A Systematic Design of High-Rate Full-Diversity Space-Frequency Codes for Multiuser MIMO-OFDM System

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Abstract: - The authors have proposed a new space-frequency (SF) code for MIMO-OFDM system in their paper. The data streams of all the users are sent simultaneously through all the OFDM sub-channels. The proposed SF code can achieve high symbol rate (rate A_t) with full diversity ($A_t A_r L$) where A_t denotes the number of transmit antenna for each user, A_r denotes number of receive antenna at the receiver and L denotes the number of independent channel taps. The threaded algebraic layering concept is used to construct this SF code which combines the space-frequency layering theory with algebraic component codes. The component code is considered to be an algebraic number theoretic constellation. Each component code is assigned to a "thread" and interleaved over space and frequency. Diophantine approximation theory is then used to make the threads transparent to each other. In addition, another approximation is used so that the users become transparent to each other. Our SF code does not require zero-padding, which always ensures high symbol rate. It is proved through simulation that the proposed coding scheme achieves higher coding and diversity gain over recently proposed space-frequency code.

Key-Words: - MIMO-OFDM systems, wideband multipath fading channels, multiple access channel (MAC), multiuser space-frequency (SF) coding, and threaded algebraic layering concept.

1 Introduction

MIMO-OFDM [1, 3-5] is one of the key focusing issues of future communication systems. Naturally this topic has drawn lots of thought among the researchers, as a result of this interest, a large number of space-time (ST) or space-frequency (SF) coding have been designed for MIMO-OFDM system [6]-[13] to exploit the spatial diversity gain, most of the codes are designed for single user system. Past work focuses mostly on employing single-user space-time codes for each of the users and separating the users in signal space or canceling multiuser interference. These approaches lead, however, to reduce transmission rate or suboptimum performance significantly, if the number of users is high. A systematic study of the general problem of space time/ frequency code design for MACs seems to be missing. Gartner and Bolcskei considered the issue and designed a multi-user space-time/frequency code [14] that was the extension of Gallager's idea in [15], which is based on the dominant error mechanisms in two-user Additive White Gaussian noise (AWGN) multiple access channel (MAC) but didn't provide a systematic code design. Recently, Zang and Letaief in [1] introduced a systematic design of full diversity multi-user space-frequency code. However, the symbol or code rate of their proposed design is 1. However, increasing the code rate is always challenging because it sometimes deteriorates the performance of the code. To achieve high speed wireless communication (telemedicine, e-governance etc) high rate data transmission is necessary. Though some attentions have been given in high rate full diversity code design, but it was not designed for multi-user MIMO-OFDM system. Therefore, to achieve high symbol rate for multiuser MIMO-OFDM system, we have proposed a systematic design of high-rate, full-diversity multi-user SF codes for MIMO frequency-selective fading MAC in our paper.

Our multi-user SF codes are constructed by exploiting the space-frequency layering concept with algebraic component code, where the component code is considered to be a algebraic number theoretic constellation. Each component code is assigned to a thread in the space-frequency matrix that provide full access to the channel frequency and spatial diversity in the absence of other threads. Diophantine approximation theory [2] is then used to make the threads transparent to each other. Further another Diophantine approximation is used to make the users transparent to each other. The numbers are referred to as "Diophantine numbers" because they are chosen in such a way that their simultaneous Diophantine approximation by algebraic number is "bad", that makes the threads and the users become transparent to each other. The authors assumed that the channel statistic is known at the transmitter and the instantaneous channel state information (CSI) at the receiver with maximum likelihood (ML) detection using the sphere decoder. The proposed multi-user SF codes for MIMO frequency-selective fading MAC achieve high rate and full-diversity for each user without deteriorating the performance. It is worth noting that the proposed coding scheme does not need the instantaneous channel side information at transmitter nor the cooperation of multiple transmitters.

This paper is organized as follows. A system model of the multi-user MIMO-OFDM system is discussed in section II. In section III, the code design criteria for multiple accesses channel (MAC) for multi-user SF codes are given. The proposed systematic design of multi-user high rate full diversity SF codes is explained with a proof in section IV. Section V provides an example to verify our proposed code design. Simulation result and discussion are given in section VI. Finally, we draw our conclusion in section VII.

The following notations are used throughout this paper: Z, Q and C stands for the integer ring, the rational number field, and complex number field respectively. $Q(j)$ represents the field generated by j and rational where j= $\sqrt{-1}$. $Z[j]$ denotes the field generated by the j and integer ring. $\lceil x \rceil$ represents the smallest integer larger than x. $\lfloor x \rfloor$ represents the largest integer smaller then x. The subscripts T and H denotes the transpose and Hermitian of a complex matrix respectively. $\otimes$ and $\circ$ denotes the Kronecker and Hadamard product respectively.

2 System Model

In a single-cell network, the MIMO-OFDM system shown in Fig.1 has total number of Z users, where each user is equipped with A_t transmit antennas, a base station (BS) with A_t receive antennas and N-tone OFDM. The complex channel between the n–th ($n = 1, \cdots, A_t$) transmit antenna of user z (z=1,…,Z), and and m-th (m=1,…, A_r) receive antenna of BS is denoted by the following equation:

$$h_{m,n}^{(z)} = \left[h_{m,n}^{(z)}(0), \ldots, h_{m,n}^{(z)}(L-1) \right], \qquad (1)$$

where, $h_{m,n}^{(z)}(a)$ are independent of any (m, n, a), where, m=1,…, A_r; $n = 1, \cdots, A_t$; and a = 0, 1, 2 ,…,L-1.

