Capacity Analysis of MIMO-OFDM Decode-and-Forward (DF) Relay Networkin the Presence of High Power Amplifiers Nonlinearity

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Abstract: - This paper presents the ergodic capacity of nonlinear MIMO-OFDM Decode-and-Forward (DF) relay network.Nonlinearities in wireless communication channels can arise at the transmitter or the receiver side. In this work, we focus on the transmitter-side nonlinearities induced by the High Power Amplifier (HPA), the last stage in the communication chain at the base station (BS) and the relay stations (RS's). We consider MIMO-OFDM relaying system where the BS, RS's, and the mobile stations (MS's) are all equipped with N_t transmitting and N_r receiving antennas, and HPAs at the BS and RS's exhibit general nonlinear behaviors. We derive closed-form expression for the ergodic capacity of the DF relaying protocol. Our results show that high-order MIMO systems suffer more capacity loss due to HPA nonlinearity than low-order MIMO systems, and that more capacity loss is experienced in multihop relaying over nonlinear channels as more relay hops are involved.

Key-Words: -Amplifier nonlinearity, MIMO-OFDM, Ergodic Capacity, Decode-and-forward, Relay Network

1 Introduction

Next generation wireless networks need to provide ultra-high data rate services in-order to meet the requirements of future high-bandwidth multimedia applications over cellular systems.Among the candidate physical layer (PHY) technologies to achieve this, MIMO-OFDM is the most potent solution that can provide such high data rates at high spectral efficiencies [1]-[2].Furthermore, to extend the coverage and enhance theQuality of Service (QoS) performance of the communication system, multi-hop functionalities have been integrated into the next generation wireless networks [3]. The multi-hop relaying communication system consists of BS, one or more RS's and MS's as shown in Fig. 1. The communication channel between BS and RS is called relay link, while the channel between the RS and MS is called access link. RS's can be classified broadly into three categories based upon their forwarding schemes: Amplify-and-Forward (AF), Decode-and-Forward (DF) and Demodulate-and-Forward relaying protocols (DemF). In this work, we consider DF relaying protocol for the analysis.

Compared to the single-carrier systems, OFDM has a large peak-to-average-power ratio (PAPR) which makes it very sensitive to high-power

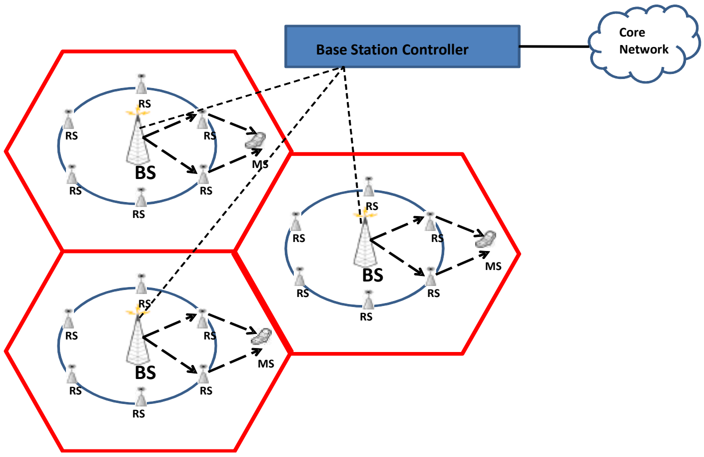

Figure 1: Multi-hop relaying system architecture

amplifier (HPA) nonlinearities at the RF stage of the transmission chain [4]. There are two importanteffects of the HPA nonlinearities introduced in the transmitted OFDM signals: in-band and out-of-band distortions. The in-band distortion degrades the own bit error rate (BER) performance or capacity of the cellular operator [5]-[6], whereas the out-of-band distortion arising from the spectral broadening effect of the HPA affects other operators operating in the adjacent frequency bands [7].

It is well-know that due to high PAPR of OFDM signal, strictly linear amplification of the transmitted OFDM signal at the RF stage is difficult to realize.

