

Joint TOA/AOA Location Algorithms with Two BSs in Circular Scattering Environments

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Abstract: In circular scattering environments, a scenario with two base stations (BSs) that the home BS measures time of arrival (TOA) and angle of arrival (AOA) while the neighboring BS only measures TOA is investigated. By incorporating the coordinates of scatterers as unknowns in the location algorithm, joint TOA/AOA Constrained Algorithm (TA-CA) is proposed. However, AOA equality constraint is not considered. By combining the AOA equality constraint into TA-CA algorithm, a new TA-CA algorithm is proposed. Due to the high computational complexity and nonconvex about TA-CA algorithm, a grid search algorithm is proposed. The grid search algorithm is formulated by introducing two scale factors and utilizing the hybrid line of position algorithm. The performance of the proposed algorithms are assessed and compared with that of existing algorithms through extensive simulation.

Key-Words: Time of arrival (TOA); Angle of arrival (AOA); Non-line-of-sight (NLOS); Algorithm

1 Introduction

The capability of accurately positioning a mobile station (MS) is one of the essential features in the existing and future wireless cellular communication systems. It was originated by Federal Communication Commission (FCC) rules, which suggested the cellular networks to provide the location for emergency calls. Besides, different service for MS location have been applied on the wireless communication networks, such as fleet management, location-based billing and intelligent transportation systems, etc[1].

The most popular methods used to estimate the MS location are time of arrival (TOA)[2], time difference of arrival (TDOA)[3], angle of arrival (AOA)[4], and received signal strength (RSS)[5]. The RSS measurement can be used to estimate the position of MS by using the path loss attenuation model of RSS with distance. However, it is difficult to obtain the accurate path loss model which is changeable in different environments. The TOA method calculates the arrival time from the MS to base station (BS), a circle is generated for each TOA measurement, and the position of MS can be estimated by the intersection of these circles. A hyperbola is generated with each TDOA measurement and the position of MS is estimated by the intersection of these hyperbolas. The AOA of

the MS to the BS is measured by an antenna array. A line from the MS to the BS can be drawn with each AOA measurement and the position of MS is calculated from the intersection of at least two lines. To improve the location accuracy of MS, the hybrid of TOA/AOA, TDOA/AOA or TOA/TDOA/AOA are also used to estimate the position of MS[6-7]. Due to hearability limitations, we investigate TOA measurements at two BSs where AOA measurement is only available at the serving BS in this paper. In fact, the hybrid location methods by combining TOA and AOA measurements can reduce the number of receiving BSs and improve the coverage of location-based service in cellular network simultaneously[8].

High accuracy can be obtained from the hybrid TOA/AOA methods with the assumption of line-of-sight (LOS) propagation. However, location errors will inevitably increase greatly as the assumption is violated by non-line-of-sight (NLOS) propagation. How to reduce the NLOS error and improve the location accuracy is a hot research topic. Taking into account the constraint on hearability when power control is employed to reduce interference, the works in [9-11] research the location scenario with three BSs to reduce the NLOS error. In [9], a location scenario with three range measurements and one AOA measurement at home BS is

considered, and proposes hybrid TOA/AOA algorithm (HTA) which utilizes non-linear constrained optimization to estimate the position of MS with bounds on the range and angle errors inferred from geometry. However, it does not use the information of scatterers. In [10], using the information of scatterers, a grid search algorithm is proposed, which generates the feasible point to judge whether it satisfies the inequality equation and averages all the potential points. However, it assumes the radius of scatter is known. Without knowing the radius of scatter, the work in [11] proposes TOA/AOA Constrained Algorithm (TA-CA) by utilizing the unknown scatterers' coordinates in the location estimation process. For two BSs location technique, the works in [8][12] have done some researches. In [8], it proposes a geometric method to locate the MS by introducing two scale factors and constructing a constrained objective function. However, it assumes the neighbouring BS experiences one bounce, which is not reliable [9][11]. Based on the ring of scatterers model[13], the work in [12] proposes hybrid TOA/AOA technique to mitigate NLOS error by introducing Taylor-Series and the multipath.

