

# Optimal Transmission Policies for Energy Harvesting Transmitter with Hybrid Energy Source in Fading Wireless Channel

DIDI LIU

School of Telecommunication Engineering  
Xidian University  
Xi'an, 710071  
CHINA  
lidd866@mailbox.gxnu.edu.cn

JIMING LIN, JUNYI WANG, YUXIANG SHEN, YIBIN CHEN

Key Lab. of Cognitive Radio and Information,  
Processing, Ministry of Education  
Guilin, 541004  
CHINA  
wangjy@guet.edu.cn

*Abstract:* Optimal energy scheduling problem for single-user wireless communication system in fading channel is studied in this paper. In this schedule, the transmitter is powered by hybrid energy sources including both the conventional grid and an energy harvester collecting energy from nature and storing energy in a rechargeable battery. In this system, data arrival process, energy harvesting process and channel state are all time-varying and possibly unpredictable. Our objective is to develop policies of transmission power to minimize the additional energy consumption from the non-renewable energy source with the constraint of data queue stability and available energy harvested. We propose an online simple optimal algorithm which provides insight into how to efficiently utilize the energy supplied by the energy harvester. We utilize the technique of Lyapunov optimization to exploit energy efficient scheduling of the transmitter by adaptively adjusting transmission power, while at the same time provided a delay guarantee less than the maximum delay spent in the data queue. The challenge in this work is to provide an efficiently low complexity algorithm without knowing priori information of probability distribution. Finally, Simulation results of this algorithm show substantial reduction of energy from the non-renewable source compared to two simple greedy algorithm.

*Key- Words:* Energy harvesting, Lyapunov optimization, Optimal transmission policy, Hybrid energy source, Wireless communication.

## 1 Introduction

The rapidly increasing mobile data has led to a high demand for energy in wireless networks. In fact, the cellular networks consume world-wide approximately 60 billion kWh per year. In particular, 80% of the electricity in cellular networks is consumed by the base stations (BSs) which produce over a hundred million tons of carbon dioxide per year [1]. These figures are projected to double by the year 2020 if no further actions are taken. Driven by environmental concerns, green communication has received considerable interest from both industry and academia [2–4]. However, a tremendous number of green technologies/methods have been proposed require the availability of an ideal power supply such that a large amount of energy can be continuously used for system operations whenever needed. In recent ten years, the energy harvesting (EH) technique advances very quickly and has attracted considerable interest as an environmentally friendlier supply of energy for communication nodes compared to traditional energy sources. In future 5G

networks, diverse base stations to support small cells and heterogeneous networks are densely deployed, EH nodes in wireless network harvest energy from their surroundings and can ensure a free and perpetual supply of energy. As a result, wireless networks with energy harvesting transmitter are not only envisioned to be energy-efficient in providing ubiquitous service coverage, but also to be self-sustained.

There has been recent research effort on understanding data transmission with solely energy harvesting transmitter that has a rechargeable battery from renewable energy sources. In [5] and [6], the authors proposed optimal power control time sequences for maximizing the throughput by a deadline with a single energy harvester. In [7] and [8], optimal packet scheduling and power allocation algorithms were proposed for energy harvesting systems for minimization of the transmission completion time, respectively. In [9–11], different optimal packet scheduling algorithms were proposed for additive white Gaussian noise (AWGN) broadcast channels for a set of pre-

lected users. But in [5]- [11] researchers consider deterministic EH model where the availability of the off-line knowledge of energy and data arrivals at the transmitter are known. Due to the intermittent nature of energy generated by a natural energy source, resulting in highly random energy availability at the transmitter, so the prior statistical information of energy and data arrivals are greatly difficult to know. For example, solar energy and wind energy are varying significantly over time because of weather and climate conditions. On the other hand, although the amount of renewable energy is potentially unlimited, transmitters powered solely by an energy harvester may not be able to maintain a stable operation and to guarantee a certain quality of service (QoS). Therefore, a hybrid energy source system design, which uses different energy sources in a complementary manner, is preferable in practice for providing uninterrupted service [12, 13]. A hybrid energy source is a combination of a constant energy source, e.g., power grid, diesel generator etc., and an EH source which harvests energy from solar, wind, thermal, or electromechanical effects. [14] considers a hybrid energy harvesting system, but it assumes the knowledge of energy and data arrivals are known as well as [5]- [11]. [22] consider two scenarios for the arrival process of the data packets into the data queue at the transmitter, and derive offline and online power allocation schemes that minimize the total amount of energy drawn from the constant energy source by Stochastic dynamic programming (DP) approach, but the online algorithm has high complete complexity.

This paper considers wireless communication using energy harvesting transmitters with hybrid energy source as shown in Fig.1. We assume that the transmitter can adaptively change its transmission power for data queue stability according to the available energy and the remaining number of bits. When energy stored in the rechargeable battery harvested from renewable source is not enough for transmission before data backlog deadline, the transmitter does not absorb additional energy from the non-renewable energy source such as power grid, in other word, constant energy source is just as a renewable energy supplement. The objective of our work is to develop methods for transmission to minimize the time average of additional energy consumption draw from the non-renewable energy source with the constraint of data queue stability and available energy harvesting, such that the harvested energy is efficiently utilized. The solution of the optimization problem considered in this paper can facilitate the design of reliable green communication systems.

