

Performance of Selected Diversity Techniques Over The α - μ Fading Channels

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Abstract: - In this paper, approximate closed-form expressions for the bit error rate (BER) of M-ary quadrature amplitude modulation (MQAM) and M-ary phase shift keying (MPSK) are derived considering independent and identically distributed (i.i.d) α - μ fading channels with a maximal ratio combining (MRC) receiver. Moreover, other closed-form expressions are obtained for the symbol error rate (SER) of both MQAM and MPSK under the same channel conditions considering dual branch selection combining (SC) receiver. The derivations for MRC are based on the exponential approximation of the coherent BER formula for both MQAM and MPSK. For dual branch SC, the derivations are based on very accurate SER approximation for both MQAM and MPSK. The derived expressions can reduce to study the BER performance over other fading channels such as; Rayleigh, Weibull, and Nakagami-m, as special cases. Numerical results are also provided for the derived expressions and they show close match with Monte-Carlo simulations, especially for the case of dual branch SC.

Key-Words: - **Maximum Ratio Combining (MRC), Selection Combining (SC), the α - μ fading channel, generalized fading model, M-ary Quadrature Amplitude Modulation (MQAM), M-ary Phase Shift Keying (MPSK), Bit Error Rate (BER), and Symbol Error Rate (SER).**

1 Introduction

The α - μ fading distribution, which was proposed in [1], is a general and a flexible fading distribution that explores the nonlinearity attributes of the propagation medium. Many other fading distribution models such as Rayleigh, Nakagami-m, Weibull, one-sided Gaussian, exponential, and Gamma can be derived from the α - μ fading distribution as special cases by substituting certain values for α and μ . Several articles have dealt with α - μ fading channels. In [2], the moment generating function (MGF) for the α - μ fading model was derived and a closed-form expression for the bit error rate (BER) of coherent binary modulation technique was obtained therein. In [3], new closed form expressions for the average amount of fading and the average normalized channel capacity for the α - μ fading channels were derived. In [4], the authors studied the asymptotic average symbol error rate

probability at high SNR in terms of α - μ parameters and extended their studies to systems employing orthogonal space time block codes (OSTBC). In [5], the author simulated the BER performance of orthogonal frequency division multiplexing (OFDM) with binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) systems for a wide range of the α and μ parameters. In [6], highly accurate closed-form approximations to the probability density function (pdf) and the cumulative distribution function (CDF) of the sum of independent and identically distributed (i.i.d) α - μ variates were provided and those approximations were used to find a precise approximation for the outage probability (OP) and the average BER of maximum ratio combining (MRC) and equal gain combining (EGC) diversity systems. In [7], the authors derived infinite series expressions for the pdf and the CDF of the output signal to interference ratio (SIR) over correlated α - μ fading channels with dual branch selection combining (SC), they also evaluated

the average error probability of coherent frequency-shift keying (CFSK), coherent phase-shift keying (CPSK), binary frequency-shift keying (BFSK) and binary differentially phase-shift keying (BDPSK). The same topic was studied in [8] but with triple SC. In [9], the authors derived an accurate approximation for the OP of EGC over the α - μ fading channel considering independent co-channel interference (CCI) paths. The approximate expressions were obtained in terms of a single fold integral. In this paper, we study the symbol error rate performance of different coherent modulation techniques i.i.d α - μ fading channels for diversity systems employing MRC and SC.

The rest of this paper is organized as follows: In Section 2, the system model is introduced. The BER expressions for MQAM and MPSK with MRC diversity combining are derived in Section 3. The SER for both MQAM and MPSK systems with dual branch SC are derived in Section 4. Numerical results are provided in Section 5 and concluding remarks are drawn in Section 6.

2 System Model

In this paper, we assume a wireless communication system that is carried over multipath small scale fading where the receiver side receives multiple replicas of the same information bearing signal. The transmitted signal is carried over L i.i.d α - μ fading channels and we assume that the receiver is employing either MRC or SC diversity schemes. For MRC diversity system, the received signals are effectively combined at the receiving side, where the overall combined SNR, at the combiner's output, is given by $\gamma = \sum_{i=1}^L \gamma_i$ where $\gamma_i = h_i^2 \frac{E_s}{N_0}$ is the instantaneous SNR for the i^{th} path, h_i is the instantaneous gain for the i^{th} path, E_s is the symbol energy, and N_0 is the single-sided power spectral density of the additive white Gaussian noise (AWGN). Whereas with SC the receiver chooses the branch with the highest SNR level $\gamma = \max(\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_L)$. The pdf of γ_i can be expressed as [2]:

$$f_{\gamma_i}(\gamma_i) = \frac{\alpha \mu^\mu}{2\Gamma(\mu)} \frac{\gamma_i^{\frac{\alpha\mu}{2}-1}}{\tilde{\gamma}_i^{\alpha\mu/2}} e^{-\mu \left(\frac{\gamma_i}{\tilde{\gamma}_i}\right)^{\frac{\alpha}{2}}} \quad (1)$$

Where $\tilde{\gamma}_i = E[r_i^2] \frac{E_b}{N_0}$ and \tilde{r} is the α -root mean value of the envelope random variable.