In this paper, we are concerned with two BSs location technique. A scenario that the home BS measures range and angle while the neighboring BS only measures range is considered. The circular scattering model is simulated as the NLOS propagation model. The main works of this paper are twofold.

1) TA-CA[11] is a nonlinear constrained algorithm, which the cost function and inequality constraints are all nonlinear. However, the inequality constraints do not consider the relation with angle. In fact, there is one hidden nonlinear equality constraint about angle. By introducing this equality constraint into TA-CA, the performance of TA-CA has improved significantly.

2) for our location scenario, we propose a grid search algorithm by introducing two scale factors. Different to the grid search algorithm in [10], the proposed algorithm does not utilize the information of scatterers. By introducing two variables (the step and number of scale factor), we can divide the scale factors into different combinations. For each combination, a possible point about MS is obtained by utilizing hybrid line of position (HLOP) algorithm[9] which has low computational complexity. A criterion for deciding whether to discard this possible point is constructed. If it satisfies the criterion, we call it the potential point and the final estimate of MS is averaging all the potential points.

The rest of the paper is described as follows. In Section 2, the circular scatterer model is presented. In Section 3, the location algorithms are proposed. Section 4 presents and comments the results of the simulation. Finally, Section 5 provides some conclusion.

2 System Model

The circular scattering model [13] is shown in Fig.1. The model assumes that the scatterers are uniformly distributed within a radius circle about the MS. The serving BS measures the range (the speed of light multiplies TOA) and angle (AOA), and the neighboring BS only measures the range. The measured ranges are the sum of the distance between the BS and scatter and the distance between the MS and scatterer. The measured angle at the serving BS is the angle between BS_1 and a scatterer located at (x_s, y_s) . The measured ranges $r_i, i = 1, 2$ and angle φ can be expressed as

$$\begin{aligned} r_1 &= \sqrt{(x-x_s)^2 + (y-y_s)^2} + \sqrt{(x_1-x_s)^2 + (y_1-y_s)^2} + n \\ r_2 &= \sqrt{(x-x'_s)^2 + (y-y'_s)^2} + \sqrt{(x_2-x'_s)^2 + (y_2-y'_s)^2} + m \\ \varphi &= \text{atan}\left(\frac{y_s - y_1}{x_s - x_1}\right) + v \end{aligned} \quad (1)$$

where (x_i, y_i) is the position of BS_i , (x_s, y_s) is the position of BS_1 scatterer, (x'_s, y'_s) is the position of BS_2 scatterer, (x, y) is the position of MS, atan is the function of inverse tangent, n , m and v are white Gaussian random variable with standard deviation σ_n , σ_m and σ_v , respectively.

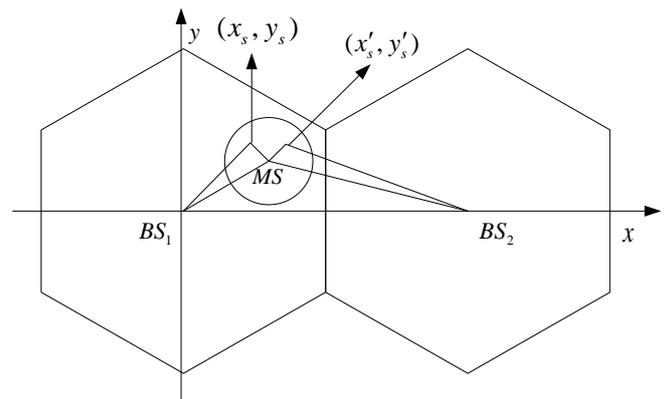


Fig.1 Geometry of circular scattering model

3 The Proposed Location Algorithms

In this section, we firstly present TA-CA algorithm, and then describe the new TA-CA algorithm. Due to high computational complexity and converge to

local optimum about TA-CA algorithm, a grid search algorithm is displayed.