In this paper we utilize the technique of Lyapunov optimization initially developed in [15–17] for dynamic control of queueing systems for wireless net-

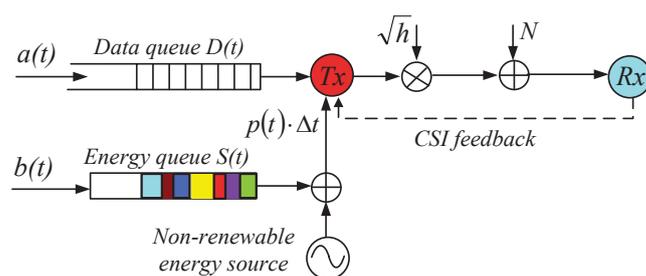


Figure 1: The model of energy harvesting transmitter with hybrid energy source

works. In [17], researchers utilize the Lyapunov optimization technique to show that the queuing model naturally fits in the renewable supplier scheduling problem and present a simple energy allocation algorithm that does not require prior statistical information and is provably close to optimal. In [17], researchers only consider a general model, in our work, we extend that approach to energy harvesting transmitter with hybrid energy source in wireless communication, a single-user fading channel with additive Gaussian noise, limited transmission power of the transmitter and the special relationship between transmission rate and power. The problem is now more complete and practical while still providing a simple approach for on-line operation.

Our main contributions in this work is that we are first to apply Lyapunov optimization to study stochastic energy efficient scheduling of the transmitter by renewable energy sources without prior information of system unknown variable, while at the same time providing a guarantee on the maximum delay  $D_{max}$  spent in the data queue. The online algorithm derived by Lyapunov optimization in this paper is relatively simple to implement compared to DP approach in [22], does not need a-priori statistical knowledge. In contrast, DP requires more stringent system modeling assumption, has a more complex solution that typical requires knowledge of energy harvesting, data arrival and channel state. Besides, DP approach involves computation of a value function that can be difficult when the state space of the system is large, and suffers from a curse of dimensionality when applied to large dimensional systems (such as systems with many queues).

## 2 System Model and Problem Statement

This paper considers wireless communication using a rechargeable battery that is able to harvest energy from nature, which operates in discrete time with unit time slots  $t \in \{0, 1, 2, 3, \dots\}$ , and harvests  $b(t)$  unit of energy at the beginning of each slot  $t$ , buffered in the rechargeable battery only for transmission. The process  $b(t)$  corresponds to the renewable supply and is assumed to be time varying and unpredictable. Every slot arrival date is stored in the data queue for transmission to the receiver. Let  $a(t)$  be new arrivals on slot  $t$ , in units of bits. The process  $a(t)$  as well as  $b(t)$  is assumed to be time varying and unpredictable.

We consider a single-user fading channel with additive Gaussian noise as shown in Fig.1, and with the perfect channel state information (CSI) known at the transmitter and the receiver. The link channels are time-varying so that we denote with  $h(t)$  as the channel state at slot  $t$  (representing, for example, attenuation values and/or noise levels), and assume it is independent and identically distributed (i.i.d.) over slots in a finite set  $H$ , i. e.  $h(t) \in H$  for all  $t$ . Channel conditions remain constant for the duration of each slot but change at slot boundaries.

We assume that the transmitter can adaptively change its transmission power for data queue stability according to the available energy and the remaining number of bits. At the beginning of each time slot  $t$ , the transmitter chooses a power  $P(t)$  to transmit data in the data queue in a First-In-First-Out(FIFO) manner, the maximum transmission power of the transmitter is limited, and denoted as  $P_{max}$ . During timeslot  $t$ , we use the amount of energy  $p(t) \cdot \Delta t$ , when the rechargeable battery energy is not enough for transmission before data backlog deadline, an amount of additional energy  $p(t) \cdot \Delta t - S(t)$  will be purchased from the non-renewable energy source, where  $\Delta t$  is the duration of one timeslot.

We assume that the transmission rate  $\mu(t)$  over the wireless link  $(a, b)$  and transmit power  $p(t)$  are related through a function in [18]:  $\mu_{ab}(t) = g(p(t), h_{ab}(t))$ , each time slot the rate-power function  $g(\cdot)$  determines the number of bits that can be transferred over the wireless link  $(a, b)$ . The rate-power curve is shown in Fig.2.

In [6] the received signal  $y$  is given by  $y = \sqrt{h}x + n$ , where  $h$  is the (squared) fading,  $x$  is the channel input, and  $n$  is a Gaussian random noise with zero-mean and unit-variance. Whenever an input signal  $x$  is transmitted with power  $p$  in the duration  $\Delta t$ ,  $\frac{\Delta t}{2} \log(1 + hp)$  bits of data is served out from the backlog with the cost of  $p \cdot \Delta t$  units of energy de-

