PEGR is getting larger as the subcarrier index becomes more far away from the center of the cluster.

4 Proposed PAPR reduction method for OFDM system with CCIF based BF

In this section, first we propose a new PAPR reduction scheme for MISO OFDM system and then, extend it to MIMO OFDM system.

4.1 PAPR reduction method for MISO OFDM system

In practice, the detection performance at the receiver is more important than PAPR reduction at the transmitter. Therefore, based on the observation in Fig. 2, we propose frequency-dependent weight $g_l(k)$ to reduce the PAPR at the transmitter and to minimize the degradation of the receiver performance caused by null space based PAPR reduction operation. For the subcarrier *k* within the *l*-th cluster, the transmitted signal with the proposed weight is represented as

$$\mathbf{s}_l(k) = \bar{\mathbf{w}}_l x(k) - g_l(k)\bar{\mathbf{P}}_l(k)\sum_{j=1}^{J} \mathbf{c}^{(j)}(k), \quad (l-1)B+1 \le k \le lB. \tag{11}$$

By designing the weight $g_l(k)$ appropriately, the PEGR shown in Fig. 2 can be equalized over all subcarriers and thus, the degradation of the detection performance can be made negligible.

First, we propose an optimal weight design method to minimize the mean square error and then, a simple suboptimal method to reduce the computational complexity.

If we define $\mathbf{g} = [g_1(1), \cdots, g_1(B), \cdots g_L(1), \cdots, g_L(B)]^T$, a proposed nonlinear optimization problem for obtaining the optimal weight vector is given by

$$\mathbf{g}_{\text{opt}} = \arg\min_{\mathbf{g}} \sum_{k=1}^{N} \varepsilon(k) \quad \text{s.t. } \mathbf{1}^T \mathbf{g} = \alpha, \tag{12}$$

where $\varepsilon(k)$ is the mean square error (MSE) between the received signal and the ideal one denoted by

$$\varepsilon(k) = E_n\left[\left|\frac{\mathbf{h}(k)^T \bar{\mathbf{P}}}{\mathbf{h}(k)^T \bar{\mathbf{w}}}\sum_{j=1}^{J} g_l(k) c^{(j)}(k) - \frac{n(k)}{\mathbf{h}(k)^T \bar{\mathbf{w}}}\right|^2\right]. \tag{13}$$

If (12) has no constraint, the optimal weight vector $\mathbf{g}_{\text{opt}}$ will be $\mathbf{0}$, i.e., a trivial solution. In order to avoid the trivial solution, we introduce a constraint on the sum of the weights in (12).

If we choose a small value for α, the power of the dummy signal which is the second term of the right-hand side in (12) will become small. Therefore, the PAPR reduction gain will become small while the degradation of the detection performance at the receiver will be small. On the other hand, if we choose a large value for α, the PAPR reduction gain will become high at the expense of the loss of the detection performance at the receiver. Therefore, α should be selected appropriately to obtain a trade-off between the PAPR reduction performance at the transmitter and the degradation of the detection performance at the receiver.

Since the optimization problem in (12) includes nonlinear operations such as clipping, it is difficult to find a closed-form solution. Hence, the optimal solution is found by resorting to the numerical method. Specifically, the problem in (12) is rewritten as

$$\mathbf{g}_{\text{opt}} = \arg\min_{\mathbf{g}} E_{\mathbf{H}}\left\{E_{\mathbf{x}}\left[\sum_{k=1}^{N} MSE(k) \,|\, \mathbf{x}\right] \,\Big|\, \mathbf{H}\right\}, \quad \text{s.t. } \mathbf{1}^T\mathbf{g} = \alpha \tag{14}$$

where

$$\mathbf{x} = [x(1), x(2), \cdots, x(N)]^T, \ \mathbf{H} = [\mathbf{h}(1), \mathbf{h}(2), \cdots, \mathbf{h}(N)]^T.$$

The detailed procedure to find the solution using the numerical method is summarized as follows:

Step 1) For a randomly generated MISO channel matrix $\mathbf{H}$, find the optimal solutions[2] that satisfy (14) over 200 samples of random input vector $\mathbf{x}$.

