

Spectral Efficiency Comparison of TDMA and DS-CDMA in Cellular Mobile Radio Systems

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Abstract: - A comparison of spectral efficiency achieved by time-division multiple-access (TDMA) and direct-sequence code-division multiple-access (DS-CDMA) systems, during cellular operation in a Rayleigh fading environment, is presented in this paper. Spectral efficiency, is evaluated in terms of the theoretically assigned downlink channel capacity per user (in the Shannon sense). In particular, in Rayleigh fading, channel capacity is estimated in an average sense considering the inherently offered diversity potential as a physical means for fading mitigation. In addition, both systems are assumed to allocate a fixed number of simultaneously transmitting users so that the total capacity of each system to be equally shared among its users. Hence, it is analytically shown that, under normalised conditions, the spectral efficiency of DS-CDMA is higher than that of TDMA regardless the fact that, in contrast to cellular TDMA, in cellular operation of DS-CDMA spectral efficiency is limited by multiple-access interference (MAI).

Key-Words: Spectral efficiency, TDMA, DS-CDMA, Rayleigh fading.

1 Introduction

TDMA is the multiple access technique of choice for several digital cellular and PCS systems since provides several advantages relative to alternative multiple access techniques, [1, 2]. One advantage is that the common radio and modem equipment, at a given carrier frequency, can be shared among the users at the base station. Another advantage, with respect to FDMA, is that data rates to and from each user can be varied according to current user needs, by allocating more or fewer timeslots to the user. With respect to CDMA, TDMA has the advantage of much less power control requirements, since interference is controlled by timeslot and frequency allocation instead of by processing gain resulting from signal spreading. However, TDMA may require equalisation against multipath fading, which is generally avoided with FDMA and CDMA techniques.

The demand for cellular and personal communications systems is the efficient use of time-varying channels. This efficiency can be quantified by the spectral efficiency: the data rate per channel bandwidth for a fixed bit-error-rate (BER). Respectively, the maximum spectral efficiency of a communication channel with bandwidth B is given by C/B where C denotes the Shannon capacity of the channel. However, in a fading channel, the maximum rate at which data can be transmitted with arbitrarily small BER, is obtained by finding the

best distribution of the transmitted power, as a function of the instantaneous signal-to-noise power ratio (SNR) and then averaging over the SNR distribution, where the maximisation is subject to the average power constraint, [3].

In an ideal non-fading additive white Gaussian noise (AWGN) environment, the theoretically achieved spectral efficiency of a cellular system utilizing any of the these three basic multiple access techniques, i.e., frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA), will be the same, since it does not matter whether the spectrum is divided into frequencies, time-slots, or codes. In contrast, in a typical mobile radio environment, physical channels exhibit randomly time-varying characteristics and, thus, the theoretical equivalence of the basic multiple-access schemes does not exist, due to channel capacity degradation. In this case, channel capacity should be estimated in an average sense, [4, 5].

In this paper, we compare the achieved spectral efficiency of pure TDMA and pure DS-CDMA based mobile radio systems for cellular operation in Rayleigh fading environment considering the examination of the inherent diversity potential that each of them can provide. The comparison, under normalized conditions, of TDMA and DS-CDMA multiple-access schemes, as described in our work, is suitable for providing quantitative results instead

of just the qualitative conclusion that is obtained for these multiple-access techniques in [6]. The approach covers the base-to-mobile units link, i.e., the downlink also called broadcast channel, while a fixed number of users per cell is assumed. Spectral efficiency is evaluated in terms of the available capacity per physical and/or logical channel for each system, i.e., in terms of channel capacity per user, on the condition that channel capacity is estimated in an average sense and used as a figure of merit of their operation in a fading environment, [4].

2 System's Considerations

Let us consider a cellular TDMA system that accommodates a fixed number K of users per cell, reflecting a static model of operation. In a TDMA cellular system, several users time-share a common carrier frequency to communicate with the base station. Each user is allocated one or more timeslots within a frame in the downlink and up link transmissions. It must be notice that, in a cellular TDMA system, a certain timeslot is allocated to one user and there are not other simultaneous users sharing the same channel within a cell. Thus, CCI is only introduced by the nearby co-channel cells. In practice, the TDMA technique is usually combined with Slow Frequency Hopping (SFH). For example, in Global System Mobile (GSM) the carriers frequencies are frequency separated and each multiplexes eight channels by time division, [7]. Assuming that q is the total number of hopping carrier frequencies assigned to each cell for frequency hopping, the totally allocated system bandwidth B_T is equal to:

$$B_T = q \cdot B_{sp,T} \quad (1)$$

where $B_{sp,T}$ is the frequency separation between the different carriers frequencies, as shown in Fig.1.