The system has been developed for the coherent scenario where the channel state information (CSI) $h_{m,n}(a)$ is perfectly known at the receiver but unknown at the transmitter. The statistics of the channel $h_{m,n}^{(z)}(t)$ are given by complex random variables with zero-mean and variance δ_a^2 , for , a = 0, 1, 2 ,…,L-1. It is assumed that the channel statistic (τ_a) is known at the transmitter. Furthermore, it is assumed that all path gains between any pair of transmit and receive antennas during any fading block follow the same power profile given by,

$$E\left[\left| h_{m,n}(a) \right|^2 \right] = \delta_a^2 > 0. \qquad (2)$$

Following the water-filling condition of the optimization problem, the power for L number of paths are normalized such that $\sum_{a=0}^{L-1} \delta_a^2 = 1$. The above assumptions regarding the multipath fading channel model have also been used in other works such as [5], [6], [8], [9], [11-13].

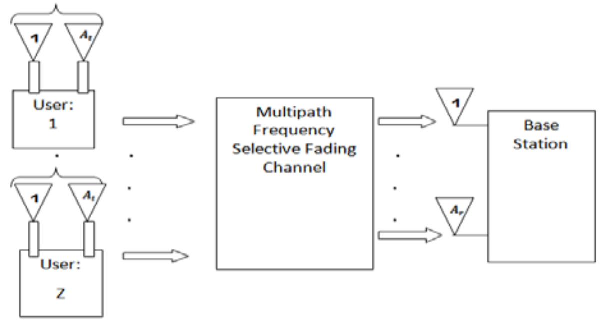

Fig-1: Multiuser MIMO OFDM System for SF code

Let, $H_{m,n}^{(z)}$ denotes the frequency response of the channel from the *n*-th transmit antenna of user z to the *m*-th receive antenna of BS during a single fading block. It can be given by the Discrete Fourier Transform (DFT) of the channel impulse response as follows,

$$H_{m,n}^{(z)} = Fh_{m,n}^{(z)} , \quad (3)$$

where,

$$H_{m,n}^{(z)} = \left[H_{m,n}^{(z)}(0) H_{m,n}^{(z)}(1), \dots, H_{m,n}^{(z)}(N-1) \right]^T , \quad (4)$$

and,

$$F = \left[f^{\tau_0}, f^{\tau_1}, \dots, f^{\tau_{L-1}} \right] . \quad (5)$$

Here, the column vector f is given by, $f = \left[1, \zeta, \dots, \zeta^{N-1}\right]^T$ and $\zeta = e^{\frac{-j2\pi}{T_s}}$, where T_s denotes the duration of one OFDM symbol.

The source generates a block of N_s information symbols from the discrete alphabet $\mathcal{T}$, which are quadrature amplitude modulation (QAM) normalized into the unit power. Using a mapping the information symbol vector $S \in \mathcal{T}^{N_s}$ is encoded into $N \times A_t$ code matrix. $C^{(z)} \in \mathcal{T}^{N \times A_t}$ .The symbol rate or code rate per channel use of the code matrix $C^{(z)}$ is given by, $\mathcal{R} = \frac{N_s}{N}$.

Let, each user has N-tone OFDM. For user z, the codeword $C^{(z)}$ can be written as [4],

$$C^{(z)} = \left[X_1^{(z)} \dots X_{A_t}^{(z)} \right], \quad (6)$$

where the OFDM symbol $X_n^{(z)}$ is assumed to be transmitted from the n^{th} transmit antenna.

Let, two codewords, $C^{(z)}$ and $C^{(z')}$ of two different users z and z' are independent. Assuming the perfect synchronization is available at the receiver, the received signal at the BS can be given by,

$$Y_m = \sum_{n=1}^{A_t} \sum_{z=1}^{Z} diag\left(X_n^{(z)} \right) H_{m,n}^{(z)} + n_m \quad , \quad (7)$$

where, $Y_m = \left[Y_m^{(z)}(0) Y_m^{(z)}(1) \dots Y_m^{(z)}(N-1) \right]^T$, and $n_m \epsilon \mathcal{T}^N$ is Additive White Gaussian Noise with zero mean and covariance $N_0 \mathrm{I}_N$ and $Y_m \in \mathcal{T}^N$ is the received signal at the m^{th} antenna of BS.

Using (3) and (5) in (7) we get,

$$Y_m = \sum_{n=1}^{A_t} \sum_{z=1}^{Z} diag\left(X_n^{(z)} \right) Fh_{m,n}^{(z)} + n_m \quad (8)$$

$$= \sum_{n=1}^{A_t} \sum_{z=1}^{Z} diag\left(X_n^{(z)} \right) \sum_{a=0}^{L-1} f^{\tau_l} h_{m,n}^{(z)}(a) + n_m \ . \quad (9)$$

Let,

$$E_a = diag(f^{\tau_a}) , \quad (10)$$

and

$$E_a X_n^{(z)} = diag\left(X_n^{(z)} \right) f^{\tau_a} \ . \quad (11)$$

Substituting (11) in (9), we get,

$$Y_m = \sum_{n=1}^{A_t} \sum_{z=1}^{Z} \sum_{a=0}^{L-1} E_a X_n^{(z)} h_{m,n}^{(z)}(a) + n_m . \quad (12)$$

Let,

$$h_m^{(z)}(a) = \left[h_{m,1}^{(z)}(a) \dots h_{m,A_t}^{(z)}(a) \right]^T . \quad (13)$$

Using (6), from (12) we get,

$$Y_m = \sum_{z=1}^{Z} \sum_{a=0}^{L-1} E_a C^{(z)} h_m^{(z)}(a) + n_m . \quad (14)$$