Step 2) Since (14) is not a convex problem, the solution obtained in Step 1 could be a local optimum. Therefore, the solutions are found for 20 randomly generated initial values and the solution causing the minimum MSE is chosen as an optimal solution.

Step 3) Repeat Step 1 and 2 over 200 randomly generated $\mathbf{H}$s.

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[2] To find the solution in (14), the function 'fmincon' in the optimization toolbox of MATLAB is used.

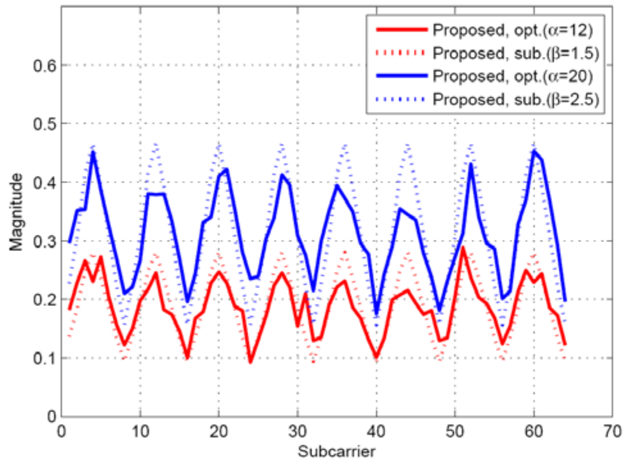

Fig. 3. Proposed optimal weights (solid line) and suboptimal weights (dotted line).

Step 4) Obtain the optimal solution $\mathbf{g}_{\text{opt}}$ by averaging the total 40,000 solutions.

Next, we propose a suboptimum method for reducing the computational complexity. In order to equalize the PEGR, the inverse of PEGR can be considered as the desired weight by intuition as follows:

$$g_l(k) = \frac{\beta}{\text{PEGR}(k)}, \qquad (15)$$

where β is a positive constant controlling total power of the dummy signal similarly to α in (12).

4.2 Extension to MIMO OFDM system

The null space based PAPR reduction method for MIMO OFDM system with BF is very similar to that for MISO OFDM system with BF. The PAPR reduction scheme proposed for MISO OFDM system can be directly applied to the MIMO OFDM system except that the MIMO channel matrix is used in the projection matrix instead of the MISO channel matrix.

Suppose that the transmitter and the receiver have M_T and M_R antennas, respectively. If $\mathbf{H}(k)$ is the $M_R \times M_T$ channel matrix at the *k*-th subcarrier, the columns of the right singular matrix of $\mathbf{H}(k)$ corresponding to nonzero eigenvalues from the optimal BF matrix $\mathbf{W}(k)$. The projection matrix orthogonal to the channel matrix is given by [14]

$$\mathbf{P}(k) = \mathbf{I} - \mathbf{W}(k)\{\mathbf{W}(k)^H \mathbf{W}(k)\}^{-1} \mathbf{W}(k)^H. \qquad (16)$$

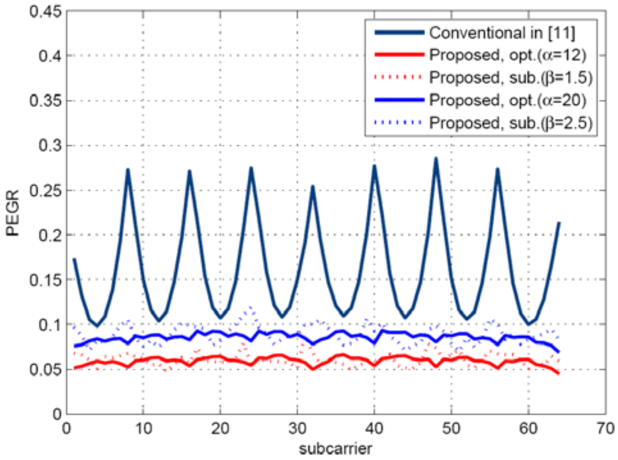

Fig. 4. PEGR of MISO OFDM system with CCIF based BF when ETSI/BRAN channel E model with $M_T = 4$ and $M_R = 1$.