The MGF of γ_i is given by [2]:

$$\mathcal{M}_{\gamma_i} = \frac{\alpha \mu^\mu}{2\Gamma(\mu)} \frac{k^{\frac{1}{2}} d^{\left(\frac{\alpha\mu-1}{2}\right)}}{\tilde{\gamma}_i^{\frac{\alpha\mu}{2}} (2\pi)^{\frac{k+d-2}{2}}} \times G_{d,k}^{k,d} \left(\left(\frac{\mu}{k \tilde{\gamma}_i^{\frac{\alpha}{2}}} \right)^k \left(\frac{d}{s} \right)^d \left| \begin{matrix} I(d, 1 - \frac{\alpha\mu}{2}) \\ I(k, 0) \end{matrix} \right. \right) \quad (2)$$

Where $G_{p,q}^{m,n}(\cdot)$ is the Meijer's G function defined in [10, pp.1032] and $(n, \xi) = \frac{\xi}{n}, \frac{\xi+1}{n}, \dots, \frac{\xi+n-1}{n}$. The integers d and k are chosen such that $\frac{\alpha}{2} = \frac{d}{k}$ and their great common divisor (gcd) is one.

3 Average BER with MRC

In this section, we consider both MQAM and MPSK modulation schemes and derive closed-form expressions for the average BER considering coherent reception for L-MRC diversity combining technique.

3.1 Average BER for MQAM

The conditional BER of MQAM modulation scheme assuming gray coding is well-defined and can be expressed as [11]:

$$P_b(E/\{\gamma_i\}_{i=1}^L) = 2 \frac{\left(1 - \frac{1}{\sqrt{M}}\right)}{\log_2 M} \text{erfc} \left(\sqrt{\frac{1.5 \gamma}{M-1}} \right) \quad (3)$$

However, in this paper we adopt the exponential approximation for the BER of coherent reception of MQAM as follows [12-13]:

$$P_b(E/\{\gamma_i\}_{i=1}^L) = 0.2 \exp\left(\frac{-1.6 \gamma}{M-1}\right) \quad (4)$$

In order to find the average BER, we need to average the conditional error rate given in (3) over the joint pdf of $\{\gamma_i\}_{i=1}^L$ as:

$$P_b = \int_0^\infty \dots \int_0^\infty P_b(E/\{\gamma_i\}_{i=1}^L) \times \prod_{i=1}^L f_{\gamma_i}(\gamma_i) d\gamma_1 \dots d\gamma_L \quad (5)$$

Substituting (1) and (4) in (5), yields:

$$P_b = 0.2 \int_0^\infty \dots \int_0^\infty \prod_{i=1}^L \exp\left(\frac{-1.6 \gamma_i}{M-1}\right) \times f_{\gamma_i}(\gamma_i) d\gamma_1 \dots d\gamma_L \quad (6)$$

It is worth mentioning here that all paths are assumed to be i.i.d. Thus, we can obtain the following expression for the BER after some mathematical manipulation:

$$P_b = 0.2 \left(\frac{\alpha \mu^\mu}{2\Gamma(\mu) \tilde{\gamma}_i^{\frac{\alpha \mu}{2}}} \frac{k^{\frac{1}{2}} d^{\frac{\alpha \mu - 1}{2}}}{2\pi^{\frac{k+d-2}{2}} \left(\frac{1.6}{M-1}\right)^{\frac{\alpha \mu}{2}}} \right)^L \times \left(G_{d,k}^{k,d} \left(\left(\frac{\mu}{k \tilde{\gamma}^{\alpha/2}} \right)^k \left(\frac{d(M-1)}{1.6} \right)^d \middle| \begin{matrix} I(d, 1 - \frac{\alpha \mu}{2}) \\ I(k, 0) \end{matrix} \right) \right)^L \quad (7)$$

It is noteworthy mentioning here that the result in (7) is a closed-form expression for the BER that is valid for both integer and non-integer values of fading parameters.

3.2 Average BER for MPSK

In this section we consider the MPSK modulation technique and derive closed-form expression for average BER considering coherent reception. We follow the same procedure as in evaluating the average BER for MQAM. The conditional BER of MPSK, with coherent reception, can be approximated in exponential form as [12]:

$$P_b(E/\{\gamma_i\}_{i=1}^L) = 0.2 \exp\left(\frac{-7 \gamma}{2^{1.9b} + 1}\right) \quad (8)$$

Where b is the number of bits per symbol. Following the same steps that led to (7), we can find the average BER MPSK to be:

$$P_b = 0.2 \left(\frac{\alpha \mu^\mu}{2\Gamma(\mu) \tilde{\gamma}_i^{\frac{\alpha \mu}{2}}} \frac{k^{\frac{1}{2}} d^{\frac{\alpha \mu - 1}{2}}}{2\pi^{\frac{k+d-2}{2}} \left(\frac{7}{2^{1.9b} + 1}\right)^{\frac{\alpha \mu}{2}}} \right)^L \times \left(G_{d,k}^{k,d} \left(\left(\frac{\mu}{k \tilde{\gamma}^{\alpha/2}} \right)^k \left(\frac{d(2^{1.9b} + 1)}{7} \right)^d \middle| \begin{matrix} I(d, 1 - \frac{\alpha \mu}{2}) \\ I(k, 0) \end{matrix} \right) \right)^L \quad (9)$$

This expression for the BER for MPSK with L-MRC diversity combiner obtained in (9) is new, and has not been reported in the literature before. Moreover, this expression is in a closed-form that is valid for both integer and non-integer values of the fading parameters.