Let,

$$P^{(z)} = \left[E_0 C^{(z)} \dots E_{L-1} C^{(z)} \right]. \quad (15)$$

and,

$$h_m^{(z)} = \left[h_m^{(z)}(0) \dots h_m^{(z)}(L-1) \right]^T \ . \quad (16)$$

Then, from (14) we get,

$$Y_m = \sum_{z=1}^{Z} p^{(z)} h_m^{(z)} + n_m . \quad (17)$$

Let,

$$P = \left[p^{(1)} \dots p^{(Z)} \right] \quad (18)$$

and,

$$h_m = \left[h_m^{(1)} \dots h_m^{(Z)} \right]^T , \quad (19)$$

Then we have,

$$Y_m = Ph_m + n_m . \quad (20)$$

Moreover, using the following notations:

$$Y = \left[Y_1^T \dots Y_{A_r}^T \right]^T, \quad (21)$$

$$h = \left[h_1^T \dots h_{A_r}^T \right]^T, \quad (22)$$

$$n = \left[n_1^T \dots n_{A_r}^T \right]^T, \quad (23)$$

and $$X = \mathrm{I}_{A_r} \otimes P \ . \quad (24)$$

We get, $$Y = \sqrt{\frac{\rho}{A_t}} Xh + n \ . \quad (25)$$

The term $\sqrt{\frac{\rho}{A_t}}$ ensures that the SNR at each receive antenna ρ, independently of the transmitter A_t.

3 SF Code design criterion

In this section, we show the criteria for SF code design which is based on the error event analysis and it was first discussed by Gallager depending on the desired transmission rate tuple [15]. Different error regions are defined by the transmission rate of users. For the rate region where user-1 or user-2 is in error, the well known single user ST/SF codes are sufficient. But, for the rate region where both users are in error, a joint code design is necessary for optimization, i.e.:- to get full-diversity for any error event. Moreover, the code design criteria were made based on high-SNR and low-SNR region. As, this effect is less pronounced for low SNR, our code design criteria for SF code is based on the high-SNR region.

For a single user codeword i.e. $C^{(z)}$, the pair wise error probability (PEP) is given by $P\left(C^{(z)} \to \hat{C}^{(z)}\right)$ where $C^{(z)} \neq \hat{C}^{(z)}$. Let, $\breve{C}^{(z)} = C^{(z)} - \hat{C}^{(z)}$. Denoting the number of OFDM tones as N, for a given channel realization $H\left(e^{j\frac{2\pi}{N}k}\right)$ on the kth tone (for, $k = 0, 1, \cdots, N-1$), the probability that the receiver decides erroneously in favor of $\hat{C}^{(z)}$ is given by [16] ,

$$P\left(C^{(z)} \to \hat{C}^{(z)} | H\left(e^{j\frac{2\pi}{N}k}\right)\right) = Q\sqrt{\frac{E_s}{2\sigma_n^2} d^2\left(C^{(z)}, \hat{C}^{(z)} | H\left(e^{j\frac{2\pi}{N}k}\right)\right)} \ , \quad (26)$$

By taking , $\bar{H} = H\left(e^{j\frac{2\pi}{N}k}\right)$,we can rewrite (26) as,

$$P\left(C^{(z)} \to \hat{C}^{(z)} | \bar{H}\right) = Q\sqrt{\frac{E_s}{2\sigma_n^2} d^2\left(C^{(z)}, \hat{C}^{(z)} | \bar{H}\right)} \ , (27)$$

where, the squared Euclidian distance between the codeword $C^{(z)}$ and $\hat{C}^{(z)}$ noted by,

$$d^2\left(C^{(z)}, \hat{C}^{(z)} | \bar{H}\right) = \left\| \left(I_{A_r} \otimes \breve{C}^{(z)} \right) \bar{H} \right\|^2, \quad (28)$$

Using Chernoff bound $Q(x) \leq e^{\frac{-x^2}{2}}$ and denoting, $\rho = \frac{E_s}{\sigma_n^2}$ as SNR, (28) can be uper bounded as,

$$P\left(C^{(z)} \to \hat{C}^{(z)} | \bar{\mathrm{H}}\right) \leq e^{\frac{-\rho}{4}\left\| \left(I_{A_r} \otimes \breve{C}^{(z)} \right) \bar{H} \right\|^2}, \quad (29)$$

since every Eigen value of the $A_t \times A_t$ matrix $\left(C^{(z)} - \hat{C}^{(z)}\right)\left(C^{(z)} - \hat{C}^{(z)}\right)^T$ is an Eigen value of $A_t A_r \times A_t A_r$ matrix, $\left[\left(C^{(z)} - \hat{C}^{(z)}\right)\left(C^{(z)} - \hat{C}^{(z)}\right)^T \right] \times I_{A_r}$ (with multiplicity A_r).

Let, $R^{(z)} = I_{A_r} \otimes \left(\breve{C}^{(z)^H} \breve{C}^{(z)} \right)$ and then using singular value decomposition, we get, $R^{(z)} = U^H \Lambda U$, where, $\Lambda = \mathrm{diag}\left(\lambda_1, \dots, \lambda_{\mathrm{W_L W_t A_r}}\right)$
with, $\lambda_{\mathrm{f}} \geq 0$ for $\mathrm{f} = 1, 2, \dots, \mathrm{w_L w_t A_r}$, where, $\mathrm{w_L} = 2^{\lceil \log_2 \mathrm{L} \rceil}, \mathrm{W_t} = 2^{\lceil \log_2 \mathrm{A_t} \rceil}$.
Therefore,

$$\left\| \left(I_{A_r} \otimes \breve{C}^{(z)} \right) \bar{H} \right\|^2 = \bar{H}^H U^H \Lambda \mathrm{U} \bar{H} \quad (30)$$

Let, $\bar{\gamma} = \mathrm{U}\bar{H}$ and $\bar{\gamma}_{\mathrm{f}}$ denotes the f^{th} element of the vector $\bar{\gamma}$ then,

$$\left\| \left(I_{A_r} \otimes \breve{C}^{(z)} \right) \bar{H} \right\|^2 = \sum_{f=1}^{W_L W_t A_r} \lambda_f \left| \gamma_f \right|^2 \quad (31)$$