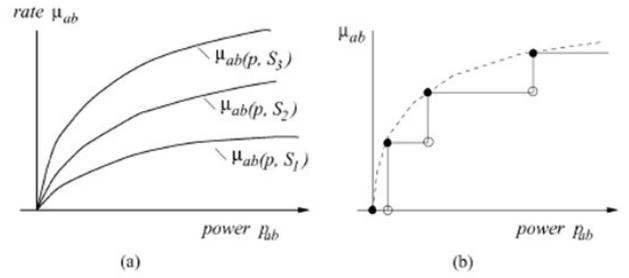


Figure 2: (a) Set of rate-power curves for improving channel conditions S1, S2, S3. (b) Curve of relationship between single power  $P_{ab}$  and transmission rate: concave function curve

pletion from the energy queue. This follows from the Gaussian channel capacity formula. If at timeslot  $t$  the transmit power of the signal is  $x^2(t) = p(t)$ , the value  $p(t)$  is a control decision on slot  $t$ , and corresponding transmission rate  $\mu(t)$  in bits per channel use is

$$\mu(t) = g(p(t), h(t)) = \frac{1}{2} \log_2 (1 + h(t)p(t)) \quad \forall t \quad (1)$$

Letting  $Q(t)$  represent the backlog of the data queue on slot  $t$  with  $Q(0) = 0$ , we have the following update equation:

$$Q(t + 1) = \max[Q(t) - \mu(t) \cdot \Delta t, 0] + a(t) \quad (2)$$

When  $Q(t) > 0$ , we must decide how much transmission power to use on the current slot or wait for a more energy-efficient future channel state.

The battery level at time  $t$  is denoted as  $S(t)$ . The battery energy is depleted due to link transmissions but is also replenished due to a recharge process. Energy queue update equation is followed as:

$$S(t + 1) = \max[S(t) - p(t) \cdot \Delta t, 0] + b(t) \quad (3)$$

We assume that energy harvesting occurs at beginning of each timeslot, and must be stored in the battery then can be used, and harvesting energy is not enough for transmission before data backlog deadline.

## 3 The Dynamic Algorithm

Suppose that the energy harvesting process  $b(t)$ , the data arrival process  $a(t)$ , and the channel state  $h(t)$ , as described previously, with some unknown probability distribution. We further assume the values of  $b(t)$ ,  $a(t)$  and  $h(t)$  are deterministically bounded by

finite constants  $b_{max}, a_{max}, h_{min}, h_{max}$ , so that:

$$\begin{aligned} 0 &\leq b(t) \leq b_{max}, & \forall t \\ 0 &\leq a(t) \leq a_{max}, & \forall t \\ h_{min} &\leq h(t) \leq h_{max} & \forall t \end{aligned}$$

The queue backlog  $Q(t)$  evolves according to equation (2). The decision variable  $p(t)$  is chosen every slot  $t$  subject to the constraint  $0 \leq p(t) \leq p_{max}$  for all  $t$ . We assume that  $g(p_{max}, h_{min}) \geq a_{max}$ , so that it is always possible to stabilize the queue  $Q(t)$ . During  $t$  slot there is the amount of energy  $\max[p(t) \cdot \Delta t - S(t), 0]$  absorbed from the non-renewable energy source.

We want to find an algorithm that choose  $p(t)$  over time to solve:

$$\begin{aligned} \text{Min} \quad & \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E\{\max[p(\tau) \cdot \Delta t - S(\tau), 0]\} \\ \text{S.t.} \quad & \bar{Q} < \infty \\ & 0 \leq p(t) \leq p_{max} \quad \forall t \end{aligned} \quad (4)$$

Where  $\bar{Q}$  is the time average expected queue backlog, defined:

$$\bar{Q} \triangleq \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E\{Q(\tau)\} \quad (5)$$

Specifically, a queue  $Q(t)$  is stable if

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E\{Q(\tau)\} < \infty$$

according to the definition of [15], hence the first constraint meets data queue stability.

### 3.1 The delay-aware virtual queue

Note that (4) does not include the terms accounting for delay constraints, we solve the above problem while also maintaining finite worst case delay using the following "virtual queue  $Z(t)$ ", in [21] which are defined as : $Z(0) = 0$ , fixed a parameter  $\epsilon > 0$  and according to the following update:

$$Z(t+1) = \max [Z(t) + \epsilon 1_{Q(t)>0} - \Delta t \cdot \mu(t), 0], \quad \forall t \quad (6)$$

where  $1_{(Q(t)>0)}$  is an indicator function that is 1 if  $Q(t) > 0$ , and 0 else. This ensures that  $Z(t)$  grows by impose a penalty  $\epsilon$  to the virtual queue backlog if there are data in  $Q(t)$  queue that has not been serviced for a long time. The constant  $\epsilon$  can adjust the growth rate of the virtual queue, if we can control the system to ensure that the queues  $Q(t)$  and  $Z(t)$  have finite upper bounds, then we can ensure all bites are served with a worst case delay given in the following lemma.