4 Average BER with Dual Branch SC

In this section, the pdf of α - μ fading channels with SC is derived considering i.i.d paths. MQAM and MPSK coherent modulation schemes are both considered in this section and we find the average SER over i.i.d α - μ fading channels for dual branch SC.

4.1 Pdf of α - μ distribution with SC

In this section, we derive the SNR main characteristics of i.i.d α - μ fading channels for diversity systems employing SC as the combining technique at the receiver side. The CDF of α - μ fading channels was defined in [2] as:

$$F(\gamma) = \frac{\gamma \left(\mu, \mu \left(\frac{\gamma}{\tilde{\gamma}} \right)^{\frac{\alpha}{2}} \right)}{\Gamma(\mu)} \quad (10)$$

where $\gamma(n, x)$ is a lower Incomplete Gamma function which is defined in [13] as $\gamma(n, x) = \int_0^x t^{n-1} e^{-t} dt$.

Following similar steps to those leading to [14, equ.4] in which the pdf for SC can be obtained by differentiating the CDF of the α - μ fading distribution raised to power L where L is the number of diversity branches. So the pdf of the combined SNR at the SC combiner output with dual i.i.d paths is expressed as:

$$f_{\gamma_{sc}}^{iid}(\gamma) = \frac{L \alpha \mu^\mu \gamma^{\frac{\alpha\mu}{2}-1}}{2\Gamma(\mu) \tilde{\gamma}^{\frac{\alpha\mu}{2}}} e^{-\mu\left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\alpha}{2}}} \times \left(\frac{\gamma \left(\mu, \mu \left(\frac{\gamma}{\tilde{\gamma}} \right)^{\frac{\alpha}{2}} \right)}{\Gamma(\mu)} \right)^{L-1} \quad (11)$$

In order to solve for the BER of coherent modulation techniques, we represent the incomplete gamma function by its equivalent finite series representation as [10, equ.8.352.6]:

$$\gamma(n, x) = (n-1)! \left[1 - e^{-x} \sum_{m=0}^{n-1} \frac{x^m}{m!} \right] \quad (12)$$

where n is an integer, $n = 1, 2, \dots$ etc. Then, replacing the incomplete gamma function by its equivalent finite series, the equation in (11) becomes:

$$f_{\gamma_{sc}}^{iid}(\gamma) = \frac{\alpha \mu^\mu}{2\Gamma(\mu)} \frac{L \gamma^{\frac{\alpha\mu}{2}-1}}{\tilde{\gamma}^{\frac{\alpha\mu}{2}}} \left(\frac{(\mu-1)!}{\Gamma(\mu)} \right)^{L-1} \times \left[1 - e^{-\mu\left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\alpha}{2}}} \sum_{m=0}^{\mu-1} \frac{\left(\frac{\mu}{\tilde{\gamma}^{\frac{\alpha}{2}}}\right)^m}{m!} \gamma^{\frac{\alpha m}{2}} \right]^{L-1} \quad (13)$$

Equation (13) represents the pdf of i.i.d α - μ fading channels for L-SC. For two paths ($L = 2$), the pdf becomes:

$$f_{\gamma_{sc}}^{iid}(\gamma) = \frac{\alpha \mu^\mu}{\Gamma(\mu)^2} \frac{(\mu-1)!}{\tilde{\gamma}^{\frac{\alpha\mu}{2}}} \gamma^{\frac{\alpha\mu}{2}-1} e^{-\mu\left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\alpha}{2}}} \times \left[1 - e^{-\mu\left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\alpha}{2}}} \sum_{m=0}^{\mu-1} \frac{\left(\frac{\mu}{\tilde{\gamma}^{\frac{\alpha}{2}}}\right)^m}{m!} \gamma^{\frac{\alpha m}{2}} \right] \quad (14)$$

4.2 Average SER for MQAM

In this paper, we adopt an approximate expression for the SER. This expression is considered as an accurate SER expression for coherent MQAM modulation scheme since it is valid for low and high SNR. This expression was derived by Lu et al [15] and it was reported in [16, equ.8.15] as:

$$P_s(E) \cong 4 \left(1 - \frac{1}{\sqrt{M}} \right) \times \sum_{i=0}^{\frac{\sqrt{M}-1}{2}} Q \left((2i+1) \sqrt{\frac{3 \log_2 M}{M-1}} \gamma \right) \quad (15)$$

We view (15) as a conditional error probability. In order to find the average SER for the MQAM modulation scheme, we average the conditional SER for MQAM which given in (15) over the pdf of the i.i.d α - μ fading channels with dual branch SC, given in (14), according to the following integral:

$$P_e = \int_0^\infty P_s(E) f_{\gamma_{sc}}^{iid}(\gamma) d\gamma \quad (16)$$

Substituting (14) and (15) in (16), yields:

$$P_e = 4 \left(1 - \frac{1}{\sqrt{M}} \right) \frac{\alpha \mu^\mu}{\Gamma(\mu)^2} \frac{(\mu-1)!}{\tilde{\gamma}^{\frac{\alpha\mu}{2}}} \sum_{i=0}^{\frac{\sqrt{M}-1}{2}} \{I_1 - I_2\} \quad (17)$$