Substituting (31) into (29) we get,

$$P\left(C^{(z)} \to \hat{C}^{(z)} | \bar{H}\right) \leq e^{-\left(\frac{\rho}{4} \sum_{f=1}^{W_L W_t A_r} \lambda_f |\gamma_f|^2\right)}, \quad (32)$$

Taking the expectation of (32), we get,

$$P\left(C^{(z)} \to \hat{C}^{(z)}\right) \leq \prod_{f=1}^{r} \left(\frac{1}{1 + \rho \frac{\lambda_f}{4}} \right) \quad (33)$$

where r is the rank of the matrix $R^{(z)}$. Our SF code design criterion is based on high-SNR. So, for high-SNR $\rho \ggg 1$, (33) reduced to,

$$P\left(C^{(z)} \rightarrow \hat{C}^{(z)}\right) \leq \left(\frac{\rho}{4}\right)^{-r_z} \prod_{f=1}^{r_z} \frac{1}{\lambda_f} \ , \qquad (34)$$

Simplifying the equation, we get,

$$P\left(C^{(z)} \rightarrow \hat{C}^{(z)}\right) \leq \rho^{-r_z} \prod_{f=1}^{r_z} \frac{1}{\lambda_f}. \qquad (35)$$

For 2-user system, let, the transmitted codeword denoted by, $C = \left[C^{(1)} C^{(2)}\right]$ and $\hat{C}$ denotes the detected codeword. It is assumed three types of error event as type-1, type-2, type-3, where type-1 and type-2 represents the error events when user-1 or user-2 is in error, and type-3 represents the error events when more than one user are in error [14]. Thus for given channel realization $\bar{H}$, the total average pair wise error probability is given by,

$$P_e = P_{e1} + P_{e2} + P_{e3} \ , \qquad (36)$$

where, $P_{eq} = E\left[P_{eq|\bar{H}}\right]$ and $P_{eq|\bar{H}}$, for q=1, 2, 3.

The components in (36) depict all error events when the pair wise error $C \neq \hat{C}$ occur. The first and second term denotes the probability of the error event that only the first or second users are in error. The last term represent the probability of the error events that both users are in error.

Now, when user-1 is in error P_{e1} is given by,

$$P_{e1} \leq \left(\rho^{-d_1}\right)\left(\prod_{f=1}^{d_1} \frac{1}{\lambda_f}\right)\left(1-\beta_1\right)\beta_2 \ . \qquad (37)$$

where, the inequality is obtained from the PEP of a single user codeword given in (35) with d_1 being the diversity gain, and $\prod_{f=1}^{d_1} \frac{1}{\lambda_f}$ being the coding gain.

Specifically, for z=1, 2, $\beta_z = P\left(C^{(z)} = \hat{C}^{(z)}\right)$, (38)

and,

$$d_1 = A_r \cdot \min_{\forall C^{(1)} \neq \hat{C}^{(1)}} rank\left(\breve{C}^{(1)}\right). \qquad (39)$$

Similarly, when user-2 is in error P_{e2} is given by,

$$P_{e2} \leq \left(\rho^{-d_2}\right)\left(\prod_{f=1}^{d_2} \frac{1}{\lambda_f}\right)\left(1-\beta_2\right)\beta_1 \ , \qquad (40)$$

where $d_2 = A_r \cdot \min_{\forall C^{(2)} \neq \hat{C}^{(2)}} rank\left(\breve{C}^{(2)}\right)$.

Furthermore, when both users are in error, P_{e3} is given by,

$$P_{e3} \leq \left(\rho^{-d_3}\right)\left(\prod_{f=1}^{d_3} \frac{1}{\lambda_f}\right)\left(1-\beta_1\right)\left(1-\beta_2\right) \ , \qquad (41)$$

where, $d_3 = A_r \cdot \min_{\forall C^{(1)} \neq \hat{C}^{(1)}, C^{(2)} \neq \hat{C}^{(2)}} rank\left(\breve{C}^{(z)}\right)$.

The single user SF codes are not optimal for multi-user MIMO-OFDM system because the optimal code should achieve full diversity for all of the cases given in (37), (40) and (41) [14]. Thus for MAC, a joint code design is necessary.

For a general case of Z users, let, the code word is $C = \left[C^{(1)} C^{(2)} \ldots C^{(z)}\right]$ and $\hat{C}$ denotes the detected code word. Assume $\breve{C} = C - \hat{C}$ and $\breve{C}^{(z)} = C^{(z)} - \hat{C}^{(z)}$. Then, the probability of symbol error can be uper bounded as [1],

$$P_e \leq \sum_{|D|=1}\left(\rho^{-r_1}\right)\left(\prod_{f=1}^{r_1} \frac{1}{\lambda_f}\right)\left(P\left(|D|=1\right)\right) + \ldots + \sum_{|D|=Z}\left(\rho^{-r_Z}\right)\left(\prod_{f=1}^{r_Z} \frac{1}{\lambda_f}\right)\left(P\left(|D|=Z\right)\right). \qquad (42)$$

Here, $\mathrm{D} = \left\{z \middle| C^{(z)} \neq \hat{C}^{(z)}\right\}$ where $|D| \geq 1$ for $C \neq \hat{C}$. And $r_w\left(w = 1, \ldots, Z\right)$ is the rank of the matrix $R = I_{A_r} \otimes \left(\breve{C}^H \breve{C}\right)$ where only w out of Z users have $\breve{C}^{(z)} \neq 0$. The probability that only w out of Z users have $C^{(z)} \neq \hat{C}^{(z)}$ is denoted by $P\left(|D| = w\right)$.

