Blind Equalization Based on Direction Gradient Algorithm under Impulse Noise Environment

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Abstract: - A new constant modulus blind equalization based on direction gradient algorithm was proposed, which can obtain robust convergence performance under impulse noise environment. The impulse noise has no more than two order moments, so constant modulus algorithm (CMA) based on stochastic gradient descent algorithm is often ill-convergence or divergence. The direction gradient algorithm uses the reliability of the output of the blind equalizer to determine the equalizer weights updating strategy, and the reliable region called decision circle is set according to the scatterplot of the send signal. If the output signal drops in the decision circle, the equalizer weights update according to the traditional CMA, otherwise it only keeps the sign of the output error, and the previous step instantaneous gradient is used to update the equalizer weights. The direction gradient algorithm can suppress the impulse noise interference effectively, which shows robust convergence performance under the impulse noise environment. Simulation results show that, compared with the fractional lower order CMA and the nonlinear transform CMA, the blind equalization based on direction gradient algorithm has the fastest convergence rate and the lowest steady state error.

Key-Words: - blind equalization, direction gradient algorithm, impulse noise, constant modulus algorithm, decision circle

1 Introduction

Blind equalization shows many advantages both in the cooperative communication system and the non-cooperative communication system for it does not require any training sequences to implement channel equalization [1][2]. In the communication system, inter symbol interference (ISI) caused by multi-path transmission degrades the quality of the received signal, and adaptive equalization is one of the effective technologies to eliminate the ISI at the receiver [3]. Blind equalization technology can save the channel bandwidth to improve the efficiency of communication [4], and it also can maintain effectiveness under the no training sequence available conditions. CMA is a widely used blind equalization algorithm for its simple and robust. However CMA only considers the ISI caused by multipath transmission and assumes that the channel is noiseless [5]. In the practical communication system, the channel noise also affects the amplitude information of the transmitted signal, and the anti-noise property is one of the key indicators for the performance evaluation of the blind equalization algorithms. White Gaussian noise (WGN) channel is the most common communication channel model. CMA blind equalization especially based on the fractionally-spaced equalizer can obtain robust convergence performance for WGN channel [6]. But in some communication systems, because of multiple user interference, atmospheric noise, discharge of automobile engine, as well as the man-made electromagnetic interference and other natural man-made noise, the wireless channel noise often shows properties of impulse noise rather than Gaussian noise [7]. Lots of experiments show that the alpha stationary distribution process is one of the most suitable models to describe the impulse noise. For impulse noise has no more than two order moments, the traditional CMA is difficult to obtain robust convergence performance under the impulse noise environment. Nonlinear transform CMA [8][9] and fractional lower order CMA [10] can suppress the impulse noise and show better performance than the traditional CMA. However, the fractional lower order CMA obtains the robust convergence at the cost of the slow convergence rate, and the nonlinear transform CMA can only suppress the distinct impulse noise, which results in its robust convergence performance cannot be ensured. According to the stochastic gradient descent algorithm, this paper proposes a CMA blind equalization based on direction gradient algorithm. The short time impulse noise with high amplitude affects the instantaneous gradient for the equalizer weights updating and the output modulus. A decision circle is set according to the constellation of the send signal. If the modulus of the output signal drops in the decision circle, which means that the effect of the impulse noise is lower, and the blind equalizer can update according to the traditional CMA. Otherwise, it means that the impulse noise seriously affects the current instantaneous gradient, and in this case it can only keep the sign of the output error, and the equalizer weights updating based on the previous step instantaneous gradient. Simulation results show that, compared with the fractional lower order CMA and the nonlinear transform CMA, the blind equalization based on direction gradient algorithm has the fastest convergence rate and the lowest steady state error.

2 Problem Formulation

CMA is a special case of the Godard algorithms which belong to Bussgang blind equalization algorithms. The cost function of Bussgang blind equalization algorithms based on the nonlinear transform on the received signal and the nonlinear transform meets the conditions of Bussgang process. The strategy of Bussgang blind equalization algorithms uses the higher-order statistics of the transmitted signal.

2.1 The principle of CMA

CMA is a blind channel equalization algorithm which is particularly suitable for the transmitted signal which has constant modulus. Fig.1 shows the baseband model of CMA blind equalization [11].