**Lemma 1 (Worst Case Delay)** Suppose the system is controlled so that the queue  $Q(t)$  and  $Z(t)$  have finite upper bounds, e.g.  $Z(t) \leq Z_{max}$  and  $Q(t) \leq Q_{max}$  for all  $t$ , for some positive constants  $Z_{max}$  and  $Q_{max}$ . Then all bits are served with a maximum delay of  $D_{max}$  slots, where:

$$D_{max} \triangleq [(Q_{max} + Z_{max})/\epsilon] \quad (7)$$

Proof 1: The proof of Lemma 1 follows the approach of Lyapunov optimization in [17, 18, 21]. We use contradiction to prove the worst delay time is less than  $D_{max}$ . The following shows that data arrivals  $a(t)$  at any slot  $t$  will be served or before timeslot  $t + D_{max}$ . Suppose not, then the queue backlog  $Q(\tau) > 0$  during slots  $\tau \in \{t + 1, \dots, t + D_{max}\}$ . In this case, for all  $\tau \in \{t + 1, \dots, t + D_{max}\}$  we have  $1_{Q(t)>0} = 1$  and from (6) have:

$$Z(t + 1) \geq Z(t) - \Delta t \cdot \mu(t) + \epsilon \quad (8)$$

Summing (8) from slot  $t+1$  to  $t + D_{max}$  yields:

$$Z(t + D_{max} + 1) - Z(t + 1) \geq D_{max} \cdot \epsilon - \sum_{\tau=t+1}^{t+D_{max}} \mu(\tau) \cdot \Delta t \quad (9)$$

Since  $Z(t + 1) \geq 0$  and  $Z(t + D_{max} + 1) \leq Z_{max}$ , (9) can be further written as:

$$Z_{max} \geq D_{max} \cdot \epsilon - \sum_{\tau=t+1}^{t+D_{max}} \mu(\tau) \cdot \Delta t \quad (10)$$

Due to FIFO service manner for data queue and  $Q(t) \leq Q_{max}$ , if the arrival data is not fulfilled on before  $t + D_{max}$ , the total data queue backlog should be more than the upper bound of queue length  $Q_{max}$ , so  $a(t)$  must be served within  $\{t + 1, \dots, t + D_{max}\}$ . Hence,  $\sum_{\tau=t+1}^{t+D_{max}} \mu(\tau) \cdot \Delta < Q_{max}$  must be held. Substituting this into (10) then rearranging and yields:

$$D_{max} < [(Q_{max} + Z_{max})/\epsilon] \quad (11)$$

(11) is contradiction with the definition (7), so the delay of data queue  $Q(t)$  should be less or equal to  $D_{max}$ .

### 3.2 Lyapunov optimization

Define  $\emptyset(t) \triangleq (Q(t), Z(t))$  as the concatenated vector of the real and virtual queues. As a scalar measure of the congestion in both the  $Z(t)$  and  $Q(t)$  queues, we define the following Lyapunov function:  $L(\emptyset(t)) \triangleq \frac{1}{2} [Q(t)^2 + Z(t)^2]$ . Define the conditional 1-slot Lyapunov drift as follows:

$$\Delta(\emptyset(t)) \triangleq E \{L(\emptyset(t + 1)) - L(\emptyset(t)) | \emptyset(t)\} \quad (12)$$

our control algorithm is designed to observe the current queue states  $Z(t)$ ,  $Q(t)$  and the current channel state  $h(t)$ , and to make a decision  $p(t)$  (where  $0 \leq p(t) \leq p_{max}$  for all slots) to minimize a bound on the following expression every slot  $t$ :

$$\min \Delta(\emptyset(t)) + VE \{ \max[p(t) \cdot \Delta t - S(t), 0] | \emptyset(t) \} \quad (13)$$

Note that the left part is the growth of the queue and the right part is the expected energy absorbed from the non-renewable energy source, and (13) is called drift-plus-penalty expression.  $V$  is a positive parameter that is used to tune performance-delay tradeoff. Intuitively, taking actions to minimize  $\Delta(\emptyset(t))$  alone would push both queues towards lower backlog but incur a large penalty, and so our approach minimizes a weighted sum of drift and penalty. The objective is to minimize the weighted sum of drift and penalty, which can be proven bounded.

**Lemma 2** *The drift-plus-penalty expression for all slots  $t$  satisfied:*

$$\begin{aligned} \Delta(\emptyset(t)) + VE \{ \max[p(t) \cdot \Delta t - S(t), 0] | \emptyset(t) \} \\ \leq B + VE \{ \max[p(t) \cdot \Delta t - S(t), 0] | \emptyset(t) \} \\ + Q(t)E \{ a(t) - \Delta t \cdot \mu(t) | \emptyset(t) \} \\ + Z(t)E \{ \epsilon - \Delta t \cdot \mu(t) | \emptyset(t) \} \end{aligned} \quad (14)$$

where the constant  $B$  is defined as:

$$B = \frac{a_{max}^2 + \Delta t^2 \cdot \mu_{max}^2}{2} + \frac{\max[\epsilon^2, \Delta t^2 \cdot \mu_{max}^2]}{2} \quad (15)$$

**Proof 2:** For real queue backlog,

$$Q^2(t+1) = \{ \max[Q(t) - \mu(t) \cdot \Delta t, 0] + a(t) \}^2 \quad (16)$$

Using the following inequality:

$$[\max(b-c, 0) + a]^2 \leq b^2 + c^2 + a^2 + 2b(a-c) \quad (17)$$

which holds for any  $a \geq 0$ ,  $b \geq 0$  and  $c \geq 0$ , then we can yield:

$$Q^2(t+1) \leq Q(t)^2 + \mu(t)^2 \cdot \Delta t^2 + a(t)^2 + 2Q(t)[a(t) - \mu(t) \cdot \Delta t] \quad (18)$$