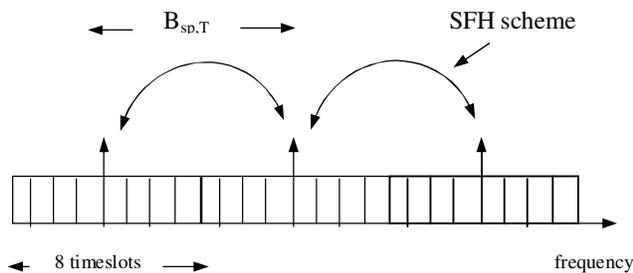


Fig1. Slow Frequency Hopping applied in a TDMA scheme.

The subscript ‘T’ used refers to the applied TDMA scheme. In this paper, following the method

described in [5], for the SFH scheme, we compare the spectral efficiency provided by cellular TDMA and DS-CDMA systems under normalized conditions i.e., for the same number of simultaneous users per cell and the same totally allocated system’s bandwidth.

2.1 Channel Capacity for Operation in an AWGN Environment

Firstly, we consider the first fourteen cells of a cellular TDMA system operating in a non-fading AWGN environment. The available channel capacity to all users of the cellular TDMA system, is limited only by CCI power since, as already mentioned, all users in each cell are mutually orthogonal, [1,8]. Clearly, transmission of a user signal (assumed Gaussian at system input) with arbitrarily small error probability, depends on CCI level. Thus, the logical channel capacity required for errorless transmission of a signal of bandwidth B_T , in each of the q frequencies, is given by the Shannon-Hartley theorem when arbitrarily complex coding and delay is applied, [9]:

$$C_{i,T} = B_T \cdot \log_2(1 + \Gamma_{i,T}) \quad (2)$$

where $\Gamma_{i,T}$, $i=[1,..,14K]$, is the signal-to-interference power ratio (SIR) received at the mobile unit as it reaches the boundary of a cell. In order to simplify the mathematical solution, all cells are approximated by circular regions of radius R . Assuming a fourth power law path loss, the average received signal power P_{av} at the cell boundary will then be:

$$P_{av} = aR^{-4} \quad (3)$$

where a is a constant factor. In the downlink, all mobile units that belong to a certain cell receive equal average signal power from their cell site, [10]. Therefore, for a TDMA cellular system with reuse factor three, $\Gamma_{i,TD}$ for each mobile unit, can readily be determined considering the average CCI power from the six nearby co-channel interfering cells as following,

$$\Gamma_{i,T} = \frac{aR^{-4}}{N_0 B_T + P_h \left[Ka(2R)^{-4} + 2Ka(2.633R)^{-4} + 2Ka(3.6R)^{-4} + Ka(4R)^{-4} \right]} = \frac{P_{av}}{N_0 B_T + P_h \cdot (0.1133K) \cdot P_{av}} \quad (4)$$

where N_0 is the power spectral density of the additive white Gaussian noise and P_h is the

probability that another user is transmitting at the same hopped frequency band as the desired user, called probability of hit. However, it must be noted that eq.(4) does not take into account the voice activity cycle or sector-reuse parameterization, as required in describing real commercial systems. Furthermore, CCI is considered as gaussian distributed interference even for small values of number of system's users, [11]. When SFH scheme is assumed, then P_h can be approximated by, [12]:

$$P_h = 1/q \quad (5)$$

Thus, for the TDMA system, eq.(4) can equivalently be written in the form:

$$\Gamma_{i,T} = \frac{\Gamma}{q + \frac{1}{q} \cdot (0.113K) \cdot \Gamma} \quad (6)$$

where $\Gamma = (P_{av}/N)$ is the received SNR over the signal bandwidth $B_{sp} = B_T/q$.

Following eq.(2), the total channel capacity (in the Shannon sense) for the fourteen cells of the cellular TDMA system under consideration, that is, the total (overall) channel capacity available to all $14K$ users, will be given by the sum of the individual rates:

$$C_T = \sum_{i=1}^{14K} C_{i,T} = B_T \cdot \sum_{i=1}^{14K} \log_2(1 + \Gamma_{i,T}) \quad (7)$$

where $\Gamma_{i,T}$ is given by eq.(6).

Let us consider a cellular DS-CDMA system that accommodates a fixed number K of users per cell, reflecting a static model of operation. Bandwidth spreading is accomplished in the transmitter by multiplying the information data by a pseudo-random sequence. Thus, a DS signal is transmitted in the form of a spread signal with bandwidth $B_{ss} = G_p \cdot B_s$, where B_s is the signal bandwidth and G_p is the processing gain or spreading factor.