Thus in practice, some levels of HPA nonlinearity have to be tolerated in MIMO-OFDM transmissions. The effects of HPA nonlinearity on the capacity of communication systems have been considered in [5]-[6],[8]. All these works, and many others in the literature, however presented the capacity analysis for the single hop communication systems. In this paper, the effect of HPA nonlinearity on the ergodic capacity of DF MIMO-OFDM signals is characterized. Closed form expression for the ergodic capacity of MIMO-OFDM DF relay network is derived. It is shown that the capacity of the MIMO-OFDM DF relaying systems degraded due do HPA nonlinearity as expected. Our results also indicate that high-order MIMO systems suffer more capacity loss due to HPA nonlinearity than low-order MIMO systems, and that more capacity loss are experienced in multihop relaying over nonlinear channels as more relay hops are involved.

The paper is organized as follows. In section 2, we discussed aboutnonlinear MIMO-OFDM relaying channel models. Closed-form expression for the ergodic capacity of DF relay network is derived in section 3. The simulation results are presented in section 4, followed by the conclusion in section 5.

2 Nonlinear MIMO-OFDM Relaying channel models

We consider an $N_r \times N_t$ MIMO-OFDM relaying system, where the BS, RS's and MS are all equipped with N_t transmitting and N_r receiving antennas. We also consider that the transmitted OFDM signal has n subcarriers, and that each transmitted data from a source node passes through a single-hop MIMO channel $\mathbf{H}_0$, and R multi-hop MIMO relaying channels $\mathbf{H}_1,\ldots,\mathbf{H}_R$, associated with R fixed or mobile relaying nodes, to the destination node as shown in Fig 2.

The MIMO channel matrix for each i-th hop transmission $\mathbf{H}_i$, $i = 0,1,\cdots,R$, is an $nN_r \times nN_t$ block diagonal matrix, with the l^{th} block diagonal entries $\mathbf{H}[l]_i$ corresponding to the fading on the l^{th} OFDM subcarrier, $l = 0,1,\ldots,n-1$, modeled as independent and identically distributed (iid) random variables taken from zero mean complex Gaussian distribution, with unit variance. The set of random matrices $\{\mathbf{H}_0,\ldots,\mathbf{H}_R\}$ are assumed independent, and DF relaying protocolcould be employed at the relay nodes [9]-[10].

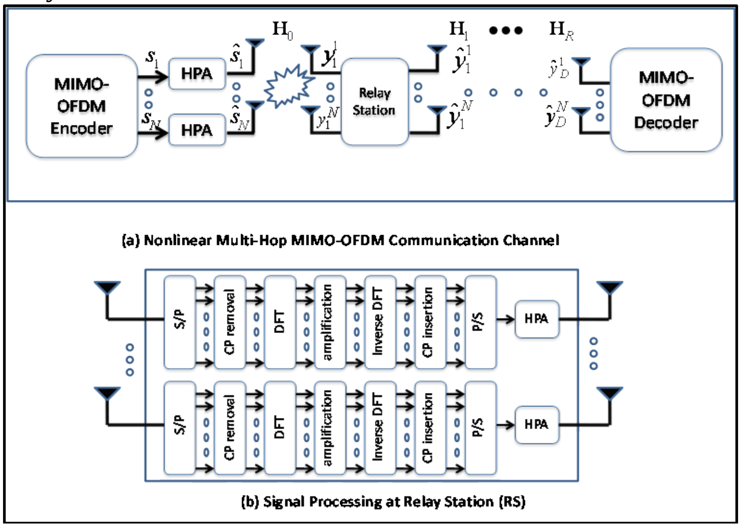

Figure 2: Nonlinear MIMO-OFDM Relaying channel model

According to the Bussgang's theorem, the output of the HPA when the input is a Gaussian process is given as [11]-[12]:

$$\hat{s}(t) = \beta s(t) + \psi(t) \quad \textbf{(1)}$$

where $s(t)$ is the input into the HPA, β (i.e. $0 \le \beta \le 1$) is an attenuation factor for the linear part and $\psi(t)$ is a nonlinear additive noise due to HPA. $\psi(t)$is a zero-mean complex Gaussian random variable (r.v,), with the in-phase and quadrature components mutually independent and identically distributed (iid), and with variance σ_ψ^2. The parameters β and σ_ψ^2 are referred to respectively as in-band and out-of-band distortion terms, and can be calculated as [13]:

$$\beta = \frac{E\{s(t)\hat{s}^*(t)\}}{E\{s(t)s^*(t)\}} = \frac{1}{P_{avg}}\left(\int_0^\infty a\hat{s}^*(a)f(a)da\right)^* \quad \textbf{(2)}$$

$$\begin{aligned}\sigma_\psi^2 &= \frac{E\{|\,\hat{s}^*(a)\,|^2\} - |\beta|^2}{E\{|\,s\,|^2\}} \\ &= \frac{1}{P_{avg}}\left(\int_0^\infty \left|\hat{s}^*(a)\right|^2 f(a)da - |\beta|^2\right)\end{aligned} \quad \textbf{(3)}$$

where '*' denotes complex conjugation, $P_{avg} = E[|s|^2]$ is the average input energy per symbol and $f(a)$ is the probability density function