Now, P_{eq} is dominated by the codeword difference matrices with minimum rank [14]. As r_w denotes the diversity order of the error event when w out of Z users are in error, the code design for MAC should guarantee the full-diversity for every error or event. Thus in this coherent scenario, the code design criteria of full diversity SF codes for MAC over MIMO frequency-selective block-fading channels as follows:

3.1. *Rank criterion:*

Maximize the transmit diversity gain over all pairs of distinct codeword C and $\hat{C}$,

$$r_w = A_r . rank\left(C - \widehat{C}\right) \ , \qquad (43)$$

when, only w out of Z users have $C^{(z)} \neq \hat{C}^{(z)}$ for w=1,…, Z.

3.2. *Block fading product criterion:*

Maximize the coding gain over all pairs of distinct code words C and $\hat{C}$;

$$C_w = \prod_{f=1}^{r_w} \lambda_f \,, \tag{44}$$

when, only w out of Z users have $C^{(z)} \neq \hat{C}^{(z)}$ For w=1,…, Z.

4 Multiuser Space-frequency code design

A systematic design of multiuser SF codes is proposed in this section. Our SF code has been constructed by applying threaded algebraic layering concept which was first used in the design of threaded algebraic space-time (TAST code) [17]. Utilizing the duality, the space-frequency coding in frequency-selective channel is constructed from space-time coding in time-selective channel. The proposed design of full diversity and high rate (rate-$\mathbf{A_t}$) SF codes has been shown in fig-2 and fig-3.

Let,
$$Q = W_Z W_L W_t \,, \tag{45}$$
where, $W_L = 2^{\lceil \log_2 L \rceil}, W_t = 2^{\lceil \log_2 A_t \rceil}, W_Z = 2^{\lceil \log_2 Z \rceil}$.

A block of NA_t information symbols $S^{(z)} = [S_1^{(z)} S_2^{(z)} \ldots S_{BQ}^{(z)}]^T$, are evenly split into $B = \frac{N}{Q}$ sub-blocks,

$$S^{(z)} = \left[\left(S_1^{(z)}\right)^T \left(S_2^{(z)}\right)^T \ldots \left(S_B^{(z)}\right)^T \right]^T, \tag{46}$$

where, $S^{(z)} \in \mathcal{T}^{NA_t}$ and $S^{(z)}$ is a QAM constellation which is normalized into the unit power.

Each sub block $S_b^{(z)} \in \mathcal{T}^{W_Z A_t W_L W_t}, b = 1, 2, \ldots, B$ is composed of the signal vectors $S_w^{(z)} \in \mathcal{T}^{\tilde{w}}$, where,w=1,2,3,… w_t, and $\tilde{w} = W_Z W_L A_t$.

Thus,

$$S_b^{(z)} = \left[\left(S_1^{(z)}\right)^T \left(S_2^{(z)}\right)^T \ldots \left(S_{w_t}^{(z)}\right)^T \right]^T. \tag{47}$$

Each one of the component vector $S_w^{(z)}$ is than encoded independently using constituent encoder $\bar{X}_w^{(z)} : \mathcal{T}^{\tilde{w}} \to \mathcal{X}^{\tilde{w}}$, where, $\mathcal{X}$ is the output alphabet and given by,

$$\bar{X}_w^{(z)} = \Theta S_w^{(z)} \,, \tag{48}$$

$$= \left[\bar{x}_{w,1}^{(z)} \ldots \bar{x}_{w,w_L}^{(z)} \right]^T, \tag{49}$$

where, Θ, is an $\tilde{w} \times \tilde{w}$ unitary matrix (rotational matrix) and it is constructed by the first principal $\tilde{w} \times \tilde{w}$ matrix of the following $\tilde{y} \times \tilde{y}$ matrix,

$$\psi = F_{\tilde{y}}^H diag\left(1, \varphi, \ldots, \varphi^{\tilde{y}}\right), \tag{50}$$

where, $\tilde{y} = 2^{\lceil \log_2 \tilde{w} \rceil}$, $F_{\tilde{y}}$ is the $\tilde{y} \times \tilde{y}$ discrete Fourier transform (DFT) matrix and

$$\varphi = e^{\left(\frac{j2\pi}{4\tilde{y}}\right)}. \tag{51}$$

It is worth noting that, multiplying information symbol vector $S_w^{(z)}$ by the rotational matrix Θ maximize the associated minimum product distance [18],

$$d_{\tilde{w}} = \min_{\bar{X}^{(z)} = \Theta\left(S^{(z)} - S'^{(z)}\right), S^{(z)} \neq S'^{(z)}} \prod_{w=1}^{\tilde{w}} \left| \bar{X}_w^{(z)} \right|, \tag{52}$$

where, $\bar{X}^{(z)} = \left[\bar{X}_1^{(z)} \bar{X}_2^{(z)} \ldots \bar{X}w_t^{(z)} \right]^T$.

The maximization of the coding gain in the code design criteria is achieved as coding gains are proportional to the minimum product distances associated with the rotational matrix used.

Each encoded vector of $S_w^{(z)}$ is multiplied by the Diophantine number $\phi_{1,w}$ to ensure the full diversity of the code and maximize the coding gain for the joint code. The numbers are referred to as “Diophantine numbers” because they are chosen in a way that their simultaneous Diophantine approximation by algebraic number is “bad”, that makes the threads and the users become transparent to each other. $\phi_{1,w}$ is chosen from the w^{th} diagonal layer of the $W_t \times A_t$ matrix Φ_1 and given by,

$$\Phi_1 = \begin{pmatrix} 1 & \phi_1^{(w_t-1)} & \ldots & \phi_1^{(w_t - A_t)+1} \\ \phi_1 & 1 & \ldots & \phi_1^{(w_t - A_t)+2} \\ \phi_1^2 & \phi_1 & \ldots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1^{(w_t-1)} & \phi_1^{(w_t-2)} & \ldots & \phi_1^{\left(1 - \frac{A_t}{W_t}\right) W_t} \end{pmatrix} \tag{53}$$

The matrix Φ_1 will be defined latter. In (49), the terms $\overline{\chi}_{w,i}^{(z)}$ is given by,

$$\overline{\chi}_{w,i}^{(z)} = \left[X_w^{(z)}\left(P_i^l+1\right)\ldots X_w^{(z)}\left(P_i^l+A_t\right)\right] \quad , (54)$$

for i=1, 2…, W_L, where the index, P_i^l is given by,

$$P_i^l = (i-1)A_t + (l-1)W_L A_t \quad (55)$$

for $l = 1, 2, ..., W_Z$.