Where

$$I_1 = \int_0^\infty \gamma^{\frac{\alpha\mu}{2}-1} e^{-\mu\left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\alpha}{2}}} \times Q \left((2i+1) \sqrt{\frac{3 \log_2 M}{M-1}} \gamma \right) d\gamma \quad (18)$$

$$I_2 = \sum_{m=0}^{\mu-1} \frac{\left(\frac{\mu}{\tilde{\gamma}^{\frac{\alpha}{2}}}\right)^m}{m!} \int_0^\infty \gamma^{\frac{\alpha(\mu+m)}{2}-1} e^{-2\mu\left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\alpha}{2}}} \times$$

$$Q \left((2i + 1) \sqrt{\frac{3 \log_2 M}{M - 1}} \gamma \right) d\gamma \quad (19)$$

First, we start by evaluating I_1 . The Gaussian Q-function can be defined (for $x \geq 0$) [16] as:

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2 \sin^2 \theta}\right) d\theta \quad (20)$$

The integration appeared in (18) becomes:

$$I_1 = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \gamma^{\frac{\alpha\mu}{2}-1} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha}{2}}} \times \exp\left(-\frac{(2i + 1)^2 3 \log_2 M}{2(M - 1) \sin^2 \theta} \gamma\right) d\gamma d\theta \quad (21)$$

To evaluate the above integration, we need to represent the exponential function by its equivalent Meijer's G-function, then with help of [17, pp.291, equ.2.24.1.1] I_1 reduces to:

$$I_1 = \frac{k^{\frac{1}{2}} d^{\left(\frac{\alpha\mu-1}{2}\right)} \left(\frac{2(M-1)}{3(2i+1)^2 \log_2 M}\right)^{\frac{\alpha\mu}{2}}}{\pi (2\pi)^{\frac{k+d-2}{2}}} \times \int_0^{\frac{\pi}{2}} (\sin^2 \theta)^{\frac{\alpha\mu}{2}} G_{d,k}^{k,d} \left(\left(\frac{\sin^2 \theta}{(2i+1)^2}\right)^d \middle| I\left(d, 1 - \frac{\alpha\mu}{2}\right) \right) d\theta \quad (22)$$

Where $w = \left(\frac{\mu}{k\bar{\gamma}^{\alpha/2}}\right)^k \left(\frac{d 2(M-1)}{3 \log_2 M}\right)^d$.

To evaluate the integration appeared in (22), we assume $y = \sin^2 \theta$. Then, from [12, pp.293, equ.2.24.2.2] I_1 becomes:

$$I_1 = \frac{k^{\frac{1}{2}} d^{\left(\frac{\alpha\mu-1}{2}\right)} \Gamma\left(\frac{1}{2}\right) \left(\frac{2(M-1)}{3(2i+1)^2 \log_2 M}\right)^{\frac{\alpha\mu}{2}}}{(2\pi)^{\frac{k+d}{2}}}$$

$$\times G_{2d,k+d}^{k,2d} \left(\frac{w}{(2i+1)^{2d}} \middle| I\left(d, \frac{1-\alpha\mu}{2}\right), I\left(d, 1 - \frac{\alpha\mu}{2}\right) \right) \quad (23)$$

To evaluate the second integral I_2 , we follow the same steps in evaluating I_1 . So I_2 becomes:

$$I_2 = \frac{k^{\frac{1}{2}} \Gamma\left(\frac{1}{2}\right) \sum_{m=0}^{\mu-1} \left(\frac{\mu}{\bar{\gamma}^2}\right)^m}{(2\pi)^{\frac{k+d}{2}} m!} d^{\left(\frac{\alpha(\mu+m)-1}{2}\right)} \left(\frac{2(M-1)}{3(2i+1)^2 \log_2 M}\right)^{\frac{\alpha(\mu+m)}{2}} \times G_{2d,k+d}^{k,2d} \left(\frac{w 2^k}{(2i+1)^{2d}} \middle| I\left(d, \frac{1-\alpha(\mu+m)}{2}\right), I\left(d, 1 - \frac{\alpha(\mu+m)}{2}\right) \right) \quad (24)$$

Now, substituting (23) and (24) in (17) we obtain:

$$P_e = \sum_{i=0}^{\frac{\sqrt{M}-1}{2}} A_i \left[B_i - \sum_{m=0}^{\mu-1} \frac{\left(\frac{\mu}{\bar{\gamma}^2}\right)^m}{m!} \left(\frac{2d(M-1)}{3(2i+1)^2 \log_2 M}\right)^{\frac{\alpha m}{2}} \times C_{i,m} \right] \quad (25)$$

Where:

$$A_i = 4 \left(1 - \frac{1}{\sqrt{M}}\right)^{\frac{1}{2}} \frac{k^{\frac{1}{2}} \Gamma(1/2)}{(2\pi)^{\frac{k+d}{2}}} d^{\left(\frac{\alpha\mu-1}{2}\right)} \left(\frac{2(M-1)}{3(2i+1)^2 \log_2 M}\right)^{\frac{\alpha\mu}{2}} \frac{\alpha\mu^{\mu} (\mu-1)!}{\Gamma(\mu)^2 \bar{\gamma}^{\frac{\alpha\mu}{2}}}$$

$$B_i = G_{2d,k+d}^{k,2d} \left(\frac{w}{(2i+1)^{2d}} \middle| I\left(d, \frac{1-\alpha\mu}{2}\right), I\left(d, 1 - \frac{\alpha\mu}{2}\right) \right)$$

$$C_{i,m} = G_{2d,k+d}^{k,2d} \left(\frac{w 2^k}{(2i+1)^{2d}} \middle| I\left(d, \frac{1-\alpha(\mu+m)}{2}\right), I\left(d, 1 - \frac{\alpha(\mu+m)}{2}\right) \right)$$

The result in (25) represents the SER for MQAM over $\alpha - \mu$ fading channels with dual branch SC considering i.i.d paths.