Therefore:

$$\begin{aligned} \frac{1}{2}[Q^2(t+1) - Q^2(t)] &\leq \frac{1}{2}(\mu^2(t) \cdot \Delta t^2 + a^2(t)) \\ &+ Q(t)[a(t) - \mu(t) \cdot \Delta t] \end{aligned} \quad (19)$$

Similar for virtual queue,

$$\begin{aligned} Z^2(t+1) &\leq [Z(t) + \epsilon 1_{Q(t)>0} - \Delta t \cdot \mu(t)]^2 \\ &= Z^2(t) + [\epsilon - \Delta t \cdot \mu(t)]^2 + 2Z(t)[\epsilon 1_{Q(t)>0} - \Delta t \cdot \mu(t)] \\ &\leq Z^2(t) + \max[\epsilon^2, (\Delta t \cdot \mu(t))^2] \\ &+ 2Z(t)[\epsilon 1_{Q(t)>0} - \Delta t \cdot \mu(t)] \end{aligned}$$

Thus we have:

$$\begin{aligned} \frac{1}{2}[Z^2(t+1) - Z^2(t)] &\leq \frac{1}{2} \max[\epsilon^2, (\mu^2(t) \cdot \Delta t^2 + a^2(t))] \\ &+ Z(t)[\epsilon - \mu(t) \cdot \Delta t] \end{aligned} \quad (20)$$

Substituting (19) and (20) into (13), then we have the expression of  $\Delta(\emptyset(t))$ , thus we can have the inequality (14).

### 3.3 On-line optimization algorithm

Due to the left-hand side of (14) tightly bounded by the right-hand side of (14), Minimizing the right-hand-side of the drift-plus-penalty bound (14) every slot  $t$  leads to the following dynamic optimization algorithm: every slot  $t$ , observe  $Z(t)$ ,  $Q(t)$ ,  $h(t)$ ,  $a(t)$  and  $b(t)$ , then choose  $p(t)$  according to the following optimization:

$$\begin{aligned} \text{Min: } &V[p(t) \cdot \Delta t - S(t)] \\ &- g(p(t), h(t)) \cdot \Delta t[Q(t) + Z(t)] \\ \text{S.t. } &0 \leq p(t) \leq p_{max} \\ &p(t) \cdot \Delta t > S(t) \end{aligned} \quad (21)$$

Then update the actual and virtual queues  $Q(t)$  and  $Z(t)$  by (2) and (6), respectively.

The problem formulation (21) is a strictly convex function for  $p(t)$ , so there is the global minimum. We denote the power used in slot  $t$  that minimize (21) as  $p^*(t)$ .

$$\begin{aligned} p^*(t) = \arg \min &V[p(t) \cdot \Delta t - S(t)] \\ &- g(p(t), h(t)) \cdot \Delta t[Q(t) + Z(t)] \end{aligned}$$

Then we have:

$$p^*(t) = \frac{Q(t) + Z(t)}{2 \ln 2 \cdot V} - \frac{1}{h(t)} \quad (22)$$

The above  $p(t)$  value drives the queue update. In fact, we choose  $p(t)$  in slot  $t$  according to:

$$p(t) = \begin{cases} 0 & p^*(t) < 0 \\ p^*(t) & 0 \leq p^*(t) \leq p_{max} \\ p_{max} & p^*(t) > p_{max} \end{cases} \quad (23)$$

If  $p(t) \cdot \Delta t \leq S(t)$  holds, the transmitter does not need to absorb additional energy from the non-renewable source; else, absorbs additional energy of  $p(t) \cdot \Delta t - S(t)$ .

### 3.4 Performance analysis

**Theorem 3** Assume  $g(p_{max}, h_{min}) \cdot \Delta t \geq \max[a_{max}, \epsilon]$ , and  $Q(0) = Z(0) = 0$ , then fixes parameter  $\epsilon \geq 0$  and  $V > 0$  for all  $t \in \{0, 1, 2, \dots\}$ , the proposed dynamic algorithm has the following properties:

1. In all timeslots, the queues  $Q(t)$  and  $Z(t)$  are upper bounded by  $Q_{max}$  and  $Z_{max}$ , where:

$$\begin{aligned} Q_{max} &\triangleq 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + p_{max} \right) + a_{max} \\ Z_{max} &\triangleq 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + p_{max} \right) + \epsilon \end{aligned} \quad (24)$$

Namely,  $Q(t) \leq Q_{max}$ ,  $Z(t) \leq Z_{max}$  for all  $t$ .

2. The maximum delay of data queue is:

$$D_{max} = \left[ 4 \ln 2 \cdot V \left( \frac{1}{h_{min}} + p_{max} \right) + a_{max} + \epsilon \right] / \epsilon \quad (25)$$

3. Given that  $\epsilon \leq E[a(t)]$ , the time-average expected energy from non-renewable source by the proposed algorithm satisfies:

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[p(\tau) \cdot \Delta t - S(\tau)] \leq C_{opt} + \frac{B}{V} \quad (26)$$

where  $C_{opt}$  is the optimal value of (4), and  $B$  is given by (15).

Proof 3.