(PDF) of the envelope of the input signal into the HPA (i.e. $a = |s|$). The closed-form expressions for the in-band and out-of-band distortion parameters for the SEL HPA model are obtained as:

$$\beta = 1 - e^{-\eta^2} + \frac{1}{2}\sqrt{\pi}\eta erfc(\eta) \quad \textbf{(4)}$$

$$\sigma_\psi^2 = P\left(1 - e^{-\eta} - \beta^2\right) \quad \textbf{(5)}$$

$$\eta = \frac{A_{is}}{\sqrt{P}} \quad \textbf{(6)}$$

where, η represents the clipping ratio (CR), $erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$ is a complementary error function, A_{is} represents the input saturation voltage of the HPA for each transmission chain, and P represents the average input power into the HPA.

3 Derivation of the Capacity of Nonlinear MIMO-OFDM Relaying channel

OFDM signal is well approximated by zero mean complex Gaussian signals for large number of subcarriers [4]. In the analysis, for the sake of simplicity, we consider the case of N transmitting and N receiving antennas ($N \le N_t, N_r$). Thus, in the ensuing analysis we focus on the $N \times N$ MIMO-OFDM relaying communication system. Using the Bussgang theorem [11]-[12], the output of the HPA on the m^{th} subcarrier of a MIMO-OFDM signal in the i^{th}-hop transmission of relaying system is given by:

$$\begin{bmatrix} \hat{s}[m]_i^1 \\ \vdots \\ \hat{s}[m]_i^N \end{bmatrix} = \begin{bmatrix} \beta_i^1 & 0 & \cdots & 0 \\ 0 & \beta_i^2 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & & \beta_i^N \end{bmatrix} \begin{bmatrix} s[m]_i^1 \\ \vdots \\ s[m]_i^N \end{bmatrix} + \begin{bmatrix} \psi[m]_i^1 \\ \vdots \\ \psi[m]_i^N \end{bmatrix}, i = 0, \ldots, R \quad \textbf{(7)}$$

where $\hat{s}[m]^j$, $m = 0, \ldots, n-1$, $j = 1, \ldots, N$, is the output of the HPA at the j^{th} transmitting antenna and the m^{th} subcarrier when the respective input to the HPA is $s[m]^j$. β_i^j, $i = 0, \ldots, R$, represents the attenuation factor for the linear part at the j^{th} transmitting antenna in the i^{th} hop transmission, and $\psi[m]_i^j$ is the nonlinear additive noise due to HPA at the j^{th} transmitting antenna and in the i^{th} hop transmission. Note that, as the clipping is performed on the Nyquist rate, therefore all the subcarriers on the j^{th} transmitting antenna experiences the same attenuation β_i^j and hence there is no index m with β_i^j . The received signal on the m^{th} subcarrier at the first relay station after propagating through $N \times N$ MIMO channel, $\mathbf{H}_i[m]$, $i = 0,1,..., R$, in the frequency domain (FD) is given by,

$$\mathbf{y}_1[m] = \boldsymbol{\beta}_0\mathbf{H}_0[m]\mathbf{s}[m] + \mathbf{H}_0[m]\boldsymbol{\psi}_0[m] + \mathbf{n}_0[m] \quad \textbf{(8)}$$

where,

$$\mathbf{y}_1[m] = \left[y_1^1[m], y_1^2[m], \cdots, y_1^{N_r}[m]\right]^T \; m = 1, \ldots, n$$

