit, related to the ability to monitor the investment in the course of its implementation. The graphical representations shown in Figures 2, 3 and 4 are the decision trees, obtained by implementing the algorithms of the options identified for each time of the analysis period. A first information immediately obtainable from the trees is qualitative, and it is related to the period in which the option is viable (or in which it is affordable to delay the start of the project, in the case of the option to defer). In this work, decision trees suggest starting the first functional lot already in the second semester of the analysis period employed and to realize the other two lots from the second year.

The other contribution of the model, of quantitative type, is connected to the opportunity to determine the *minimum* value of the underlying project for the activation of the options. It should be noted that, for the same period *dt* when the algorithm indicates a possibility of exercise, the corresponding nodes do not provide all the same result in dichotomous terms (the option should/should not enable). For example, with reference to the option to defer and the period *dt* = 2, since the evolution of the decision tree is a function of the present value (*VA*) of the underlying project, there will be a *limit* value of the underlying asset below the which should still wait before investing in the project. Therefore, for each decision tree and in respect of each period of activation, it is possible to identify, in an iterative way, the *minimum limit* value of the revenues generated by the transformation (and therefore the market values of the building products) that makes it convenient to enable the project or to exercise the corresponding option to expand. Thus, the limit value of revenues to enable the main project at the second semester is € 10,860,000, whereas it becomes € 9,680,000 if the operator prefers to postpone the execution; the limit value which makes it convenient to carry out the 2° functional lot at the second year is € 9,585,000, whereas it is amount to € 10,070,000 if the entrepreneur decides to observe the evolution of the market; the limit value to activate the 3° functional lot in the second year is € 9,585,000 and it is equal to € 10,670,000 if the entrepreneur waits another six months.

Option to expand 3° lot	
$t = 8$	$VA_t^{a3} = \max(a_3 \cdot VA_t - I_{a3}),0]$
$3 \le t < 8$	$VA_t^{a3} = \max[(a_3 \cdot VA_t - I_{a3}), \frac{p \cdot VA^{a3+}{}_{t+dt} + (1-p) \cdot VA^{a3-}{}_{t+dt}}{e^{r_f \cdot dt}}]$
$0 \le t < 3$	$VA_t^{a3} = \frac{p \cdot VA^{a3+}{}_{t+dt} + (1-p) \cdot VA^{a3-}{}_{t+dt}}{e^{r_f \cdot dt}}$
Option to expand 2° lot	
$t = 5$	$VA_t^{a2} = \max(a_2 \cdot VA_t - I_{a2} + VA^{a3}{}_t),0]$
$3 \le t < 5$	$VA_t^{a2} = \max[(a_2 \cdot VA_t - I_{a2} + VA^{a3}{}_t), \frac{p \cdot VA^{a2+}{}_{t+dt} + (1-p) \cdot VA^{a2-}{}_{t+dt}}{e^{r_f \cdot dt}}]$
$0 \le t < 3$	$VA_t^{a2} = \frac{p \cdot VA^{a2+}{}_{t+dt} + (1-p) \cdot VA^{a2-}{}_{t+dt}}{e^{r_f \cdot dt}}$
Option to defer	
$t = 5$	$VA_t^{d} = \max(VA_t - I_d + VA^{a2}{}_t),0]$
$2 \le t < 5$	$VA_t^{d} = \max[(VA_t - I_d + VA^{a2}{}_t), \frac{p \cdot VA^{d+}{}_{t+dt} + (1-p) \cdot VA^{d-}{}_{t+dt}}{e^{r_f \cdot dt}}]$
$0 \le t < 2$	$VA_t^{d} = \frac{p \cdot VA^{d+}{}_{t+dt} + (1-p) \cdot VA^{d-}{}_{t+dt}}{e^{r_f \cdot dt}}$

Table 8 - Algorithms of the sequential options identified

VA_t^i	value of the *i*-th option at time *t*
VA_t	value of the underlying asset (1° functional lot) at time *t*
$VA^{i+}{}_{t+dt}$	value of the *i*-th option at time *t+dt* and assuming favorable evolution of the value at time *t*
$VA^{i-}{}_{t+dt}$	value of the *i*-th optionat time *t+dt* and assuming unfavorable evolution of the value at time *t*