4.3 Average SER for MPSK

Actually, there is no exact closed-form expression for the BER of coherent MPSK modulation schemes in AWGN channel, however an accurate approximate expression was derived by Lu et al in [15]. This expression is precise at low and high SNR and valid for large values of M. This expression is well- defined and can be expressed as:

$$P_s(E) \cong \frac{2 \log_2 M}{\max(\log_2 M, 2)} \times \sum_{i=0}^{\max(\frac{M}{4}, 1)} Q \left(\sin \left((2i-1) \frac{\pi}{M} \right) \sqrt{2 \log_2 M \gamma} \right) \quad (26)$$

Following the same steps that led to (25), we find the average BER MPSK to be:

$$P_e = \sum_{i=1}^{\max(\frac{M}{4}, 1)} A_i \left\{ D_i - \sum_{m=0}^{\mu-1} \frac{\left(\frac{\mu}{\gamma^2}\right)^m}{m!} \left(d \frac{\left[\sin \left((2i-1) \frac{\pi}{M} \right) \right]^{-2}}{\log_2 M} \right)^{\frac{\alpha m}{2}} E_{i,m} \right\} \quad (27)$$

where

$$A_i = \frac{2 \log_2 M}{\max(\log_2 M, 2)} \frac{k^{\frac{1}{2}} \Gamma(1/2)}{(2\pi)^{\frac{k+d}{2}}} d^{\left(\frac{\alpha\mu}{2}-1\right)} \left(\frac{\left[\sin \left((2i-1) \frac{\pi}{M} \right) \right]^{-2}}{\log_2 M} \right)^{\frac{\alpha\mu}{2}} \frac{\alpha\mu^{\mu} (\mu-1)!}{\Gamma(\mu)^2 \gamma^{\alpha\mu/2}}$$

and

$$D_i = G_{2d, k+d}^{k, 2d} \left(\frac{w}{\left(\sin \left((2i-1) \frac{\pi}{M} \right) \right)^{2d}} \left| \begin{array}{l} I \left(d, \frac{1-\alpha\mu}{2} \right), I \left(d, 1-\frac{\alpha\mu}{2} \right) \\ I(k, 0), I \left(d, 1-\frac{\alpha\mu}{2} \right) \end{array} \right. \right)$$

$$E_{i,m} = G_{2d, k+d}^{k, 2d} \left(\frac{w 2^k}{\left(\sin \left((2i-1) \frac{\pi}{M} \right) \right)^{2d}} \left| \begin{array}{l} I \left(d, \frac{1-\alpha(\mu+m)}{2} \right), I \left(d, 1-\frac{\alpha(\mu+m)}{2} \right) \\ I(k, 0), I \left(d, 1-\frac{\alpha(\mu+m)}{2} \right) \end{array} \right. \right)$$

Equation (27) represents the average SER for the MPSK modulation schemes over i.i.d $\alpha - \mu$ fading channels for diversity systems employing dual branch SC. It should be noted that the results in (25) and (27) are general expressions and have not been reported in the literature before. These expressions can be reduced to other fading distributions (i.e. Rayleigh,

Negative exponential, Weibull, and Nakagami-m) by substituting the proper values of α and μ , in addition these expressions are valid for integer and non-integer values of α and μ .

5 Numerical Results

In this section, we evaluate the derived expressions for a wide range of the fading parameters (α and μ) and the constellation size for both MQAM and MPSK. We also compare the evaluated results to simulation results to show the tightness of the derived expressions. Figs. 1 and 2 show the average SER over Rayleigh fading channels ($\alpha=2, \mu=1$) with MRC combining scheme. Fig. 1 shows that analytical results are almost matched with simulation results for 4QAM with two and three i.i.d paths. For 16QAM, analytical and simulation results are in close agreement over Rayleigh fading channels as shown in Fig. 2. The average SER for MQAM over i.i.d Weibull fading channels ($\alpha=3, \mu=1$) with MRC is also shown in Figs. 3 and 4. Figs. 5 and 6 show the SER for 4QAM and 16QAM assuming Nakagami-2 channels respectively, it is clear from Figure 5 that analytical results are almost matched with simulation results for 4QAM for L=2 and 3. Fig. 6 shows that analytical results are in close agreement with simulation results for 16QAM. Figs. 7-9 show the average SER for 4PSK with MRC over different fading channels. Figure 7 shows that analytical results are in very close agreement with simulation results over Rayleigh fading channels ($\alpha=2, \mu=1$) for both two and three i.i.d paths. Fig. 8 shows that analytical results are in close agreement with simulation results for 4PSK over i.i.d Weibull fading channels with $\alpha=3$ and $\mu=1$. The same situation is considered for Nakagami-2 fading channels as shown in Fig. 9.