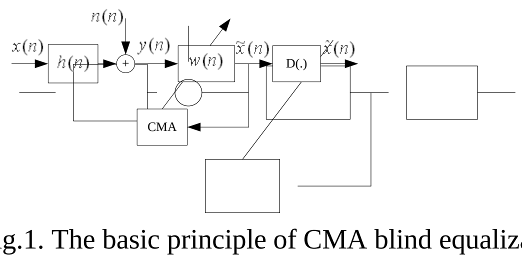

Fig.1. The basic principle of CMA blind equalization

where

$x(n)$ = The send signal.

$h(n)$ = The unknown channel.

$n(n)$ = Additive noise usually assumed to be the White Gaussian noise.

$y(n)$ = The received signal before blind equalizer.

$w(n)$ = The weights of the blind equalizer.

$\tilde{x}(n)$ = The output of the blind equalizer.

$D(.)$ = The decision function.

$\hat{x}(n)$ = The output symbol after symbol detection.

According to the communication signal transmission theory, the signal transmit through the channel is given by

$$y(n) = h(n) \otimes x(n) + n(n) \quad (1)$$

where the symbol ‘ $\otimes$ ’ denotes convolution operation. The objective of blind equalization is that recover the send signal rely solely on the observed signal. The output signal of blind equalizer is given by

$$\tilde{x}(n) = w(n) \otimes h(n) \otimes x(n) + w(n) \otimes n(n) \quad (2)$$

In the research of blind equalization theory and algorithm, the convolution noise $w(n) \otimes n(n)$ is often not being considered, and then the zero forcing condition of blind equalization [12] is given by

$$w(n) \otimes h(n) = \delta(n - D) e^{j\varphi} \quad (3)$$

where D is a constant time delay which doesn’t affect the signal recovery, ϕ is a constant phase shift which can be got rid of by the decision device.

CMA updates the weights of the blind equalizer based on mean square error to meet the zero forcing condition and the cost function is given by [13]

$$J_{CMA}(n) = \frac{1}{2} E\left[(|\tilde{x}(n)|^2 - R_2)^2 \right] \quad (4)$$

where R_2 is the constant modulus is given by

$$R_2 = \frac{E\left[|x(n)|^4 \right]}{E\left[|x(n)|^2 \right]} \quad (5)$$

From the cost function of CMA we can see that CMA uses the higher-order statistics of the observed signal indirectly, and the blind equalizer weights updating often use the stochastic gradient descent algorithm. Based on the stochastic gradient descent algorithm, the blind equalizer weights updating formula is given by

$$w(n+1) = w(n) + \mu \nabla J_{CMA}(n) \quad (6)$$

where μ is the study step which control the convergence rate and the convergence precision. Let the error $e(n)$ is

$$e(n) = R_2 - |\tilde{x}(n)|^2 \quad (7)$$

The blind equalizer weights updating formula can be rewritten as

$$w(n+1) = w(n) + \mu e(n)\tilde{x}(n)y^*(n) \quad (8)$$

2.2 The model of impulse noise

CMA blind equalization updates the equalizer based on the stochastic gradient descent algorithm to meet the zero forcing condition without considering the noise interference. The adding noise $n(n)$ at the communication channel is usually assumed to be the White Gaussian noise. Under the White Gaussian noise environment, CMA can obtain robust convergence performance. In particular, CMA blind equalization can obtain better performance by using the fractionally spaced equalizer which can suppress the White Gaussian noise effectively. But in some communication systems, the channel noise is often shows impulse noise characteristics. The alpha stationary distribution is often used to describe the impulse noise model and it has been applied to many communication systems in noise modeling as a class of heavy tailed distribution. Because there is no closed mathematical expression, the alpha stationary distribution commonly described by characteristic function which is given by [14]

$$\varphi(t) = \begin{cases} \exp\left\{ \begin{aligned} & jat - \gamma |t|^\alpha \Big[1 \\ & + j\beta \,\mathrm{sgn}(t) \tan\left(\frac{\alpha\pi}{2}\right)\Big]\end{aligned} \right\} & \alpha \neq 1 \\ \exp\left\{ \begin{aligned} & jat - \gamma |t|^\alpha \Big[1 \\ & + j\beta \,\mathrm{sgn}(t) \frac{2}{\pi} \lg(t)\Big]\end{aligned} \right\} & \alpha = 1 \end{cases} \quad (9)$$