Table 9 - Meaning of the terms in the algorithms of the binomial model implemented

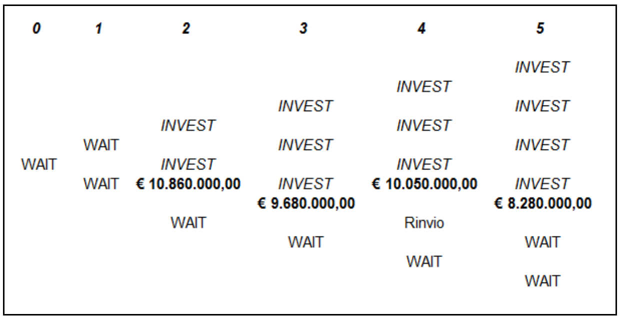

Figure 2 - Decision tree for the option to defer

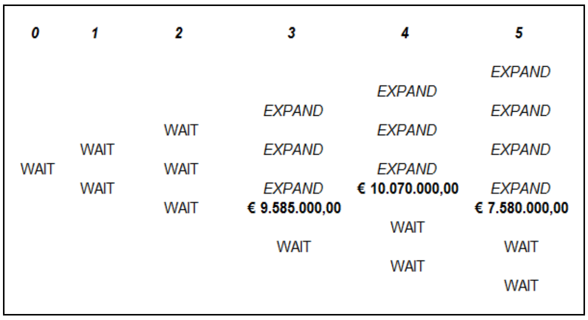

Figure 3 - Decision tree for the option to expand (2° lot)

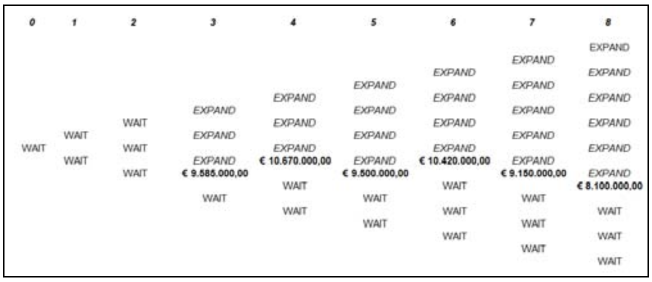

Figure 4 - Decision tree for the option to expand (3° lot)

5 Conclusions

In recent years the collapse of real estate values and financial stocks traditionally considered safe, has strongly influenced the economic behavior of the operators, generating the feeling that there is not an investment decision immune from risk.

In this context of uncertainty, the ability to adapt and modify the economic choices in a following step to the start of the investment and in relation to the opportunities that might arise during the course of the project, constitutes an added value with respect to a "rigid" project. The ROA is therefore an inescapable support for investment decisions: on the one hand, it enables to identify the flexibility inherent in the project and to use it at the most convenient time; on the other hand, it allows to quantify and include in the assessment the greater value associated to the opportunities identified.

In the field of real estate investments, the potentialities of the ROA have been enucleated applying the technique to the financial evaluation of a urban redevelopment project of an abandoned industrial complex. Even in an economic environment characterized by high market risks, in which the traditional assessment techniques suggest not to implement the project in analysis, the use of ROA leads to an opposite result. Enhancing the of factor "wait" to start the initiative and identifying the relationship of complementarity between the different functional lots that compose the project, the options identified return an added value that enhances the financial feasibility of the investment.

Moreover, the binomial approach employed has two advantages over other procedures of the ROA. In fact, it is easy to use and to read for the operator. Secondly, the decision trees allow to build a real *panel control* to guide the choices of the investor in relation to the results sought periodically from the realization of the project.

The "finance static", even if it is complemented by simulations on the sensitivity of some input variables or by probabilistic scenarios, is not able to enhance the flexibility in the management decisions. The real options belong to the "dynamic finance" because their value and their exercise depend on the evolution of the uncertain variables: the task of management is to control and to exercise these options at the best time.

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