1. We use induction method to show that:

$$Q(t) \leq Q_{max} = 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + p_{max} \right) + a_{max}, \quad \forall t$$

It holds clearly for  $t = 0$  (because  $Q(0) = 0$ ). Next assume:

$$Q(t) \leq 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + p_{max} \right) + a_{max}, \quad \forall t$$

what we can do is to prove it also true for slot  $t + 1$ . If

$$Q(t) \leq 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + p_{max} \right)$$

the maximum queue backlog growth is  $a_{max}$ , then

$$Q(t) \geq 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + p_{max} \right)$$

since  $Z(t) \geq 0$ , we have:

$$\begin{aligned} Q(t) + Z(t) &\geq 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + p_{max} \right) \\ &\geq 2 \ln 2 \cdot V \left( \frac{1}{h(t)} + p_{max} \right) \end{aligned}$$

In this case, according to the algorithm proposed above we will have  $p^*(t) > P_{max}$  by formula (22). Then we will choose  $p(t) = P_{max}$  on slot  $t$  according to (23), thus the data queue is served by at least  $a_{max}$ , because

$$g(p_{max}, h_{min}) \cdot \Delta t \geq \max[a_{max}, \epsilon]$$

hence the data queue backlog cannot grow on the next slot, i.e.,

$$Q(t+1) \leq Q(t) \leq 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + P_{max} \right) + a_{max}$$

Therefore, we have  $Q(t) \leq 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + P_{max} \right) + a_{max}$  for all slot  $t$ .

The proof that  $Z(t) \leq 2 \ln 2 \cdot V \left( \frac{1}{h_{min}} + P_{max} \right) + \epsilon$  is similar above.

2. It is very easy to prove according to Lemma 1 and the conclusion of Theorem 3.
3. The proof follows the drift-plus-penalty presented in [19–21].

The performance analysis shows that  $V$  as tune parameter balance energy efficiency and delay. The  $V$  value is larger, the performance will close to the optimal infinitely, but the queue backlog is longer. Thus we should choose appropriate  $V$  value. To reduce  $D_{max}$  value, we should use  $\epsilon$  as large as possible while still meet  $\epsilon \leq E[a(t)]$ . We can choose  $\epsilon = E[a(t)]$  if this expectation is given. Using  $\epsilon = 0$  does not provide a finite delay guarantee according to (25), it still can preserves part (1) and (3) of Theorem 3.

In latter simulation, we relax the original condition:

$$g(p_{max}, h_{min}) \cdot \Delta t \geq \max[a_{max}, \epsilon]$$

as  $g(p_{max}, E(h(t))) \cdot \Delta t \geq \max[a_{max}, \epsilon]$ , so we obtain the mean worst delay  $D_{max}$ .

## 4 Simulation Results

We consider a fading additive Gaussian channel with bandwidth  $W$  where the transmission rate at slot  $t$  is

$$\mu(t) = W \log_2 (1 + h(t)p(t))$$

$h(t)$  is the channel signal-noise-ratio(SNR), i.e., the actual channel gain divided by the noise power spectral density multiplied by the bandwidth, bandwidth is chosen as  $W = 1\text{MHz}$  for the simulations.

We have performed simulations on data sets with 1 second timeslot interval and use solar energy as renewable energy source. The related simulation settings are summarized in Table.1:

Table 1: SIMULATION SETTINGS

Parameters	Value
Bandwidth $W$	1 MHz
Frame length	1 s
Noise power spectrum	$10^{-19}W/Hz$
Average Path loss	-110 dB
Channel Fading	Gaussian
Avg. harvesting rate	100 mJ/frame
Harvest process	I.I.D. poisson process
Max transmission power	2W

To better evaluate the performance of our proposed algorithm, three scenarios are considered for simulations. The first scenarios use Lyapunov optimization algorithm with a balance between delay time and performance, where  $\epsilon = E\{a(t)\}$  and  $V$  is set to 500. The latter two scenarios use simple greedy algorithms. The second scenario deploys "absorb-upon-arrival" strategy, when energy in rechargeable battery cannot meet the need of the transmitter, the transmitter absorbs energy from the non-renewable source immediately to send data, which results in the least delay time, but possibly higher cost. The final scenario deploys the strategy "absorb-at-deadline" means that the transmitter absorbs energy at deadline when if no renewable energy is available after deadline. Before the deadline, the transmitter use only renewable energy, where the deadline is set to 25.

#### 4.1 Performance on amount of absorbing additional energy

Fig.3 shows the costs of three different scenarios for each strategy. From the results, we can see Lyapunov optimization achieves the minimum cost among the three scenarios, the reason is that Lyapunov optimization algorithm enables the transmitter to send more data when channel state is better. The average maximum delay  $D_{av}$  is 9 slots used this algorithm. If  $\epsilon = 0$ , the cost used proposed algorithm will be smaller, but the average maximum delay will be larger.