where $\alpha \in (0, 2]$ is the characteristic exponent which controls the degree of the impulse of the stochastic process. The smaller is α, the stronger impulse will be the noise. The alpha stationary distribution is called the fractional low order stationary distribution (FLOA) if $0 < \alpha < 2$. $\beta \in [-1,1]$ is the symmetry coefficient, and the distribution is the symmetrical distribution if $\beta = 0$. There is a special case that the alpha stationary distribution is the same as Gauss distribution if $\beta = 0$ and $\alpha = 2$. $\gamma > 0$ is the dispersion coefficient which is almost same as the variance of the Gauss distribution. $a \in R$ is the location parameter which expresses the mean or median value of the stationary distribution. Alpha stationary distribution is the symmetry alpha stationary distribution ($S\alpha S$) if $\beta = 0$ and $a = 0$ which has some same properties as Gauss distribution such as slickness, unimodality and bell type, etc. The study also shows that the alpha stationary distribution with the characteristic exponent $1 < \alpha < 2$ can sufficiently describes the impulse noise in the real world. Therefore the channel noise is assumed to be the FLOA- $S\alpha S$ with $1 < \alpha < 2$. The important difference between the FLOA distribution and the Gauss distribution is that the FLOA distribution has no more than α order moments which result in failure of CMA blind equalization based on stochastic gradient descent algorithm.

3 Problem Solution

Blind equalization under the impulse noise environment is one of the difficult problems which hinders the technology apply to the engineering. So far, the more effective blind equalization methods under impulse noise environment include the fractional low order CMA and the nonlinear transform CMA. Fractional low order CMA uses the fractional low order moments information of the observed signal based on the redefined cost function as follow

$$J(p,q) = E\left[(|\tilde{x}(n)|^p - R_2)^q \right] \quad (10)$$

where p/q is a fraction between 0 and α, at the same time, $pq < \alpha$ condition is set to ensure that the cost function Eq.10 is limited. Blind equalizer weights updating can use stochastic gradient descent algorithm based on the cost function Eq.10. The nonlinear transform CMA blind equalization doesn't change the cost function of CMA, which do the nonlinear transform on the received signal to suppress the impulse noise. The point of view of the nonlinear transform CMA is that the impulse noise causes the amplitude distortion of the signal in the transmission process, and the nonlinear transform can suppress the impulse noise by nonlinear filtering with soft limiting. The nonlinear transform function is given by

$$x_f(n) = 2/(1 + \exp(-2\tilde{x}(n))) - 1 \quad (11)$$

After the nonlinear transform, the iterative error and the gradient for blind equalizer weights updating can be calculated according to $x_f(n)$.

Although the fractional low order CMA blind equalization can ensure the robust convergence, it suppresses the impulse noise at the cost of the loss of the high-order statistical information of the received signal at the same time, which results in the slow convergence rate. The nonlinear transform CMA blind equalization uses the soft limiting on the observed signal to obtain the partly impulse noise suppression result, while the impulse cannot be suppressed thoroughly. Therefore, the nonlinear transform CMA often shows instability in some cases, furthermore, its convergence rate is the same as CMA.

This paper proposes a CMA blind equalization based on direction gradient algorithm according to the implement process of the stochastic gradient descent algorithm. The blind equalizer updating gradient according to CMA cost function is given by

$$\frac{\partial J_{CMA}(n)}{\partial w(n)} = 2\left[\left|\tilde{x}(n)\right|^2 - R_2\right]\tilde{x}(n)y^*(n) \quad (12)$$

If the convolutional noise $w(n)\otimes n(n)$ is not ignored, the modulus of the output of the blind equalizer will be affected. For the short time impulse noise interference, the constellation of the output signal will be distortion. Assuming that the blind equalization algorithm can obtain robust convergence, the relationship between $w(n-1)$ and $w(n)$ is given by

$$\lim_{n\to\infty}[w(n) - w(n-1)] = 0 \quad (13)$$

Eq.13 shows that the convolutional noise affects the modulus of the output of the blind equalizer only depend on the impulse noise at the current iterative time. That is the modulus of $\tilde{x}(n)$ is mainly affected by the current impulse noise $n(n)$ which can be illustrated as follow

$$\begin{aligned}\Delta x(n) = [\hat{C}(n) - C(n)] \otimes x(n) \\ + w(n)\otimes n(n)\end{aligned} \quad (14)$$