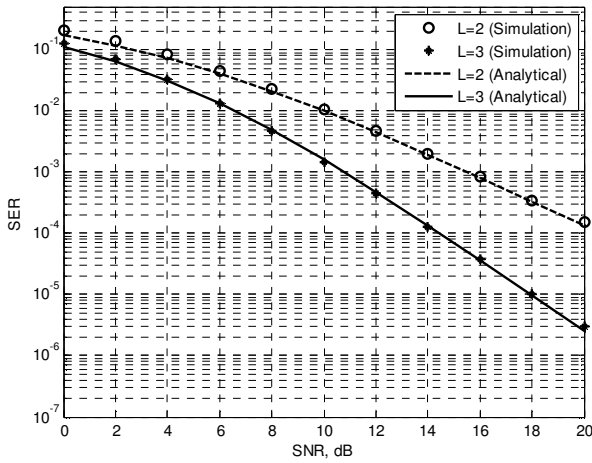


Fig. 1: SER for 4QAM over Rayleigh fading channels ($\alpha=2$, $\mu=1$) with MRC.

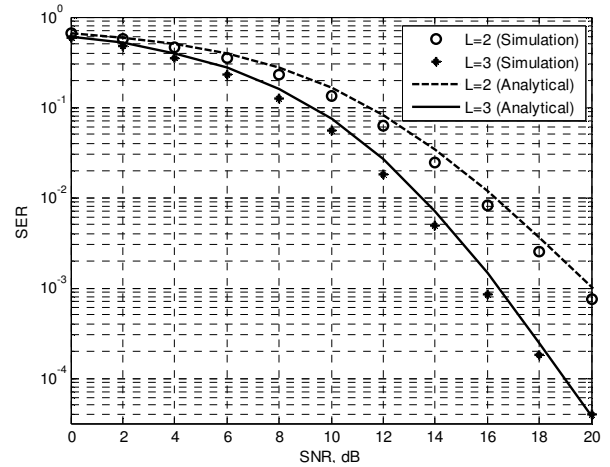


Fig. 4: SER for 16QAM over Weibull fading channels ($\alpha=3$, $\mu=1$) with MRC.

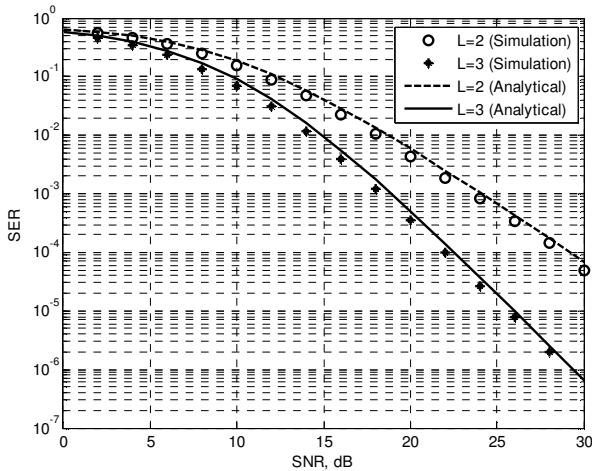


Fig. 2: SER for 16QAM over Rayleigh fading channels ($\alpha=2$, $\mu=1$) with MRC.

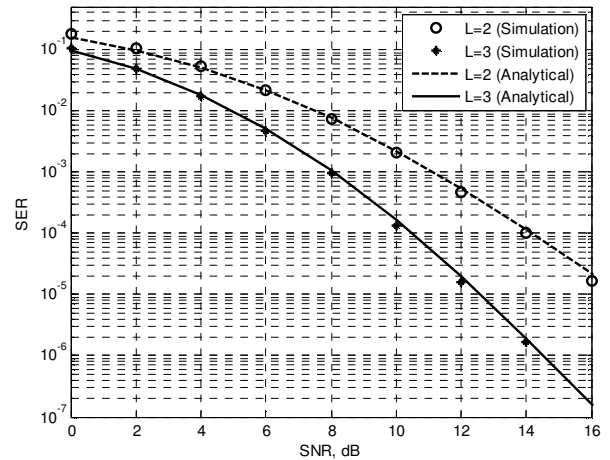


Fig. 5: SER for 4QAM over Nakagami-2 fading channels ($\alpha=2$, $\mu=2$) with MRC.

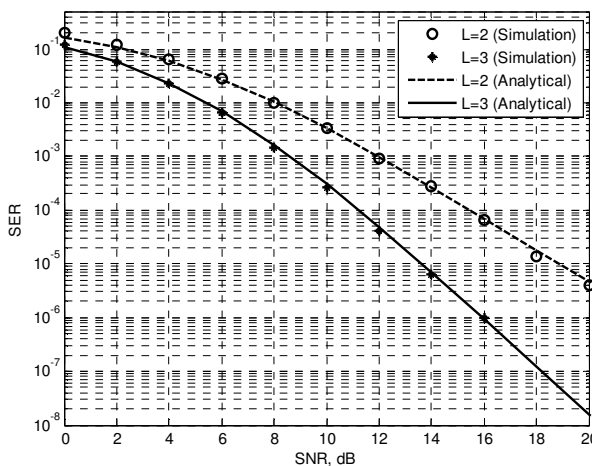


Fig. 3: SER for 4QAM over Weibull fading channels ($\alpha=3$, $\mu=1$) with MRC.