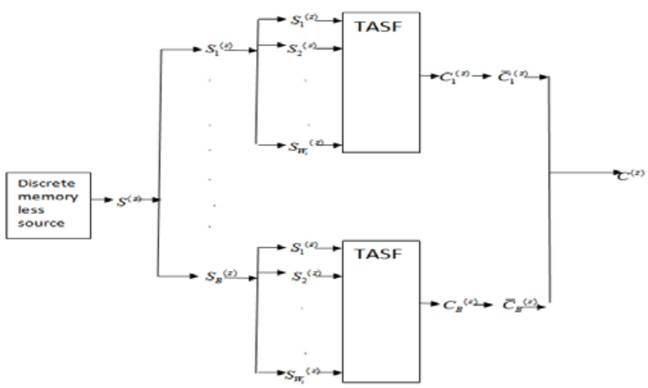

Figure 2. SF coding structure in MIMO-OFDM system

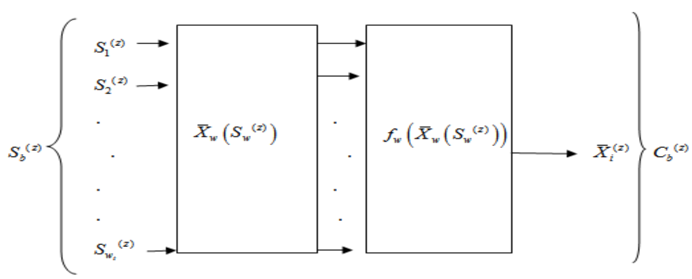

Figure 3. Threaded algebraic SF code design

Next, a space-frequency formatter $f_W\left(\overline{X}_w^{(z)}\left(S_w^{(z)}\right)\right)$ assigns the w^{th} code symbols $X_w^{(z)}\left(P_i^l+n\right)$ (where, $n = 1, 2, \ldots, A_t$) of the row vector $\overline{x}_{w,i}^{(z)}$ on the w^{th} layer of the $W_t \times A_t$ matrix, $\overline{X}_{i,l}^{(z)}$. The construction of the matrix is given by,

$$\overline{X}_{i,l}^{(z)} = \chi_i^{(z)} \circ \Phi_1. \quad (56)$$

Here, the $W_t \times A_t$ matrix $\chi_i^{(z)}$ is given by,

$$\chi_i^{(z)} = \begin{pmatrix} X_1^{(z)}\left(P_i^l+1\right) & X_{W_t}^{(z)}\left(P_i^l+1\right) & \ldots\ldots & X_{(W_t-A_t)+2}^{(z)}\left(P_i^l+1\right) \\ X_2^{(z)}\left(P_i^l+\left\lfloor \frac{A_t}{W_t}\right\rfloor+1\right) & X_1^{(z)}\left(P_i^l+2\right) & \ldots\ldots & X_{(W_t-A_t)+3}^{(z)}\left(P_i^l+2\right) \\ X_3^{(z)}\left(P_i^l+2\right) & X_2^{(z)}\left(P_i^l+\left\lfloor \frac{A_t}{W_t}\right\rfloor+2\right) & \ddots & X_{(W_t-A_t)+4}^{(z)}\left(P_i^l+3\right) \\ \vdots & \vdots & \vdots & \vdots \\ X_{W_t}^{(z)}\left(P_i^l+A_t\right) & X_{W_t-1}^{(z)}\left(P_i^l+A_t\right) & \ldots\ldots & X_{\left(1-\frac{A_t}{W_t}\right)W_t+1}^{(z)}\left(P_i^l+A_t\right) \end{pmatrix} \quad (57)$$

The diagonal layer index of $\overline{X}_{i,l}^{(z)}$ is shown in fig-(4) A_t=5 and W_t=8

$$\begin{pmatrix} 1 & 8 & 7 & 6 & 5 \\ 2 & 1 & 8 & 7 & 6 \\ 3 & 2 & 1 & 8 & 7 \\ 4 & 3 & 2 & 1 & 8 \\ 5 & 4 & 3 & 2 & 1 \\ 6 & 5 & 4 & 3 & 2 \\ 7 & 6 & 5 & 4 & 3 \\ 8 & 7 & 6 & 5 & 4 \end{pmatrix}$$

Figure 4. Symbols placement in the diagonal layers of $\overline{X}_{i,l}^{(z)}$ matrix for $A_t = 5$ *and* $W_t = 8$.