3.1 TA-CA Algorithm

Since the TOA/AOA measurements are noisy, the following nonlinear least squares method is proposed. The unknown parameters (x, y) are estimated by minimizing the following objective function:

$$F_{TA-CA}(x, y, x_s, y_s, x'_s, y'_s) = \alpha_1(r_1 - \sqrt{(x-x_s)^2 + (y-y_s)^2} - \sqrt{(x_1-x_s)^2 + (y_1-y_s)^2})^2 + \alpha_2(r_2 - \sqrt{(x-x'_s)^2 + (y-y'_s)^2} - \sqrt{(x_2-x'_s)^2 + (y_2-y'_s)^2})^2 + \beta(\varphi - \text{atan}(\frac{y-y_1}{x-x_1}))^2 \quad (2)$$

where $\alpha_i, i=1,2$ and β are the weight factors that depend on the measurement accuracy.

In addition, constraints on the MS position can be deduced by using the fact that the distance between the MS position (x, y) and the $BS_i, i=1,2$ position should be less than or equal to r_i

$$\sqrt{(x-x_i)^2 + (y-y_i)^2} \leq r_i \quad (3)$$

Thus, the TA-CA algorithm is formulated as follows:

$$\begin{aligned} & \text{Minimize } F_{TA-CA}(x, y, x_s, y_s, x'_s, y'_s) \\ & \text{Subject to } \sqrt{(x-x_i)^2 + (y-y_i)^2} \leq r_i, i=1,2 \end{aligned} \quad (4)$$

3.2 New TA-CA Algorithm

The TA-CA algorithm only considers the inequality constraints about range r_i in (3). It does not consider the angle constraint. In fact, there is one hidden equation constraint about angle. As shown in Fig 2, AOA seen at BS_1 is φ and the corresponding angle spread is $\theta = \text{atan}(\frac{y-y_1}{x-x_1}) - \varphi$.

From the geometry relationships shown in Fig.2, it can be found that the equation $BC=AC-AB$ holds where $AC = (x-x_1)\sin(\varphi)$, $AC = (y-y_1)\cos(\varphi)$, and $BC = \sqrt{(x-x_1)^2 + (y-y_1)^2} \sin(\theta)$.

The following equation about angle and the position of MS is obtained as

$$(y-y_1)\cos(\varphi) - (x-x_1)\sin(\varphi) = \sqrt{(x-x_1)^2 + (y-y_1)^2} \sin(\theta) \quad (5)$$

Putting (5) into (4), a new TA-CA algorithm is formulated as

$$\begin{aligned} & \text{Minimize } F_{TA-CA}(x, y, x_s, y_s, x'_s, y'_s) \\ & \text{Subject to } \begin{cases} \sqrt{(x-x_i)^2 + (y-y_i)^2} \leq r_i, i=1,2 \\ (y-y_1)\cos(\varphi) - (x-x_1)\sin(\varphi) = d_i \sin(\theta) \\ d_i = \sqrt{(x-x_1)^2 + (y-y_1)^2} \end{cases} \end{aligned} \quad (6)$$

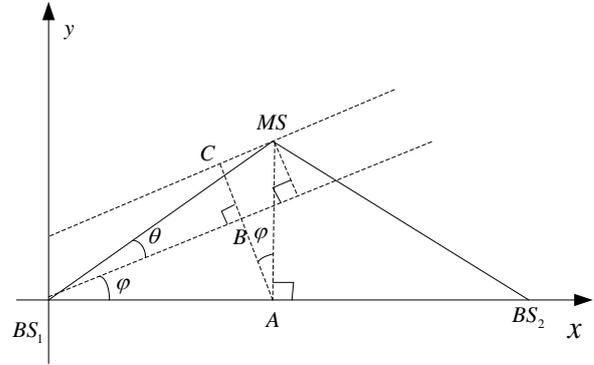


Fig 2. Constraint from angle measurement

3.3 Grid Search Algorithm

The solution space in (6) is six dimension, which has high computational complexity. Moreover, due to the nonconvex optimization about (6), the optimization algorithm may converge to local optimum. Therefore, it is not a good choice to adopt the optimization method in real implementation. In order to find better method, a grid search algorithm is proposed. The main idea of the grid search algorithm is described as following:

Based on the range model in[14-15], the true ranges can be expressed as the form of the measured ranges r_i by introducing two scale factors $\alpha_i, i=1,2$.

$$\alpha_i r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}, i=1,2 \quad (7)$$

Due to the NLOS propagation, the upper bounds of two scale factors are

$$\begin{cases} \alpha_{1,upp} = \min(1, \frac{R_c}{r_1}) \\ \alpha_{2,upp} = 1 \end{cases} \quad (8)$$

where R_c is the cell radius.

Without loss of generality, we assume two scale factors have the same number and step. Then two variables δ (the step of scale factor) and k (the number of scale factor) are introduced. The possible combinations about $\alpha_i, i=1,2$ are

$$\begin{cases} \alpha_1 = (\alpha_{1,upp} - (k-1)\delta) : \delta : \alpha_{1,upp} \\ \alpha_2 = (\alpha_{2,upp} - (k-1)\delta) : \delta : \alpha_{2,upp} \end{cases} \quad (9)$$

where $\alpha_i = p : j : h$ denotes α_i can choose the value from p to h with step j .

For each combination of scale factor $(\hat{\alpha}_1, \alpha_2)$, following HLOP method in [9], the system equations can be expressed as matrix form

$$AX = b \tag{10}$$

where $A = \begin{bmatrix} x_1 - x_2 & y_1 - y_2 \\ \tan(\varphi) & -1 \end{bmatrix}$,

$$b = \frac{1}{2} \begin{bmatrix} \hat{\alpha}_2^2 r_2^2 - \alpha_1^2 r_1^2 + K_1 - K_2 \\ 2(x_1 \tan(\varphi) + y_1) \end{bmatrix}, X = [x, y]^T.$$

$K_i = x_i^2 + y_i^2, i=1,2$ and \tan is the function of tangent.

The MS position can be estimated with a least square approach

$$X = A^{-1}b \tag{11}$$

The estimated error $\varepsilon_i, i=1,2$ is a criterion for deciding whether to discard the estimated position, which is calculated by HLOP method with the combination $(\hat{\alpha}_1, \alpha_2)$

$$\varepsilon_i = r_i - \sqrt{(\hat{x} - x_i)^2 + (y - y_i)^2} \tag{12}$$

where (\hat{x}, y) is the estimate of MS position (x, y) .

If $\varepsilon_i \geq 0$, then (\hat{x}, y) is called the potential point and saved, else discard. The final estimate of MS is averaging all the potential points.

For the AOA measurement φ in home BS, there are lower and upper bounds[9]

$$\varphi_{\min} \leq \varphi \leq \varphi_{\max} \tag{13}$$

where φ_{\min} and φ_{\max} are the corresponding minimum and maximum angles, respectively. The angles are the orientation of line joining the serving BS and the intersection point of the two range circles.

Thus, the proposed grid search algorithm can be summarized as follow:

1. Given the measured distances and cell radius, the upper bounds of $\alpha_i, i=1,2$ are computed in (8).
2. Given the AOA measurement φ , if it is less than φ_{\min} , then set $\varphi = \varphi_{\min}$, else if it is more than φ_{\max} , then set $\varphi = \varphi_{\max}$.
3. For each combination of $\alpha_i, i=1,2$ in (9) and the AOA measurement, we can obtain the MS position estimation (\hat{x}, y) with (10) and (11).
4. Determining the MS position estimation (\hat{x}, y) whether satisfying the inequations $\varepsilon_i \geq 0, i=1,2$ in (12), if it is satisfied, the point (\hat{x}, y) is called the potential point and saved; else go to step 3.