Fig.4 gives the total additional energy absorbed from non-renewable source in three cases: the amount

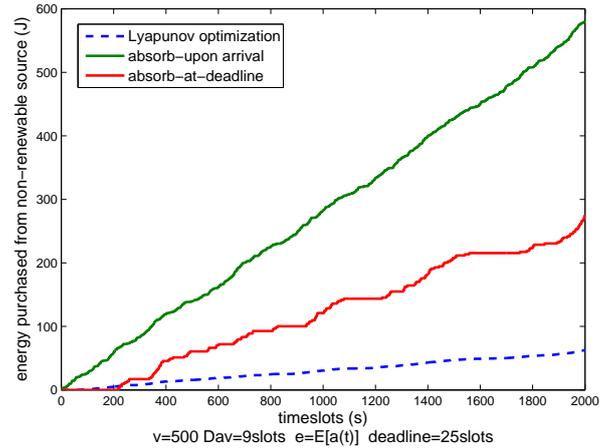


Figure 3: Comparison of additional energy absorbed from non-renewable source using different strategy

of harvesting energy is large, moderate and small respectively (2000 slots). In case 3, when the amount of harvesting energy is small, the strategy "absorb-at-deadline" has worse performance than "absorb-upon-arrival". The reason is that in this case a good deal of data is backlogged till the deadline, so the transmitter send data in the maximum power at deadline slot, and the relationship between the transmission rate and power is concave function.

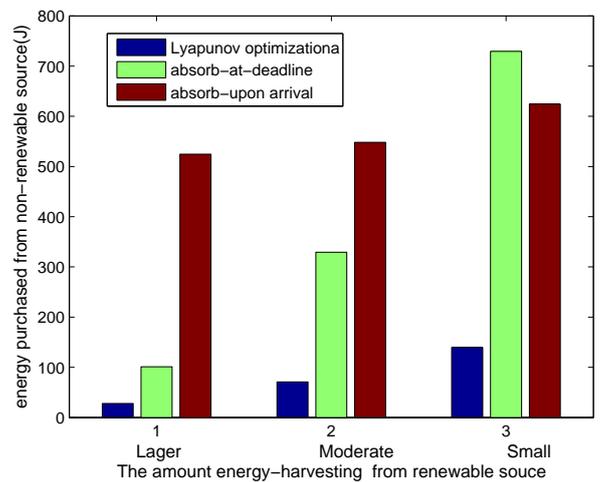


Figure 4: Comparison of additional energy cost in different case

#### 4.2 Performance on delay time

To have a better insight of impact of delay-time reduction, we have shown simulation results on the fraction

of waiting data in Fig.5, not change the parameters in Fig.4. Seen from Fig.5, Lyapunov optimization algorithm has on average a much smaller delay than the deadline. The arrival date on each slot waits mostly about 9 slots used proposed algorithm, while the strategy "absorb-at-deadline" waits mostly 24 slots.

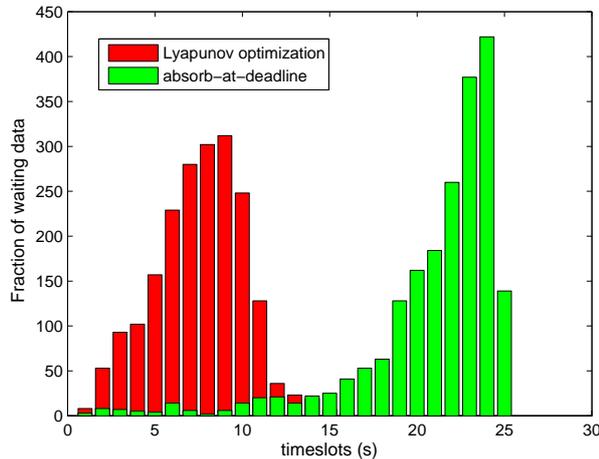


Figure 5: Histogram of delay for data waiting in the service queue under two algorithm

### 4.3 Balance between performance and delay

In order to study the impact of parameter  $V$  on the cost and mean delay, we have plotted Fig.6 showing the relationship between the cost and the value of  $V$  and the relationship between the mean delay time and the value  $V$ . We can see that as we expected, the mean delay increases non-linearly with the value of  $V$ , while the cost decreases with  $V$ . The cost and mean delay reach saturation when  $V$  is larger than a certain value, which illustrates that when  $V$  is large enough, the mean delay will reach its maximum and the cost is close the optimal value ( $C_{opt}$ ).

## 5 Conclusion and Discussion

In this paper, we focus on single-user wireless communication system using hybrid energy harvesting transmitter, in this system data arrival process, energy harvesting process and channel state are all time-varying and possibly unpredictable, we utilize Lyapunov optimization to exploit an efficient power scheduling algorithm, the additional energy cost can close to the minimum value infinitely by tuning the parameter  $V$ , while gives the worst case delay  $D_{max}$ .

To evaluate proposed Lyapunov optimization algorithm, simulations with different profiles are per-

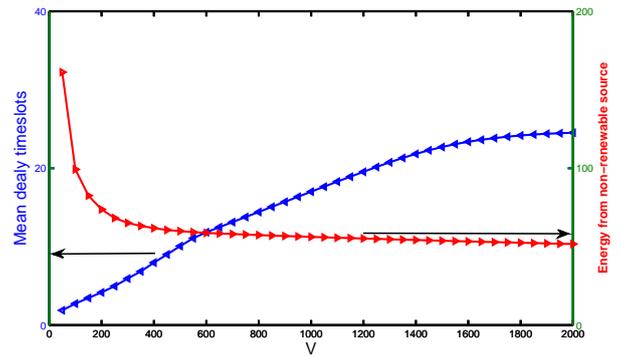


Figure 6: the amount of additional energy and mean delay time for different  $V$  value

formed and analyzed, a comparison is done with two other simple greedy algorithms presented in simulation part. Simulation results demonstrate that our optimization algorithm provided both obviously better performance and less delay as compared to the greedy algorithms. The proposed optimization algorithm in this paper is a robust energy efficiency approach, without knowing the statistics of underlying processes, which is dominant position of this optimization algorithm. In this paper we consider only single user data queue, further, we can extend this approach to multiple data queues corresponding to different users with different delay requirement.