Thus each sub-block, $S_b^{(z)} \epsilon \mathcal{T}^{W_Z A_t W_L W_t}$ b=1, 2, 3,…., B; is encoded into an SF code matrix $c_b^{(z)}$ of size $Q \times A_t$ matrix, where,

$$C_b^{(Z)} = \left[\left(\overline{X}_{1,1}^{(1)}\right)\ldots\left(\overline{X}_{1,W_Z}^{(Z)}\ldots\right)\ldots\left(\overline{X}_{W_L,1}^{(1)}\right)\ldots\left(\overline{X}_{W_L,W_Z}^{(Z)}\right)\right]^T. \quad (58)$$

For multi-user MIMO-OFDM MAC, the encoded codeword $\overline{C}_b^{(z)}$ is given by,

$$\overline{C}_b^{(z)} = C_b^{(z)} \circ \left(\Phi_{2,z} \otimes 1_{1\times A_t}\right) \quad (59)$$

where, $\Phi_{2,z}$ is the z^{th} column of the $Q \times W_Z$ matrix Φ_2, which is given by,

$$\Phi_2 = \begin{pmatrix} 1 & \phi_2^{(Q-1)} & \ldots & \phi_2^{(Q-Z)+1} \\ \phi_2 & 1 & \ldots & \phi_2^{(Q-Z)+2} \\ \vdots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \phi_2^{(Q-1)} & \phi_2^{(Q-2)} & \ldots & \phi_2^{\left(1-\frac{Z}{Q}\right)Q} \end{pmatrix} \quad (60)$$

The choice of Φ_1 and Φ_2 as follows:

- $\phi_1 = \theta^{\frac{1}{W_t}}$, where θ is an algebraic clement with degree at least $W_L W_t$ over $\mathcal{A}$ and $\phi_2 = \phi^{\frac{1}{Q}}$, where ϕ is an algebraic number with degree of at least QW_L over $\mathcal{A}$. Here $\mathcal{A}$ is the extension field of $\mathbb{Q}$ which contains the signal alphabet $\mathcal{T} \subset \mathbb{Z}[j]$, all the entries of Θ and $e^{\frac{-j2\pi\tau_l}{T_s}}$, for $(l = 0,1,\ldots,\mathrm{L}-1)$.

It is worth noting that, according to the *Theorem* 1 and 2 given in [1], the coding gain expresses the simultaneous Diophantine approximation of the Diophantine numbers

$$\left\{\phi_1^0 = 1, \phi_1^1 = \theta^{\frac{1}{W_t}}, \ldots, \phi_1^{(W_t-1)} = \theta^{\frac{(W_t-1)}{W_t}}\right\}$$

and

$$\left\{\phi_2^0 = 1, \phi_2^1 = \phi^{\frac{1}{Q}}, \ldots, \phi_2^{(Q-1)} = \phi^{\frac{(Q-1)}{Q}}\right\}$$

by other algebraic numbers, depending on the constellation used. This observation implies that optimizing the coding gain is equivalent to choosing these Diophantine numbers to be "badly approximated" by other algebraic number.

The proposed SF coding applies the same coding strategy to every encoded sub-block $\bar{C}_b^{(z)}$, $b = 1, 2, \cdots, B$. So, for convenience, we have just showed the SF coding of one sub block $\bar{C}_b^{(z)}$ as illustrated in Fig-3.Thus, the proposed rate - A_t SF codes $C^{(z)} \in \mathcal{T}^{N \times A_t}$ for the z^{th} user is of the form:

$$C^{(z)} = \left[\left(\bar{C}_1^{(z)}\right)^T \left(\bar{C}_2^{(z)}\right)^T \ldots \left(\bar{C}_B^{(z)}\right)^T\right]^T \qquad (61)$$

Note that, the difference of the SF code between any two users lies in the design of $\Phi_{2,z}$ in (60) which is selected from the different columns of matrix Φ_2. Since Φ_2 is fixed in advance, each transmitter know it's $\Phi_{2,z}$ before transmitting the data and $\Phi_{2,z}$ is fixed for each user, therefore the co-operation between the users is not necessary in the uplink process.

It is also worth noting that, the proposed SF code achieve high symbol rate, i.e rate- A_t per channel use. This can be seen from (61) that NA_t encoded independent information symbol are sent over N OFDM tones used in the system. Furthermore, as TASF code confirms that N is the integer multiple of Q, no zero-padding matrix is required in our proposed code structure. Thus the high rate (rate- A_t) is always guaranteed. Now, we give the following theorem:

Theorem: Suppose that in a single cell network, a multi-user MIMO-OFDM system with Z users, each having A_t transmit antennas, one BS having A_r receiver antennas, has N OFDM tones; such that the MIMO channels that are spatially uncorrelated experience the wideband block-fading characterized by L independent paths. Then the proposed multi-user SF codes designed in (59), (61) achieve full diversity - $A_t A_r L$ and the symbol rate- A_t per channel use.

Proof: The proof of high rate and full-diversity of proposed code are given in the Appendix [19].

The maximum likelihood (ML) decoding is often used for full-diversity performance of STF codes. However, the decoding complexity is very large with the increase of the value of L and A_t [5]. To remove the burden of ML, sphere decoder can be used to achieve the approximate performance of ML [17], [20].

5 SF code design example

In this section, the sub-block $\bar{C}_1^{(z)}$ has been shown as an example. This is more convenient because the same coding strategies can be applied to every sub-block $\bar{C}_b^{(z)}$, b=1,..., B. **Example- :** Let, $A_t = 2, L = 2, Z = 2$. Hence, we obtain,