5. The final MS position estimation is obtained by averaging all the potential points.

4 Simulation Results

Simulations are carried out to determine the performance of the proposed techniques and the conventional techniques. For simplicity, the standard deviations of the two range measurements are assumed to have the same value $\sigma_n = \sigma_m = \sigma$. In the simulation, the positions of BSs are (0,0) and $(1000\sqrt{3}, 0)$, the radius of cell $R_c = 1000m$. The position of MS is (x, y) , where $x = 250\sqrt{3} + 250\sqrt{3} \cdot u$, $y = \frac{\sqrt{3}}{3}x \cdot u$, and u is a random variable uniformly distributed in the region [0,1]. Each algorithm does 1000 independent runs and the scatterers are randomly and uniformly generated within the scattering circle.

4.1 Comparison of Different Algorithm

The HTA and TA-CA algorithm are presented as a meaningful comparison. The range measurement error is assumed to have 30m standard deviation, and the AOA error is 3° standard deviation. The weight factors are all set to 1. The effect of the radius of scatterer on the average location error is shown in Fig.3. As the radius of scatterers increases, the NLOS range and angle errors increases and leads to less accurate location results. It can be observed that Grid search algorithm gives better performance than New TA-CA algorithm and that the New TA-CA algorithm outperforms the HTA and TA-CA algorithm. Fig. 4 and Fig. 5 present the location error of different algorithms versus the standard deviation of range and angle error, respectively. It is shown that two parameters have little effect on the performance of different algorithms. Fig.6 shows the cumulative distribution function (CDF) of the different algorithms, where the radius of scatterer is set to 200m for the serving BS and 300m for the other BS. It indicates that the grid search algorithm gives a better performance than new TA-CA algorithm, and new TA-CA algorithm is better than HTA and TA-CA algorithm.

4.2 The Analysis of Grid Search Algorithm

For our proposed grid search algorithm, there are two important factors to affect the performance. One is the step δ , the other is the number of scale factors k . Therefore, it is necessary to know how the two factors affect the performance of grid search algorithm. As shown in Fig.7, we can see that

increasing the number of scale factors k can remarkably improve the performance, when the step δ is small. However, when the step δ increases to 0.05, the performance begins to decrease. The reason is lower scale factor, which leads to the adjusted range is smaller than the true distance. Therefore, the step δ should not choose too big or too small.

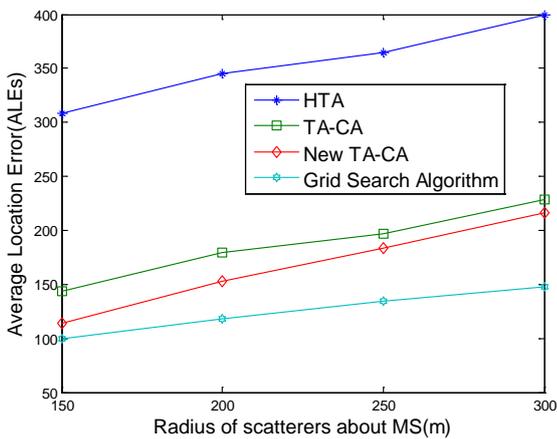


Fig. 3. Comparisons of the location error for various scatterer radius

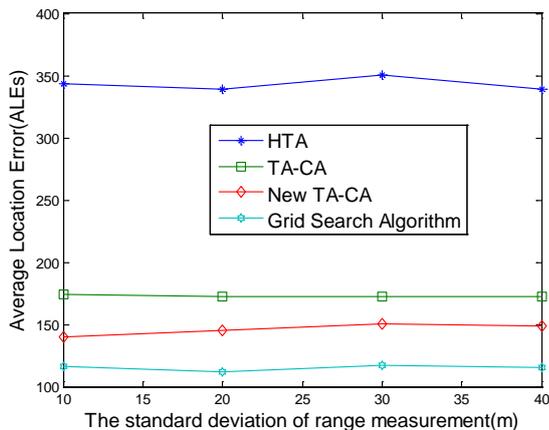


Fig. 4. Comparisons of the location error for different standard deviation of range measurement