and $y_i^j[m]$ is the received OFDM symbol at the j^{th} receiving antenna in the i^{th} hop transmission. The relay node demodulates the received signal and then normalize each subcarrier separately in the FD with normalization parameter $\mathbf{\Lambda} = diag\{\Lambda_i^j\}_{j=1}^N, i = 1, \ldots, R$. The normalization parameter $\Lambda_i^j[m] = \sqrt{P_R / E\left[\left|y_i^j[m]\right|^2\right]}$ is selected such that the total transmit power at the relay station is constrained to P_R (i.e. total power available at the relay station) for all subcarriers. Before transmitting to the next hop, the OFDM signal is passed through HPA at the RF stage of the relay station as shown in Fig 1. The received signal at the destination after propagating through R relays in the FD after the FFT, is given by

$$\mathbf{y}_D = \prod_{i=0}^{R}\boldsymbol{\beta}_i\prod_{k=1}^{R}\mathbf{\Lambda}_k(\mathbf{H}_R\mathbf{H}_{R-1}.......\mathbf{H}_1\mathbf{H}_0)\mathbf{s} + \mathbf{n}_D \quad \textbf{(9)}$$

where $\boldsymbol{\beta}_i, i = 0.....R$, is an $nN \times nN$ block diagonal matrix of the attenuation factors for the linear part of HPA , $\mathbf{\Lambda}_k$, $k = 1, \ldots, R$, is an $nN \times nN$ block diagonal matrix of the amplification parameter in the in i^{th} hop transmission, and $\mathbf{n}_D$ is the $nN \times 1$ complex additive noise vector that captures the over-all noise in the multi-hop channel. The over-all complex additive noise vector $\mathbf{n}_D$ can be modeled as:

$$\mathbf{n}_D = \mathbf{n}_0 + \mathbf{n}_1 + ... + \mathbf{n}_R \quad \textbf{(10)}$$

where

$$
\begin{aligned}
&\mathbf{n}_R = \tilde{\mathbf{n}}_R + \mathbf{H}_R \boldsymbol{\psi}_R,\\
&\mathbf{n}_{R-1} = \boldsymbol{\Lambda}_R \boldsymbol{\beta}_R \mathbf{H}_R \tilde{\mathbf{n}}_{R-1} + \boldsymbol{\Lambda}_R \boldsymbol{\beta}_R \mathbf{H}_R \mathbf{H}_{R-1} \boldsymbol{\psi}_{R-1},\\
&\mathbf{n}_{R-2} = \boldsymbol{\Lambda}_R \boldsymbol{\Lambda}_{R-1} \boldsymbol{\beta}_R \boldsymbol{\beta}_{R-1} \mathbf{H}_R \mathbf{H}_{R-1} \tilde{\mathbf{n}}_{R-2} + \boldsymbol{\Lambda}_R \boldsymbol{\Lambda}_{R-1} \boldsymbol{\beta}_R \boldsymbol{\beta}_{R-1} \mathbf{H}_R \mathbf{H}_{R-1} \mathbf{H}_{R-2} \boldsymbol{\psi}_{R-2},\\
&.\\
&.\\
&\mathbf{n}_1 = \boldsymbol{\Lambda}_R \cdots \boldsymbol{\Lambda}_2 \boldsymbol{\beta}_R \cdots \boldsymbol{\beta}_2 \mathbf{H}_R \cdots \mathbf{H}_2 \tilde{\mathbf{n}}_1 + \boldsymbol{\Lambda}_R \cdots \boldsymbol{\Lambda}_2 \boldsymbol{\beta}_R \cdots \boldsymbol{\beta}_2 \mathbf{H}_R \cdots \mathbf{H}_1 \boldsymbol{\psi}_1,\\
&\mathbf{n}_0 = \boldsymbol{\Lambda}_R \cdots \boldsymbol{\Lambda}_1 \boldsymbol{\beta}_R \cdots \boldsymbol{\beta}_1 \mathbf{H}_R \cdots \mathbf{H}_1 \tilde{\mathbf{n}}_0 + \boldsymbol{\Lambda}_R \cdots \boldsymbol{\Lambda}_1 \boldsymbol{\beta}_R \cdots \boldsymbol{\beta}_1 \mathbf{H}_R \cdots \mathbf{H}_0 \boldsymbol{\psi}_0
\end{aligned}
$$