It is well known that, in a cellular DS-CDMA system, the main interference sources are CCI and MAI. For such an operation mode the totally available capacity to all of the system users is limited by MAI as well as CCI. Clearly, errorless transmission of a user signal (assumed gaussian at the system input) in either form, i.e., unspread with bandwidth B_s or spread with bandwidth B_C depends on the level of the total interference. In a cellular DS-CDMA system, the broadband frequency channel B_c can be reused in every adjacent cell so that the cell reuse factor is close to 1. In each of

these cells, different code sequences use the same radio channel to carry different traffic channels.

Assuming a fixed number of simultaneous users per cell, the total capacity of the broadband channel B_c can be defined by the Shannon's formula since the channel capacity does not change in a dynamic fashion. We consider, as in the TDMA case, the first fourteen cells of a cellular DS-CDMA system operating in a non-fading AWGN environment. The logical channel capacity (in the Shannon sense) required for errorless transmission of a spread signal of bandwidth B_{ss} to each user i of the system, $i=[1, \dots, 14K]$, will be given by, [13]:

$$C_{i,C} = B_C \cdot \log_2(1 + \Gamma_{i,ss}) \quad (8)$$

where $\Gamma_{i,ss}$ is the spread signal-to-interference power ratio (SIR) received at the mobile unit as it reaches the boundary of a cell and the subscript 'C' used refers to the applied DS-CDMA scheme. Therefore, for each mobile unit, $\Gamma_{i,ss}$ can readily be determined by considering the average interference power from all the fourteen interfering co-channel cells as following, [13]:

$$\Gamma_{i,ss} = \frac{P_{av}}{N_0 B_C + (2.45K - 1)P_{av}} \quad (9)$$

Furthermore, both MAI and CCI are considered as gaussian interference. Such an approximation can hold reasonably well provided that each user's pseudorandom signal waveform has "noise-like" properties. Then, the broadband signal would appear "noise-like" to other receivers. Eq.(9) can equivalently be written in the form:

$$\Gamma_{i,ss} = \frac{\Gamma}{G_p + (2.45K - 1) \cdot \Gamma} \quad (10)$$

where $\Gamma = (P_{av}/N)$ is the received SNR over the signal bandwidth $B_s = B_C/G_p$. Thus, the total channel capacity (in the Shannon sense) for the 14 cells of the cellular DS-CDMA system under consideration, that is, the total capacity available to all $14K$ users, will be equal to:

$$C_C = \sum_{i=1}^{14K} C_{i,C} = B_C \cdot \sum_{i=1}^{14K} \log_2(1 + \Gamma_{i,ss}) \quad (11)$$

while, based on the assumption that $\Gamma_{i,ss} = \Gamma_{ss}$ for all $i=[1, \dots, 14K]$ and the fact that Γ_{ss} is well below unity (in linear scale), eq.(11) can be approximated by:

$$C_C = B_C \cdot \log_2(1 + 14K \cdot \Gamma_{ss}) \quad (12)$$

2.1.1 Channel Capacity for Operation in a Rayleigh Fading Environment

We consider a cellular DS-CDMA system operating in a Rayleigh fading environment. We assume that the physical channel of bandwidth B_C is greater than the coherence bandwidth B_{coh} of the urban Rayleigh fading channel and, thus, it appears to be frequency-selective to the transmitted DS signals, [10]. The maximum number M_C of the inherent diversity branches provided by the DS transmission, will then be given by, [4, 14]:

$$M_C = [B_C \cdot \Delta] + 1 = \left\lfloor \frac{\Delta}{T_C} \right\rfloor + 1 \quad (13)$$

where $T_C = 1/B_C$ is the "chip" duration of the DS spreading sequence, Δ is the time-delay or total multipath spread of the fading channel (assumed known or measurable), and $[.]$ returns the largest integer less than, or equal to, its argument. However, if Δ is sufficiently small compared to T_C , or, equivalently, if the bandwidth B_C is smaller than the coherence bandwidth B_{coh} in a Rayleigh fading environment, there will be no inherent diversity potential, and, if fading reduction is seek through diversity techniques, space diversity or a hybrid scheme shall be used, [15].