**Acknowledgements:** The research was supported by the National Natural Science Foundation of China (Grant No.61261017) and Guangxi Natural Science Foundation 2014GXNSFAA118387, 2013GXNSFAA019334.

### References:

- [1] C. Han, T. Harrold, S. Armour, I Krikidis, S. Videv, P. Grant, H. Haas, J. Thompson, I. Ku, C.-X. Wang, T. A. Le, M. Nakhai, J. Zhang, and L. Hanzo, "Green radio: radio techniques to enable energy-efficient wireless networks," *IEEE Commun. Mag.*, vol.49, pp.46-54, Jun.2011.
- [2] H. Bogucka and A. Conti, "Degrees of freedom for energy savings in practical adaptive wireless systems," *IEEE Commun. Mag.*, vol.49, pp.38-45, Jun.2011.
- [3] G. P. Fettweis and E. Zimmermann, "ICT energy consumption-trends and challenges," in *2008 International Symposium on Wireless Personal Multimedia Communications*, pp.1-5. 2008 .

- [4] Y. Chen, S. Zhang, S. Xu, and G. Li, "Fundamental trade-offs on green wireless networks," *IEEE Commun. Mag.*, vol. 49, pp.30-37, Jun.2011.
- [5] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 11, pp. 1180-1189, Mar. 2012.
- [6] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: optimal policies," *IEEE J. Sel. Areas Commun.*, vol. 29, pp. 1732-1743, Sep. 2011.
- [7] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. Commun.*, vol. 60, pp. 220-230, Jan. 2012.
- [8] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *IEEE Trans. Signal Process.*, vol. 60, pp. 4808-4818, Sep. 2012.
- [9] M. Antepi, E. Uysal-Biyikoglu, and H. Erkal, "Optimal packet scheduling on an energy harvesting broadcast link," *IEEE J. Sel. Areas Commun.*, vol. 29, pp. 1721-1731, Sep. 2011.
- [10] O. Ozel, J. Yang, and S. Ulukus, "Optimal broadcast scheduling for an energy harvesting rechargeable transmitter with a finite capacity battery," *IEEE Trans. Wireless Commun.*, vol. 11, pp. 2193-2203, Jun. 2012.
- [11] J. Yang, O. Ozel, and S. Ulukus, "Broadcasting with an energy harvesting rechargeable transmitter," *IEEE Trans. Wireless Commun.*, vol. 11, pp. 571-583, Feb. 2012.
- [12] Huawei Technologies Co., Ltd., "Green communications, green Huawei, green world: renewable energy," <http://www.greenhuawei.com/green/greenenergy.html>
- [13] C. Wang and M. Nehrir, "Power management of a stand-alone wind/ photovoltaic/fuel cell energy system," *IEEE Trans. Energy Conversion*, vol. 23, pp. 957-967, Sep. 2008.
- [14] D. W. K. Ng, E. S. Lo, and R. Schober, "Energy-Efficient Resource Allocation in OFDMA Systems with Hybrid Energy Harvesting Base Station," *IEEE Trans. Wireless Commun.*, Vol. 12, No. 7, July. 2013.pp.3412-3426.
- [15] L. Georgiadis, M. J. Neely, and L. Tassiulas, "Resource allocation and cross-layer control in wireless networks," *Foundations and Trends in Networking*, vol. 1, no. 1, pp. 1-149, 2006.
- [16] M. J. Neely, "Energy optimal control for time varying wireless networks," *IEEE Transactions on Information Theory*, vol. 52, no. 7, 2006, pp. 2915-2934.
- [17] M. J. Neely, A. S. Tehrani and A. G. Dimakis, "Efficient algorithms for renewable energy allocation to delay tolerant consumers," *IEEE International Conference on Smart Grid Communications*, 2010, pp. 549- 554.
- [18] M. J. Neely, E. Modiano and C. E. Rohrs, "Dynamic power allocation and routing for time-varying wireless networks," *IEEE J.Sel. Areas Commun*, vol.23. No.1. January 2005. pp. 89-102.
- [19] Y. Guo, M. Pan and Y. Fang, "Optimal Power Management of Residential Customers in the Smart Grid," *IEEE Trans. on Parallel and Distributed Systems*, vol. 23, no. 9, pp. 1593-1606, 2012.
- [20] C. Jin, X. Sheng and P. Ghosh, "Optimized electric vehicle charging with intermittent renewable energy sources," *IEEE J.Sel. Topics in Signal Processing*, pp.1-10, 2014.
- [21] M. J. Neely, *Stochastic network optimization with application to communication and queueing systems*, Morgan & Claypool Publishers, 2010.
- [22] I. Ahmed, A. Ikhlef, D. W. K. Ng, R. Schober, "Power allocation for an energy harvesting transmitter with ybrid energy sources," *IEEE Trans. Wireless Commun.*, Vol. 12, No. 12, pp.6255-6267. Dec. 2013.