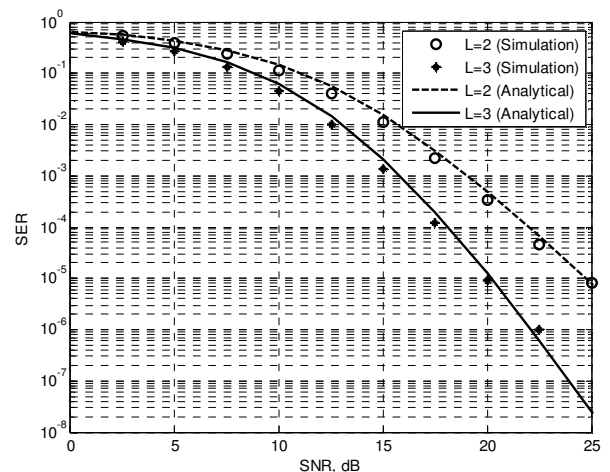


Fig. 6: SER for 16QAM over Nakagami-2 fading channels ($\alpha=2$, $\mu=2$) with MRC.

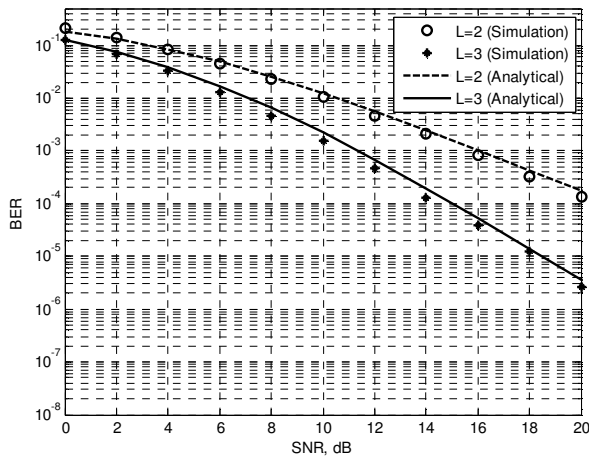


Fig. 7: SER for 4PSK over Rayleigh fading channels ($\alpha=2, \mu=1$) with MRC.

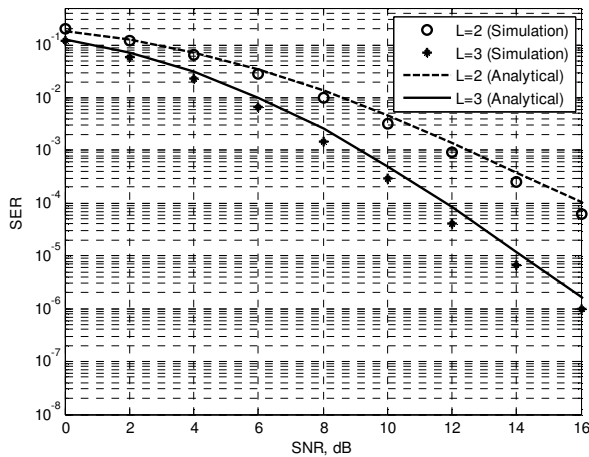


Fig. 8: SER for 4PSK over Weibull fading channels ($\alpha=3, \mu=1$) with MRC.

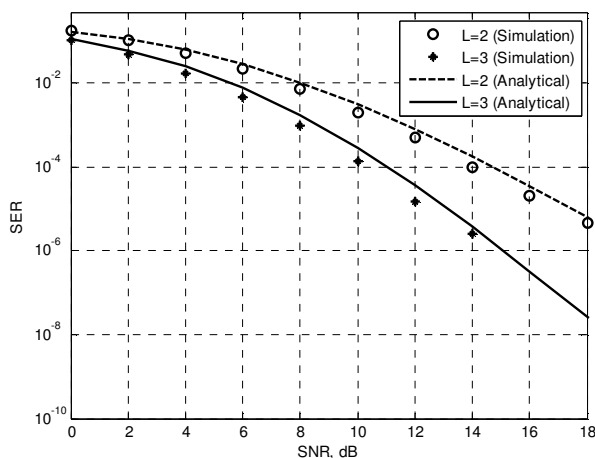


Fig. 9: SER for 4PSK over Nakagami-2 fading channels ($\alpha=2, \mu=2$) with MRC.

For dual branch SC, Figs. 10-15 show the average SER for both MQAM and MPSK over different fading channels. Figs. 10 - 12 shows the average SER for 16QAM and 64QAM over Rayleigh, Weibull and Nakagami-m ($m=2$). Fig. 10 shows that simulation and analytical results are matched for 16QAM and 64QAM. The same situation is considered for Weibull fading channel as shown in Fig. 11. For Nakagami-m ($m=2$) fading channels, analytical results are in total agreement with simulation results for both 16QAM and 64QAM as shown in Fig. 12. Figs. 13 - 17 show the average SER for 16PSK and 64PSK. In Fig. 15, analytical results are almost matched with simulation results for 16PSK over Rayleigh fading channels and are in close agreement with simulation results for 64PSK especially at high SNR. For Weibull fading channels, Fig. 11 shows that simulation and analytical results are approximately matched for 16PSK. For 64PSK simulation and analytical results are in close agreement especially at high SNR. The same case can be observed over Nakagami-m ($m=2$) fading channels as shown in Fig. 15.