$\tilde{\mathbf{n}}_i$ is the $nN \times 1$ iid zero-mean complex AWGN vector, introduced at the i^{th} hop transmission on the n subcarriers. $E\left[\tilde{\mathbf{n}}_i \tilde{\mathbf{n}}_i^H\right] = diag\left\{\sigma_i^2 \mathbf{I}[m]_N\right\}_{m=0}^{n-1}, i = 0,..., R,$, where $\mathbf{I}[m]_N$ is an $N \times N$ identity matrix, and σ_i^2 is the variance of each elements of $\tilde{\mathbf{n}}_i$. $\boldsymbol{\psi}_i$ is an $nN \times 1$ additive zero-mean complex Gaussian distortion noise vector due to HPA at the i^{th} hop transmission on the n subcarriers uncorrelated with the input OFDM symbol $\mathbf{s}$. Also, $E\left[\boldsymbol{\psi}_i \boldsymbol{\psi}_i^H\right] = diag\left\{\sigma_{\psi_i}^2 [m] \mathbf{I}[m]\right\}_{m=0}^{n-1}, i = 0,..., R$, where $\sigma_{\psi}^2[m]$ is the variance of the element of ψ on the m^{th} subcarrier and $E[\psi_i] = 0$. We assume that $\mathbf{n}_i$ and $\boldsymbol{\psi}_j$ are mutually independent so that $E[\mathbf{n}_i \boldsymbol{\psi}_j] = 0$, $\forall\ i$ and j. Since $E[\tilde{\mathbf{n}}_i] = 0$, for all i and $\tilde{\mathbf{n}}_i$ is independent of $\mathbf{H}_j$ and $\boldsymbol{\psi}_k$ for all i, j and k, we have that $E[\mathbf{n}_i] = 0$ for all i. The covariance matrix of $\mathbf{n}_D$ can thus be computed as:

$$
\begin{aligned}
E[\mathbf{n}_D \mathbf{n}_D^H] &= E[\mathbf{n}_0 \mathbf{n}_0^H] + E[\mathbf{n}_1 \mathbf{n}_1^H] + E[\mathbf{n}_2 \mathbf{n}_2^H] + \cdots \\
&+ E[\mathbf{n}_{R-1} \mathbf{n}_{R-1}^H] + E[\mathbf{n}_R \mathbf{n}_R^H]
\end{aligned} \tag{11}
$$

Under the above-mentioned assumptions, the covariance matrix is calculated and given by:

$$
\begin{aligned}
\sigma_{n_D}^2 \mathbf{I}_{nN} &= \sigma_0^2 \left(1 + \sum_{i=1}^{R} \left[\prod_{j=1}^{R-i+1} \Lambda_{R+1-j}^2 \beta_{R+1-j}^2 N^{R+1-i}\right]\right) \mathbf{I}_{nN} \\
&+ \left(\sum_{i=0}^{R-1} \left[\prod_{j=1}^{R-i+1} \Lambda_{R+1-j}^2 \beta_{R+1-j}^2 N^{R+1-i} \sigma_{w_i}^2\right]\right) \mathbf{I}_{nN} + N\sigma_{w_R}^2 \mathbf{I}_{nN}.
\end{aligned}
$$

(12)

Since the covariance matrix of $\mathbf{n}_D$ is a diagonal, then the elements of $\mathbf{n}_D$ are iid. If we assume similar behaviour HPA models at the BS and R RS's, then $\boldsymbol{\beta}_0 = \boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_R = \boldsymbol{\beta}$ and same amplification factor at all R RS then $\boldsymbol{\Lambda}_1 = \boldsymbol{\Lambda}_2 = \cdots = \boldsymbol{\Lambda}_R = \boldsymbol{\Lambda}$. Eq. (12) can be modified as:

$$
\begin{aligned}
E\left[\mathbf{n}_D \mathbf{n}_D^H\right] = \sigma_{n_D}^2 \mathbf{I}_{nN} &= \sigma_0^2 \left(1 + \sum_{i=1}^{R} \left[(\Lambda\beta)^{2(R+1-i)} N^{R+1-i}\right]\right) \mathbf{I}_{nN} \\
&+ \left(\sum_{i=0}^{R-1} \left[(\Lambda\beta)^{2(R-i)} N^{R+1-i} \sigma_{w_i}^2\right]\right) \mathbf{I}_{nN} + N\sigma_{w_R}^2 \mathbf{I}_{nN}.
\end{aligned} \tag{13}
$$