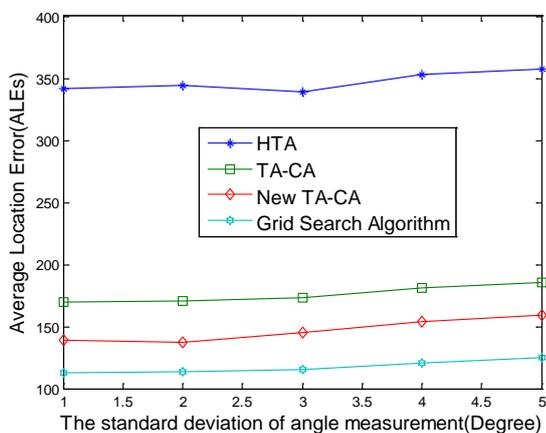


Fig.5 Comparisons of the location error for different standard deviation of angle measurement

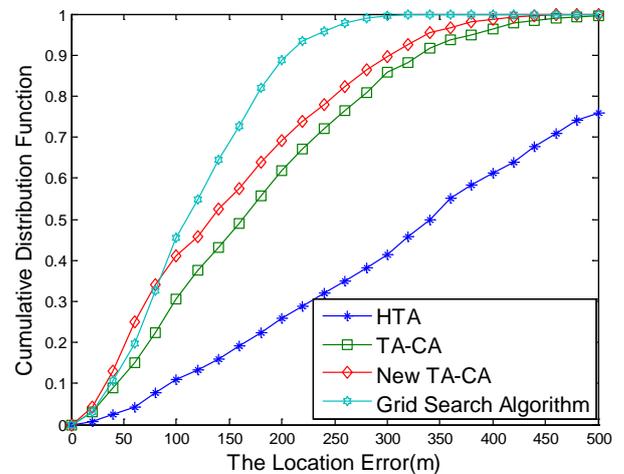


Fig.6 Comparison of error CDFs with different algorithms

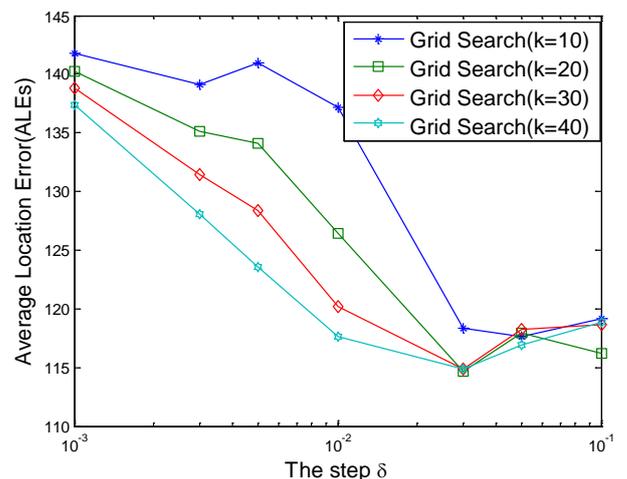


Fig.7 The analysis of grid search algorithm

4 Conclusion

In circular scattering environment, this paper proposes new TA-CA algorithm with only two BSs by building the equality constraint about the AOA measurement. Due to the high computational complexity and nonconvex of TA-CA algorithm, a grid search algorithm is proposed. Simulation results demonstrate: 1) the proposed grid search algorithm has the best performance, which is better than the new TA-CA algorithm, TA-CA algorithm and HTA algorithm. 2) the new TA-CA algorithm has better performance than TA-CA and HTA algorithm, which confirms the equality constraint can improve the performance of optimization method. 3) the standard deviation of range and angle measurement has little effect on the performance of different algorithms. 4) the step and number of scale

factor can remarkably affect the performance of the grid search algorithm .

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