$$\bar{C}_1^{(1)} = \begin{pmatrix} X_1^{(1)}(1) & \phi_1 X_2^{(1)}(1) \\ \phi_2\phi_1 X_2^{(1)}(2) & \phi_2 X_1^{(1)}(2) \\ \phi_2^2 X_1^{(1)}(5) & \phi_2^2\phi_1 X_2^{(1)}(5) \\ \phi_2^3\phi_1 X_2^{(1)}(6) & \phi_2^3 X_1^{(1)}(6) \\ \phi_2^4 X_1^{(1)}(3) & \phi_2^4\phi_1 X_2^{(1)}(3) \\ \phi_2^5\phi_1 X_2^{(1)}(4) & \phi_2^5 X_1^{(1)}(4) \\ \phi_2^6 X_1^{(1)}(7) & \phi_2^6\phi_1 X_2^{(1)}(7) \\ \phi_2^7\phi_1 X_2^{(1)}(8) & \phi_2^7 X_1^{(1)}(8) \end{pmatrix} and, \bar{C}_1^{(2)} = \begin{pmatrix} \phi_2^7 X_1^{(2)}(1) & \phi_2^7\phi_1 X_2^{(2)}(1) \\ \phi_1 X_2^{(2)}(2) & X_1^{(2)}(2) \\ \phi_2^1 X_1^{(2)}(5) & \phi_2^1\phi_1 X_2^{(2)}(5) \\ \phi_2^2\phi_1 X_2^{(2)}(6) & \phi_2^2 X_1^{(2)}(6) \\ \phi_2^3 X_1^{(2)}(3) & \phi_2^3\phi_1 X_2^{(2)}(3) \\ \phi_2^4\phi_1 X_2^{(2)}(4) & \phi_2^4 X_1^{(2)}(4) \\ \phi_2^5 X_1^{(2)}(7) & \phi_2^5\phi_1 X_2^{(2)}(7) \\ \phi_2^6\phi_1 X_2^{(2)}(8) & \phi_2^6 X_1^{(2)}(8) \end{pmatrix},$$

where, the sub-block $\bar{C}_1^{(1)}$ is for user, z=1 and the sub-block $\bar{C}_1^{(2)}$ is for user, z=2.

Moreover,

$$\left[X_w(1)\ldots\ldots\ldots\ldots X_W(8)\right]^T = \Theta\left[S_{8(W-1)+1}\ldots\ldots S_{8W}\right]^T, W = 1, 2.$$

6 Simulation results

In this section, simulation result is presented to show the performance comparison of different coding schemes. Two users scenario has been considered where each user has two transmit antennas and the base station has two receive antennas with equal power gain. A two ray channel model has been simulated for each pair of transmit-receive antennas. The length of the cyclic prefix is chosen as 16 and N=64 OFDM tones are used for each transmit antenna. The second path delay is assumed to be 0.5µs that is, 10 times the sampling interval. In the simulation, the channel coefficients are independent from one OFDM block to other block but remain constant during one OFDM block. The simulation result shows the performance comparison among the multiuser SF code [1], Alamouti code [21] and the proposed multiuser space-frequency code with 16-QAM.

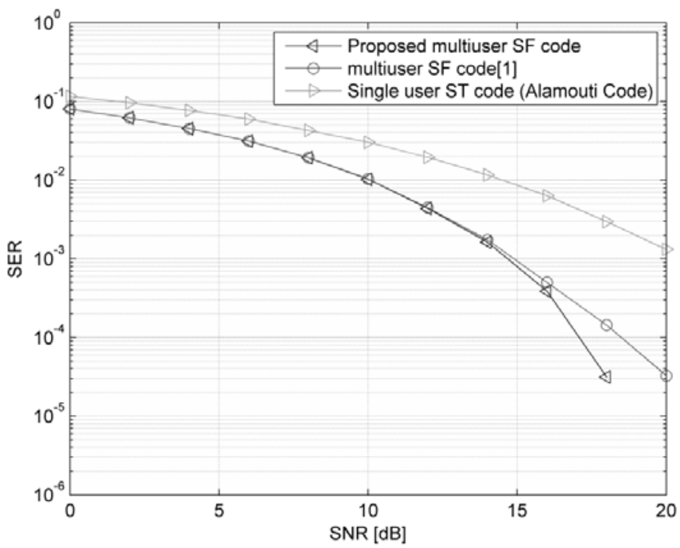

Fig-5: SER performance comparison among the multiuser SF code [1], Alamouti code [21] and the proposed multiuser space-frequency code.

Fig-5 shows that the symbol error probability (SER) P_{e3} for both the single user ST code [21] (Alamouti code) and the proposed multiuser SF code with 16-QAM. It is apparent that the Alamouti code can achieve full special diversity gain of 4; on the other hand the proposed multiuser SF code achieve full spatial and frequency diversity gain of $A_t A_r L$=8. From the graph, it can be seen that the proposed code have larger slope curve, compared with Alamouti code. This implies that the proposed SF code achieve a larger diversity gain than the Alamouti code.

Another SER performance comparison between the proposed code and the multiuser SF code in [1] has been given in fig-5. From the analysis it can be seen that the both codes achieve full spatial and frequency diversity gain of 8. It is observed that the proposed rate - A_t(in this case A_t=2) SF code achieve the best SER performance among the simulated cases. In the high SNR region the proposed code has better performance with a gap of about 2dB than the rate-1 code in [1], but it outperforms with a gap of about 2 dB over all examined SNR region when it is compared with Alamouti code. The analysis implies that the proposed multiuser rate- A_t SF code has a better coding gain than the both rate-1 SF code [1] and Alamouti code [21]. The diversity and coding gain advantages are attributed to the proposed multiuser SF codes which can obtain full diversity and high rate (rate - A_t) for every error event of the two user system which indicates that the increment of code rate does not deteriorate the performance.

7 Discussion and Conclusion

We have proposed a systematic design of high-rate; full-diversity multi-user space-frequency codes for MIMO-OFDM systems over frequency-selective block-fading multiple access channels. The proposed codes can achieve high-rate (rate-A_t) and full-diversity $A_t A_r L$ which validate the theoretical analysis. The presented multi-user SF code is bandwidth efficient and always ensures high rate-A_t . A few examples of our proposed code design have also been given. Simulation result showed that in the high SNR region, the proposed code has a better coding gain in compare to [1] and over the all examined SNR values, the proposed code achieve higher diversity and coding gain than [20], which indicates that the code performance does not affected while the symbol rate is increased to A_t. For channels with low correlations, it results in better performance in low to moderate SNR regions. Our proposed SF coding can guarantee the full diversity and high bandwidth efficiency over recently proposed space-frequency codes.

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