3.1 Capacity of Decode-and-Forward (DF) Relaying Protocol

In DF relay network, the signal at each relay node is fully decoded, re-encoded and then re-transmitted to the next terminal [9]-[10]. The transmitted nonlinearly amplified signal in multi-hop DF relaying system at the i^{th} relay node, $\hat{\mathbf{s}}_i$, $i = 1, \cdots, R$, is an estimate of the source signal $\mathbf{s}_i$, that has been obtained by decoding the received signal at the i^{th} relay node given by

$$
\mathbf{y}_i = \mathbf{H}_i \hat{\mathbf{s}}_{i-1} + \mathbf{n}_i \tag{14}
$$

Since $\hat{\mathbf{s}}_{i-1} = \boldsymbol{\beta}_{i-1} \mathbf{s}_{i-1} + \boldsymbol{\psi}_{i-1}$, we can express $\mathbf{y}_i$ as

$$
\begin{aligned}
\mathbf{y}_i &= \boldsymbol{\beta}_{i-1} \mathbf{H}_i \mathbf{s}_{i-1} + \mathbf{H}_i \boldsymbol{\psi}_{i-1} + \mathbf{n}_i \\
&= \boldsymbol{\beta}_{i-1} \mathbf{H}_i \mathbf{s}_{i-1} + \hat{\mathbf{n}}_i
\end{aligned} \tag{15}
$$

where $\hat{\mathbf{n}}_i = \mathbf{H}_i \boldsymbol{\psi}_{i-1} + \mathbf{n}_i$ is complex additive noise vector that captures the overall noise in the i^{th} transmission hop, and

$$
\begin{aligned}
E\left[\hat{\mathbf{n}}_i \hat{\mathbf{n}}_i^H\right] &= \hat{\sigma}_i^2 = E[(\mathbf{H}_i \boldsymbol{\psi}_{i-1} + \mathbf{n}_i)(\mathbf{H}_i \boldsymbol{\psi}_{i-1} + \mathbf{n}_i)^H] \\
&= N\sigma_{\psi_{i-1}}^2 \mathbf{I}_{nN} + \sigma_o^2 \mathbf{I}_{nN}
\end{aligned} \tag{16}
$$

Therefore, the capacity of the i^{th} hop channel in a nonlinear multi-hop DF MIMO-OFDM system is given by

$$
C_i = \frac{1}{n} E_{H_i} \left[\log_2 \left\{\det\left[\mathbf{I}_{nN} + \frac{\beta_i^2 \Lambda_i^2}{\hat{\sigma}_i^2} \mathbf{H}_i \mathbf{Z} \mathbf{H}_i^H\right]\right\}\right] \tag{17}
$$

For uniform power allocation across all space-frequency subchannels in the i^{th} hop transmission, $E[\hat{\mathbf{s}}_i\hat{\mathbf{s}}_i^H] = \mathbf{Z}$, and for the m^{th} subcarrier $Z_m = \frac{P_{r,i}}{nN} I_N$, where $P_{r,i}$ is the total available transmit power at i^{th} relay station. Hence,

$$C_i = \frac{1}{n} E_{H_i}\left[\log_2\left\{\det\left(\mathbf{I}_{nN} + \frac{\beta_i^2\Lambda_i^2}{\hat{\sigma}_i^2}.\frac{P_{r,i}}{nN}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}\right] \quad \textbf{(18)}$$

Using the eigenvalue decomposition of $\mathbf{H}_i\mathbf{H}_i^H$ we can express Eq. (18) in terms of the eigenvlaues as,

$$C_i = \frac{1}{n}\sum_{m=0}^{n-1}\sum_{l=1}^{N} E_{\lambda_{m,i}^l}\left[\log_2\left(1+\frac{\beta_i^2\Lambda_i^2}{\hat{\sigma}_i^2}.\frac{P_{r,i}}{nN}\lambda_{m,i}^l\right)\right] \quad \textbf{(19)}$$

which can be written as

$$C_i = \frac{N}{n}\sum_{m=0}^{n-1} E_{\lambda_{m,i}}\left[\log_2\left(1+\frac{\beta_i^2\Lambda_i^2}{\hat{\sigma}_i^2}.\frac{P_{r,i}}{nN}\lambda_{m,i}\right)\right] \quad \textbf{(20)}$$

where $\lambda_{m,i}$ is a randomly selected eigenvalue from the set of eigenvalues $\left\{\lambda_{m,i}^1,\cdots,\lambda_{m,i}^N\right\}$ of $\mathbf{H}_i[m]\mathbf{H}_i^H[m]$. The pdf of $\lambda_{m,i}$ can be expressed as [14]:

$$f_{\lambda_{m,i}}(\lambda) = \frac{1}{N}\sum_{p=0}^{N-1}\sum_{q=0}^{p}\sum_{r=0}^{2q}\frac{(-1)^r(2q)!}{2^{2p-l}q!r!(q)}\binom{2p-2q}{p-q}\binom{2q}{2q-r}\lambda^r e^{-\lambda} \quad \textbf{(21)}$$