where $\hat{C}(n)$ is the union impulse response of the equalizer $w(n)$ and the channel $h(n)$ after perfect equalization, and $C(n)$ is the union impulse response of the equalizer $w(n)$ and the channel $h(n)$ at current iterative time. If $\hat{C}(n)\to C(n)$ with the blind equalizer updating process, the modulus of convolutional noise $w(n)\otimes n(n)$ is the mainly factor affects the modulus of the output of the blind equalizer. Therefore, a decision circle is set according to the constellation of the send signal. If the output of the blind equalizer drops in the decision circle, it can be consider that the effect of the impulse noise interference at current iterative time is small. Otherwise, it considers that the effect of the impulse noise interference is very serious. Fig.1 shows the decision circle diagram, where r_d is the radius of the decision circle, R_d is the modulus of the constellation point of the send signal and r_x is the modulus of the output of the blind equalizer.

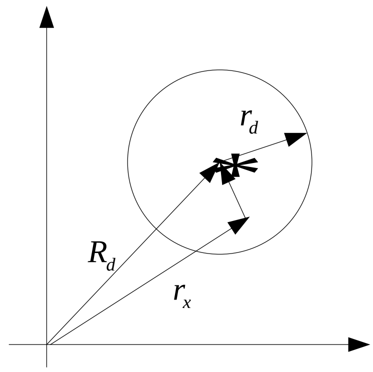

Fig.2 The decision circle diagram

Based on this idea, the CMA blind equalization based on direction gradient algorithm is proposed. The impulse noise at the current iterative time affects the modulus of the output of the blind equalizer which reflect in the error $e(n)$, and according to Eq.13, the equalizer weights difference between the adjacent iterative times is very small. Therefore, if the impulse noise affects the output seriously which judgment according to the decision circle, the blind equalizer weights can be updated by the previous instantaneous gradient and only keep the sign of the error $e(n)$. The updating method of the CMA blind equalization based on direction gradient algorithm is given by

$$\begin{cases} w(n+1) = w(n) \\ \quad +\mu e(n)\tilde{x}(n)y^*(n) & if\ \left|R_d - r_x\right| < r_d \\ w(n+1) = w(n) + \mu \\ \times sign(e(n))\tilde{x}(n-1)y^*(n-1) & if\ \left|R_d - r_x\right| \geq r_d \end{cases} \quad (15)$$

The CMA blind equalization based on direction gradient algorithm can suppress the impulse noise effectively and ensure the algorithm convergence to the correct direction to obtain robust convergence performance.

4 Simulations and Analysis

In the simulations, the send signal is the binary probability random sequence using QPSK modulation. The channel modes adopt the typical telephone channel and the mixed phase channel. The equivalent baseband impulse response of the typical telephone channel is given by

$$\begin{aligned}h_T = [0.005,0.009,\text{-}0.024,0.854, \\ \text{-}0.218,0.049,\text{-}0.016]\end{aligned} \quad (16)$$

The equivalent baseband impulse response of the mixed phase channel is given by

$$h_M = [0.3132,0.1040,0.8908,0.3143] \quad (17)$$

Because the α-stationary distribution with the characteristic exponent α has no statistical moments above α order, the signal-to-noise (SNR) defined by the two order statistics cannot describe the degree of the impulse noise interference. Therefore, the generalized SNR (GSNR) which can measure the impulse noise in the signal is defined as

$$GSNR = \frac{10\lg|x(n)|^2}{\gamma} \tag{18}$$

where γ is the dispersion coefficient of the α-stationary distribution impulse noise and $|x(n)|^2$ is the signal energy. The performance is evaluated according to the residual inter symbol interference (*ISI*) which is defined as [15]

$$ISI = \frac{\sum_i |C_i|^2 - |C_i|^2_{\max}}{|C_i|^2_{\max}} \tag{19}$$

where C is the union impulse response of the channel and the blind equalizer.

To verify the effectiveness of the CMA blind equalization based on direction gradient algorithm (DSGD-CMA), the CMA blind equalization based on stochastic gradient descent algorithm (SGD-CMA), fractional low order CMA (RSGD-CMA) blind equalization and the nonlinear transform CMA (NSGD-CMA) blind equalization are done in the simulations for comparison. The blind equalizer length is 28 and 20 in typical telephone channel and mixed phase channel simulations respectively. The step size is set to $\mu = 0.002$ in both simulations. Fig.3 and Fig.4 show the ISI curve with $GSNR = 20$ dB by 500 times Monte Carlo simulations.