If we define the singular value decomposition of $\mathbf{H}(k)$ as $\mathbf{H}(k) = \mathbf{U}(k)\mathbf{\Sigma}(k)\mathbf{V}^H(k)$ where $\mathbf{U}(k)$ and $\mathbf{V}^H(k)$ denote the left and right singular matrices, respectively, and $\mathbf{\Sigma}(k)$ is a diagonal matrix, the BF matrix $\mathbf{W}(k)$ is composed of the columns of $\mathbf{V}(k)$ and the projection matrix in (16) holds $\mathbf{H}(k)^T \mathbf{P}(k) = \mathbf{0}$. The procedures to generate the dummy signal and to find the weight vector $\mathbf{g}$ are the same as the MISO OFDM case. Here, we note that the sufficient condition for the null space to exist is $M_T > M_R$. If this condition is not satisfied, the null space may not exist.

4 Simulation results

In order to evaluate the BER and PAPR performance of the proposed methods, we have performed computer simulation over 10^5 Monte Carlo trials. The system parameters are as follows: $M_T = 4$ and $M_R = 1$ for MISO system, $M_R = 2$ for MIMO, $N = 64$, and $B = 8$. The transmitted data are encoded by rate 1/2 convolutional code, and modulated by quadrature phase shift keying (QPSK). The oversampling ratio is 4. For the channel model, ETSI/BRAN Channel Model E in [15] is employed and the channels from different transmit antennas are generated independently. The channel is assumed to be quasi-static, *i.e.*, the channel is time-invariant within an OFDM symbol period but varies independently between symbols. The noise is complex additive white Gaussian random variable with zero mean. For the detection of the transmitted

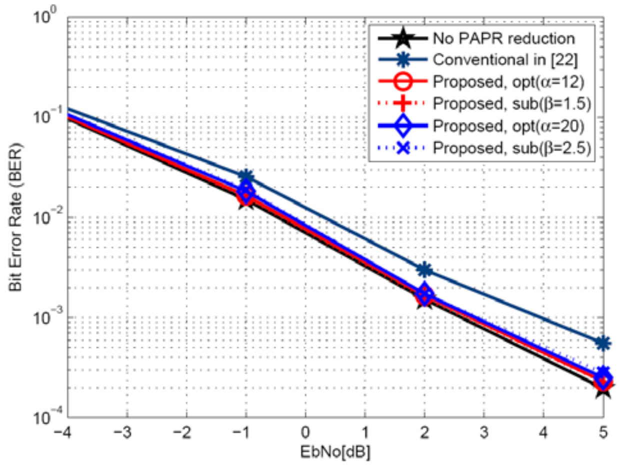

Fig. 5. BER performance of MISO OFDM system with CCIF based BF.

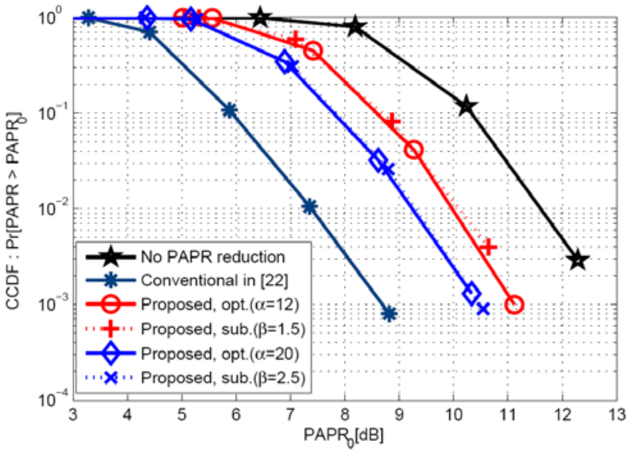

Fig. 6. PAPR performance of MISO OFDM system with CCIF based BF.