The ergodic capacity can now be expressed as:

$$C_i = \frac{n}{N}\sum_{m=0}^{n-1}\int_0^{\infty}\left[\log_2\left(1+\frac{\beta_i^2\Lambda_i^2}{\hat{\sigma}_i^2}.\frac{P_{r,i}}{nN}\lambda_{m,i}\right)\right] f_{\lambda_{m,i}}(\lambda)d\lambda_{m,i} \quad \textbf{(22)}$$

Substituting Eq.(21) in Eq.(22) and solving the integration, we come up with the following expression for the ergodic capacity of the i^{th} hop nonlinear MIMO-OFDM DF relaying system [15],[16] (see Appendix-A for proof):

$$C_i \approx \sum_{p=0}^{N-1}\sum_{q=0}^{p}\sum_{r=0}^{2q}\frac{(-1)^r(2q)!}{2^{2p-l}q!r!q}\binom{2p-2q}{p-q}\binom{2q}{2q-r}r! \left[\ln\left(\frac{\beta_i^2\Lambda_i^2}{\hat{\sigma}_i^2}.\frac{P_{r,i}}{nN}\right) - 0.577 + \sum_{m=1}^{r} m^{-1}\right] \quad \textbf{(23)}$$

Finally the overall capacity of the DF relaying system is the minimum of the achievable capacities over each individual hop [9]-[10]. Therefore, the ergodic capacity of the nonlinear MIMO-OFDM DF relaying system is given by,

$$C_{DF} = \min\{C_1, C_2, \cdots, C_R\} \quad \textbf{(24)}$$

4 Simulation Results

In this section, we consider multi-hop MIMO-OFDM system model for different number of subcarriers and different number of transmit and receive antennas. The HPA model employed in the simulation studies is SEL model, which is widely used model both in the field and in the literature.

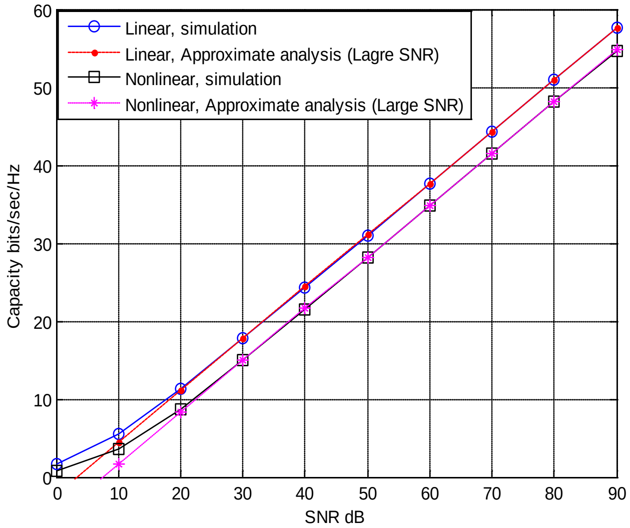

Figure 3: Simulation and Analytical results for the i^{th} hop ergodic capacity of the Linear and Nonlinear $2x2$ MIMO-OFDM DF relaying system

Fig. 3 compares simulation results with the approximate analysis, for the cases of linear and nonlinear DF relaying systems, for the multi-hop capacity of MIMO-multiplexing system using SEL HPA model. It can be observed that the analysis and simulation results agree closely in high-SNR region for both the linear and nonlinear cases.As expected,