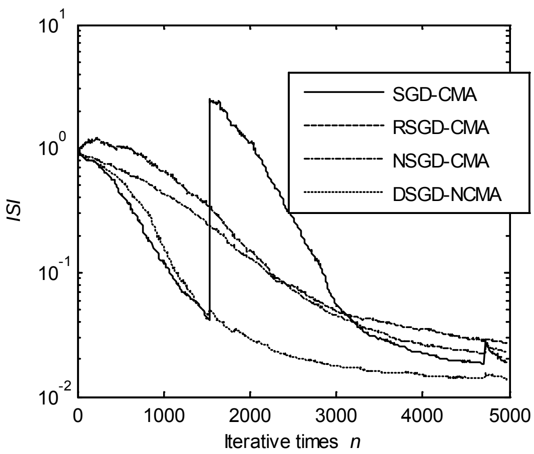

Fig.3 ISI in typical telephone channel

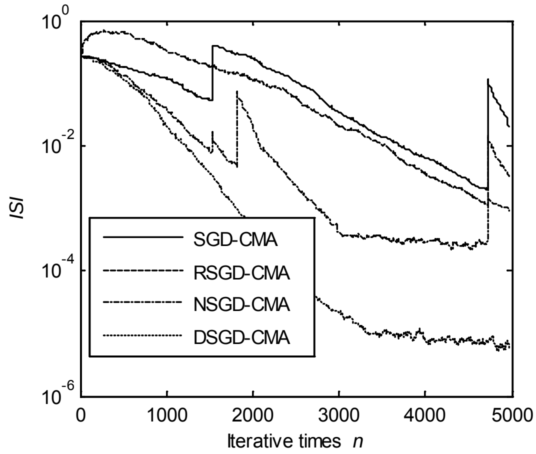

Fig.4 ISI in mixed phase channel

From Fig.3 and Fig.4 can see that the SGD-CMA cannot ensure the robust convergence due to the impulse noise interference and the ISI curve shows large jitter during the convergence process. Although RSGD-CMA can obtain robust convergence performance, its convergence rate is very slow and the steady residual error is biggest among the four algorithms. NSGD-CMA only consider the short time distinct impulse noise interference in the received signal, although the convergence rate and stability has been certain improved, the ISI still has jitter during the convergence process. DSGD-CMA proposed in this paper can obtain robust convergence performance. Meanwhile, it has fastest convergence rate and lowest steady state residual error among the four algorithms.

To further verify the performance of DSGD-CMA, the bit error rate (BER) of the four algorithms is counted by 500 time's Monte Carlo simulations under different *GSNR* conditions. Fig.5 and Fig.6 show the BER in the simulations under the two type channels respectively.

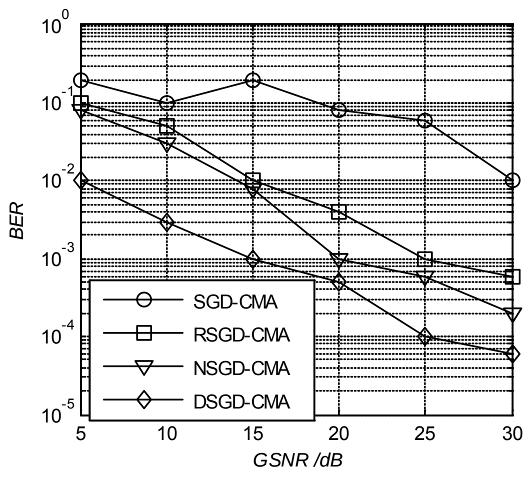

Fig.5 *BER* in typical telephone channel

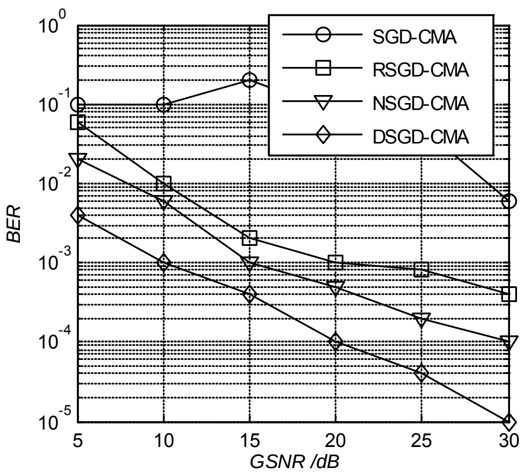

Fig.6 *BER* in mixed phase channel

From Fig.5 and Fig.6 can see that DSGD-CMA proposed in this paper has the lowest BER among the four algorithms under different *GSNR* conditions for the two type channels, which proves the effectiveness of DSGD-CMA blind equalization.