In the following, we estimate the average channel capacity per user in a DS-CDMA cellular system, as affected by the inherent diversity potential of the DS transmission and assuming operation in Rayleigh fading. In addition, we assume that all users of the system equally share the capacity of the entire physical channel of bandwidth B_C and that all users in all cells operate under similar fading conditions. Thus, we write:

$$\langle C_i \rangle_{C, \text{Rayleigh}} = \frac{1}{14K} \cdot \langle C \rangle_{C, \text{Rayleigh}} \quad (14)$$

where $\langle C_i \rangle_{C, \text{Rayleigh}}$ is the i -th user's portion of the totally available average capacity $\langle C \rangle_{C, \text{Rayleigh}}$ of the channel B_C . Note also that, in this case, the number of effective users equals to $14K$, since the remaining users of the system do not affect the totally available channel capacity.

In general, the multipath-intensity profile (MIP) in a fading environment is exponential but, here, it is assumed discrete and constant, so that the "resolvable" path model can be considered to have equal path strengths on the average. Hence, path-diversity reception of the DS transmitted signals,

achieved by a maximal-ratio RAKE receiver, can be seen as equivalent to a M -branch space diversity maximal-ratio combining technique, with $M=M_C$. However, the performance of a RAKE receiver depends on the degree of accuracy of fading channel estimation. In order to simplify further our analysis, we also assume perfect estimation of the channel tap weight coefficients. An expression for the average channel capacity $\langle C \rangle_{C, \text{Rayleigh}}$ of the channel B_C can then be written as, [5]:

$$\begin{aligned} \langle C \rangle_{C, \text{Rayleigh}} &= \\ &= B_C \cdot \int_0^\infty \log_2(1 + \gamma_t) \cdot \frac{(\gamma_t)^{M_C - 1}}{(M_C - 1)! (\Gamma_t)^{M_C}} \exp\left(-\frac{\gamma_t}{\Gamma_t}\right) d(\gamma_t) \end{aligned} \quad (15)$$

where Γ_t is the total average received spread SIR, from all users as it appears at the output of the channel B_C , i.e., $\Gamma_t = \langle \gamma_t \rangle$, and M_C is obtained from eq.(13). Clearly, eq.(15) is an estimation of the effective average channel capacity that appears at each receiver output in the form of the average best recovered data rate from all users of the cellular system. If we assume that the signal to each user appears at the receiver input with the same average spread SIR $\Gamma_{i,ss}$, then the totally received average spread SIR from all users will be equal to $14K\Gamma_{i,ss}$. Thus, following eq.(10),

$$\Gamma_t = \frac{14K\Gamma}{G_p + (2.45K - 1) \cdot \Gamma} \quad (16)$$

Then, combining eqs (14) and (15), the spectral efficiency $SE_{C, \text{Rayleigh}}$ achieved by a cellular DS-CDMA system operating in a Rayleigh fading environment and measured in (bits/sec/Hz) over B_C , is found to be:

$$\begin{aligned} SE_{C, \text{Rayleigh}} &= \frac{\langle C_i \rangle_{C, \text{Rayleigh}}}{B_C} = \\ &= \frac{1}{14K \cdot (M_C - 1)! (\Gamma_t)^{M_C}} \cdot \int_0^\infty \log_2(1 + \gamma) \cdot \gamma^{M_C - 1} \exp\left(-\frac{\gamma}{\Gamma_t}\right) d\gamma \end{aligned} \quad (17)$$

where the notation for the instantaneous spread SIR γ_t , used in eq.(15), has been changed to γ and the average SIR value Γ_t is given by eq.(16).

We now examine the case of a cellular TDMA system operating in a Rayleigh fading environment. The symbol duration in the TDMA frame, in GSM case, is smaller than the coherence time T_{coh} of the fading channel, [7]. Therefore, no inherent diversity potential, is provided by the pure TDMA system

along the time axis. However, diversity potential is offered by a discontinuous exchange of carriers frequencies between different users of the system i.e. when SFH is applied. Assuming that all users of the system equally share the capacity of the entire physical channel of bandwidth B_T and that all users in all cells operate under similar Rayleigh fading conditions. Thus, we can write:

$$\langle C_i \rangle_{T, \text{Rayleigh}} = \frac{1}{K_T} \cdot \langle C \rangle_{T, \text{Rayleigh}} \quad (18)$$

where $\langle C_i \rangle_{T, \text{Rayleigh}}$ is the i -th user's portion of the totally available average capacity $\langle C \rangle_{T, \text{Rayleigh}}$ of the channel B_T and K_T is the total number of effective users equals to:

$$K_T = (r-1)K \cdot q \quad (19)$$

where r equals to fourteen. Then, combining eqs (18) and (1), the spectral efficiency $SE_{T, \text{Rayleigh}}$ achieved by a cellular TDMA system operating in an urban Rayleigh fading environment and measured in (bits/sec/Hz) over B_T , is found to be:

$$SE_{T, \text{Rayleigh}} = \frac{\langle C_i \rangle_{T, \text{Rayleigh}}}{B_T} = \frac{1}{K_T} \cdot \int_0^\infty \log_2(1 + \gamma) \cdot \left[\frac{1}{\Gamma_{t,T}} \cdot \exp\left(-\frac{\gamma}{\Gamma_{t,T}}\right) \right] d\gamma \quad (20)$$

considering the probability density function (p.d.f. of the SIR at the output of the Rayleigh fading channel, [4,16], and where the average received SNR value $\Gamma_{t,T}$, from all users, is given by:

$$\Gamma_{t,T} = \frac{K_T \cdot \Gamma}{q + \frac{1}{q} \cdot (0.113 K) \cdot \Gamma} \quad (21)$$

Eqs (17) and (20), are plotted in Figs 2 and 3 respectively, against Γ (expressed in dB) for $K=8$ users per cell. In addition, the following values are assumed: (i) DS-CDMA and TDMA cellular systems with total bandwidth $B_C=B_T=25\text{MHz}$ (ii) signal bandwidth: $B_s=30\text{KHz}$, (iii) frequency separation between the carriers frequencies $B_{sp,T}=200\text{KHz}$ (iv) total number of hopping carrier frequencies assigned to each cell: $q=124$, (v) time-delay spread: $\Delta \approx 3\mu\text{sec}$. Moreover, the integrals in eqs (17) and (20), are calculated numerically since they cannot be expressed in closed form.

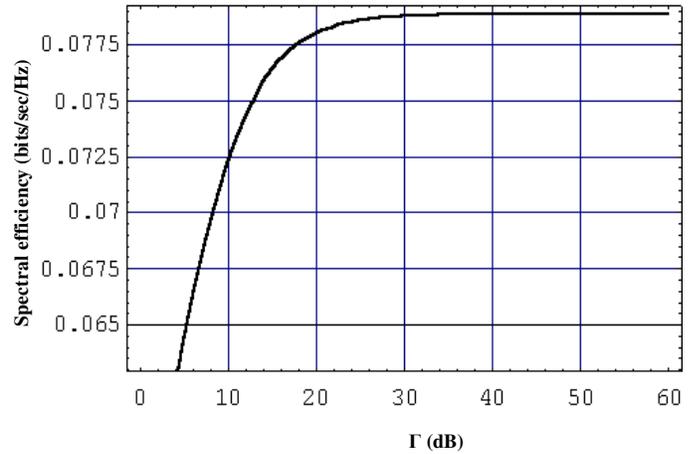


Fig.2. Spectral efficiency of cellular DS-CDMA operating in an urban Rayleigh fading environment [$\Delta \approx 3 \mu\text{sec}$] according to eqs (17).

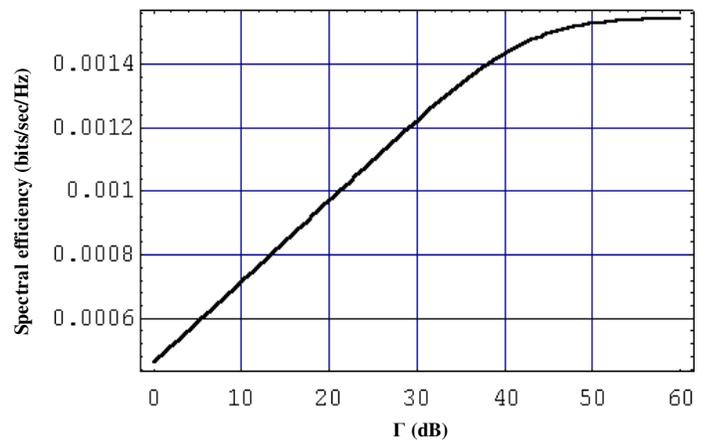


Fig.3. Spectral efficiency of cellular TDMA system operating in an urban Rayleigh fading environment [$\Delta \approx 3 \mu\text{sec}$] according to eqs (20).

As it can readily be seen, for cellular operation in a Rayleigh fading environment and under normalized conditions, DS-CDMA offers higher spectral efficiency than TDMA.

3 Conclusion

The estimation of the spectral efficiency achieved by cellular DS-CDMA and TDMA systems operating in a Rayleigh fading environment is discussed. Spectral efficiency is examined in terms of channel capacity per user considering a static model of operation. As it is shown, under

normalized conditions, the spectral efficiency of a cellular DS-CDMA system, is greater than that of TDMA regardless the fact that, in contrast to a cellular TDMA, in cellular operation of DS-CDMA spectral efficiency is seriously limited by MAI.

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