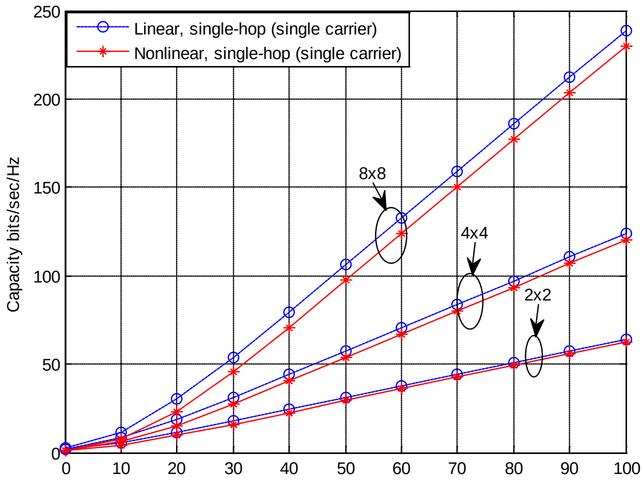

Figure 4: Effect of HPA nonlinearity on the ergodic capacity of different MIMO configurations for the SEL HPA model, 1-hop transmission

the capacity of MIMO-multiplexing relaying system degrades due to HPA nonlinearity.

Next, the effect of HPA nonlinearity on the capacity of different MIMO configurations is illustrated in Fig. 4. Here we observed that, as the dimensionof the MIMO system increases the gap between the linear and nonlinear capacity also increases. From this it can be concluded that the capacity of the high-order MIMO-multiplexing relaying systems degrade more in the presence of HPA nonlinearity.

5 Conclusion

This studypresents the capacity of nonlinear MIMO-OFDM DF relaying channels. Closed-form expressions for the ergodic capacity of the decode-and-forward (DF) relaying protocolis presented. The derived expressions are validated using extensive computer simulations. Our results show that high-order MIMO systems suffer more capacity loss due to HPA nonlinearity than low-order MIMO systems, and that more capacity loss are experienced in multihop relaying over nonlinear channels as more relay hops are involved. This analysis complements recent works in the literature, where the effects of HPA on the error rate performance of MIMO systems have been documented. It also helps to stress the need for highly linearized HPA when relaying methods of wireless transmissions are desired.

Appendix 1: Proof of Eq. (23)

Let $\Omega = \frac{\beta_i^2 \Lambda_i^2}{\hat{\sigma}_i^2} \cdot \frac{P_{r,i}}{nN}$ and substituting Eq. (21) into Eq. (22), we get the following expression:

$$C_i = \left[\frac{1}{n} \sum_{p=0}^{N-1} \sum_{q=0}^{p} \sum_{r=0}^{2q} \frac{(-1)^r (2q)!}{2^{2p-l} q! r! (q)} \binom{2p-2q}{p-q} \binom{2q}{2q-r} \cdot \sum_{m=0}^{n-1} \left\{ \int_0^\infty \log_2 (1+\Omega\lambda_{m,i})(\lambda_{m,i})^r (e^{-\lambda})_{m,i} \, d\lambda_{m,i} \right\} \right] \quad \text{(A.1)}$$

The integral in (A.1) can be represented as $\mathrm{T}_r(\Omega) = \int_0^\infty \log_2 (1+\Omega\lambda_{m,i})(\lambda_{m,i})^r (e^{-\lambda})_{m,i} \, d\lambda_{m,i}$,

The closed-form solution of this integral is given as [16]-[18]:

$$\mathrm{T}_r(\Omega) = \sum_{\kappa=0}^{r} \frac{r}{(r-\kappa)!} (-1)^{r-\kappa-1} \left(1/\Omega\right)^{r-\kappa} e^{1/\Omega} Ei(-1/\Omega) + \sum_{\varepsilon=1}^{r-\kappa} (\varepsilon-1)!(-1)^{r-\kappa-1} (-1/\Omega)^{r-\kappa-\varepsilon} \quad \text{(A.2)}$$

where $Ei(a) = \int_{-\infty}^{a} \frac{e^t}{t} dt$ which can be approximated by [16]-[18]:

$$Ei(a) \approx c + \ln(-a) \quad |a| << 1, \qquad c \approx 0.577 \quad \text{(A.3)}$$

Using the above approximation, Eq. (A.2) can be simplified as

$$\mathrm{T}_r(\Omega) = r! \left[\ln(\Omega) - c + \sum_{\kappa=1}^{r} \frac{1}{\kappa} \right] \quad \text{(A.4)}$$

By putting Eq. (A.4) into Eq. (A.1), we get Eq. (23).

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