symbols, zero-forcing equalization is employed, *i.e.*, at the receiver, the received signal is divided by the channel coefficient at each subcarrier. For the BF codebook, 4 bit codebook for MISO system and 6 bit for MIMO system are used [16]. It is assumed that the feedback channel has no time delay and no transmission error. The PAPR performance is examined through the complementary cumulative distribution function (CCDF) defined as [16]

$$CCDF(PAPR_o)=\Pr(PAPR>PAPR_o)\,. \qquad (17)$$

Fig. 3 shows the optimum weights given in (12) for $\alpha=12$ and 20, and the suboptimum weights given in (15) for $\beta=1.5$ and 2.5 in case of MISO OFDM system. Fig. 4 represents the PEGR after applying the proposed weights. It is observed that the PEGR is equalized over all subcarriers. Also, it

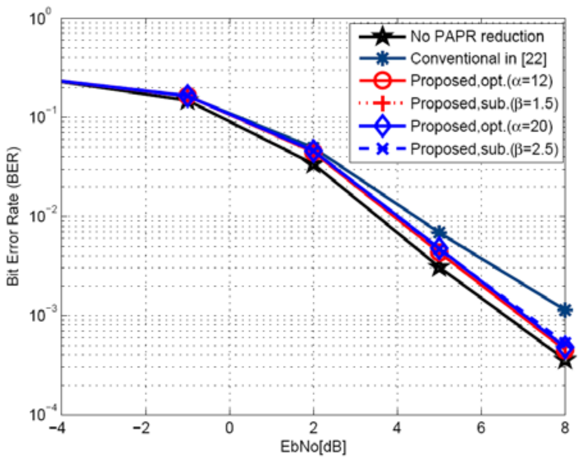

Fig. 7. BER performance of MIMO OFDM system with CCIF based BF when $M_T=4$ and $M_R=2$.

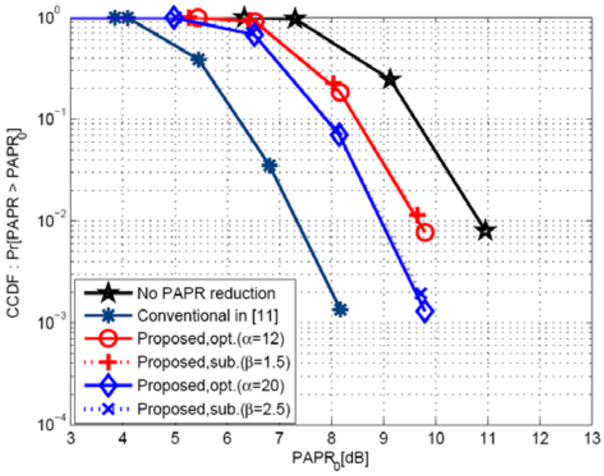

Fig. 8. PAPR performance of MIMO OFDM system with CCIF based BF when $M_T=4$ and $M_R=2$.

is shown that by varying α in (12) and β in (15), the total amount of PEGR can be controlled.

Figs. 5 and 6 show the BER and PAPR performances for MISO OFDM system, respectively. By applying the conventional method in [1], a significant BER loss about 1.5dB at $BER=10^{-3}$ is observed while the PAPR is reduced by 3.5dB at $CCDF=10^{-2}$. In contrast, the proposed methods mitigate the BER loss at the expense of the sacrifice of the PAPR reduction gain. The suboptimum weights $\beta=1.5$ and 2.5 provide almost the same performance as the optimum weights $\alpha=12$ and 20, respectively.

Figs. 7 and 8 show the BER and PAPR performances when the proposed techniques are applied to MIMO OFDM system. It is observed that the behavior is quite similar to the MISO OFDM case. The proposed methods relieve the BER loss significantly while providing less PAPR reduction performance than the conventional method.

Through the simulation results, it is confirmed that the proposed weights can significantly reduce the BER loss at the expense of the small degradation of the PAPR reduction performance when the null space based PAPR reduction technique is used for OFDM system with CCIF based BF.

4 Conclusion

A new PAPR reduction method was proposed for MISO/MIMO OFDM systems with the limited feedback based BF. In the proposed scheme, frequency domain weights are introduced to mitigate the BER performance loss at the receiver caused by the inaccuracy of the BF weight vector and projection matrix in CCIF based BF system. Simulation results showed that the proposed method has better BER performance than the conventional method at the expense of the small PAPR performance degradation

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