4 Conclusions

This paper proposed a blind equalization based on direction gradient algorithm to solve the problem of blind equalization under impulse noise environment. The decision circle is set to judge the degree of the impulse noise interference on the output signal, based on which to design a direction gradient algorithm to update the blind equalizer weights. Simulation results under the typical telephone channel and the mixed phase channel show the effectiveness of the DSGD-CMA.

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References:

[1] Di XueJing, Tong Cheng, Zhang Xia et al., Adaptive step-size constant-modulus algorithm for high-speed optical coherent communication system, *Acta Optical Sinica*, Vol.32, No.10, 2012, pp. 1006004.

[2] M. Pinchas, B.Z. Bobrovsky, A novel HOS approach for blind channel equalization, *IEEE Transaction on Wireless Communications*, Vol.6, No.3, 2007, pp. 875-886.

[3] Banani S.A., Vaughan R.G., Blind channel estimation and discrete speed tracking in wireless systems using independent component analysis with particle filtering, *IET Communications*, Vol.6, No.2, 2012, pp. 224-234.

[4] Ning Xiaoling, Liu Zhong, Luo Yasong, et al., Improved super-exponential iteration blind equalization algorithm for carrier phase recovery in underwater acoustic channels, *Xi'an Dianzi Keji Daxue Xuebao/Journal of Xidian University*, Vol.39, No.1, 2012, pp. 151-156.

[5] Niroomand M., Derakhtian M., Masnadi-Shirazi M.A., Steady-state performance analysis of a generalised multimodulus adaptive blind equalisation based on the pseudo Newton algorithm, *IET Signal Processing*, Vol.6, No.1, 2012, pp. 14-26.

[6] Fijalkow I., Fractionally spaced equalization using CMA: robust-ness to channel noise lack of disparity, *IEEE Transaction on Signal Processing*, Vol.45, 1997, pp. 56-66.

[7] Liu Jianqing, Feng Dazheng, Blind sources separation in impulse noise, *Journal of Electronics and Information Technology*, Vol.25, No.7, 2003, pp. 896-900.

[8] Zhang Yinbing, Zhao Junwei, Guo Yecai, et al., Adapative error-constrained constant modulus algorithm for blind equalization to make it suitable in α -stable noise, *Journal of Northwestern Polytechnical University*, Vol.28, No.2, 2010, pp. 202-206.

[9] Li Jinming, Zhao Junwei, Guo Yecai, et al., A novel robust constant modulus blind equalization algorithm in heavy-tailed noise environment, *Applied Acoustics*, Vol.29, No.1, 2010, pp. 17-22.

[10] Tsakalides P, C. L. Nikias, Maximum likelihood localization of sources in noise modeled as a stable process, *IEEE Transaction on signal processing*, Vol.43, No.11, 1995, pp. 2700-2713.

[11] Xie Ning, Hu Hengyun, Wang Hui, A new hybrid blind equalization algorithm with steady-state performance analysis, *Digital Signal Processing: A Review Journal*, Vol.22, No.2, 2012, pp. 233-237.

[12] Hwang Kyuho, Choi Sooyong, Blind equalizer for constant-modulus signals based on Gaussian process regression, *Signal Processing*, Vol.92, No.6, 2012, pp. 1397-1403.

[13] Ozen A., Kaya I., Soysal B., A supervised constant modulus algorithm for blind equalization, *Wireless Personal Communications*, Vol.62, No.1, 2012, pp. 151-166.

[14] Guo Ying, Qiu Tianshuang, Tang Hong, et al., Constant modulus algorithm for blind equalization under impulsive noise environments. *Journal on Communications*, Vol.30, No.4, 2009, pp. 35-40.

[15] Mitra R., Singh S., Mishra A., Improved multi-stage clustering-based blind equalization. *IET Communications*, Vol.5, No.9, 2011, pp. 1255-1261.