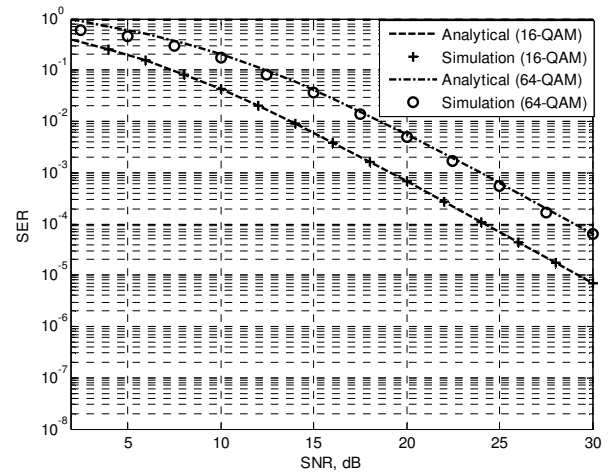


Fig. 10: SER over Rayleigh fading channels ($\alpha=2, \mu=1$) with dual-SC.

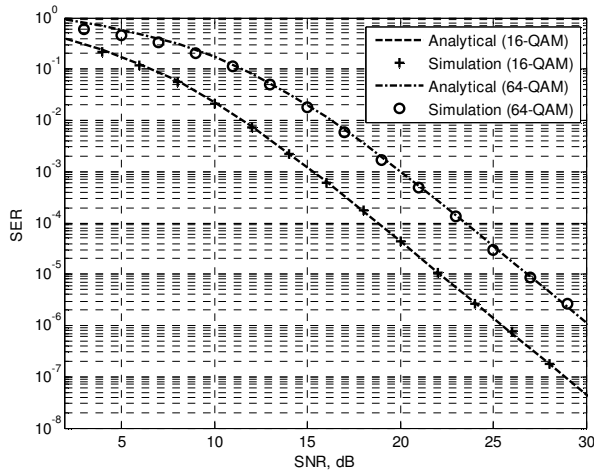


Fig. 11: SER over Weibull fading channels ($\alpha=3, \mu=1$) with dual-SC.

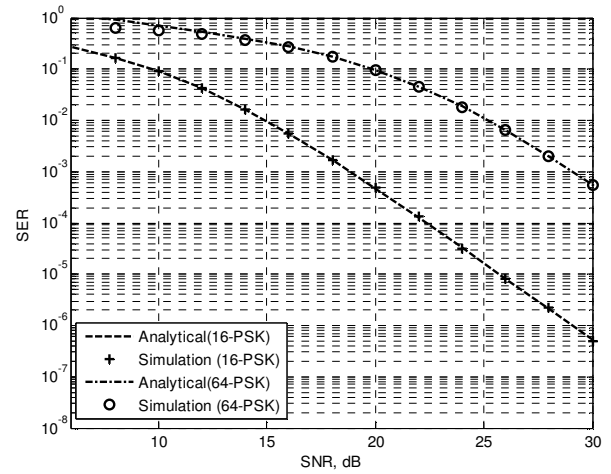


Fig. 14: SER over Weibull fading channels ($\alpha=3, \mu=1$) with dual-SC.

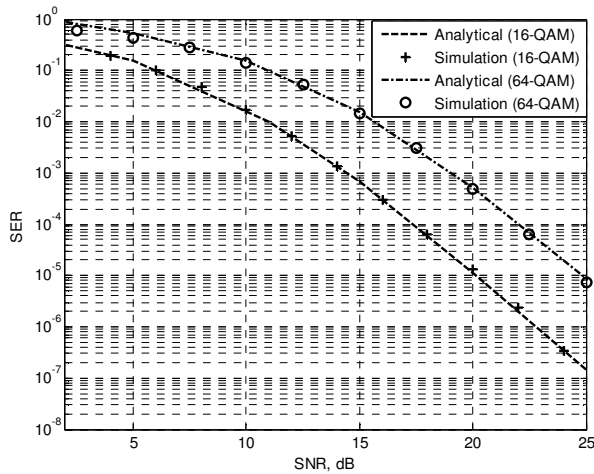


Fig. 12: SER over Nakagami-2 fading channels ($\alpha=2, \mu=2$) with dual-SC.

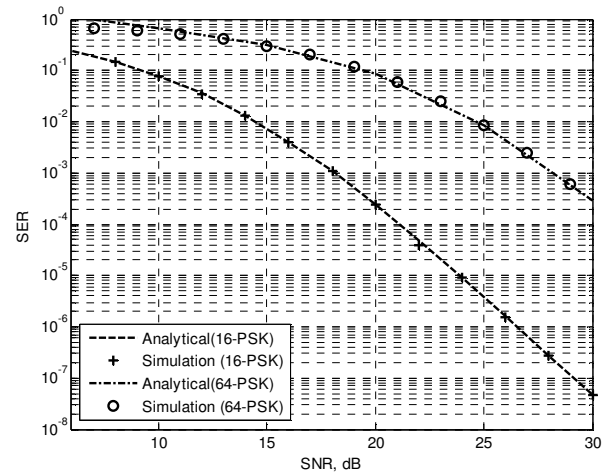


Fig. 15: SER over Nakagami-2 fading channels ($\alpha=2, \mu=2$) with dual-SC.

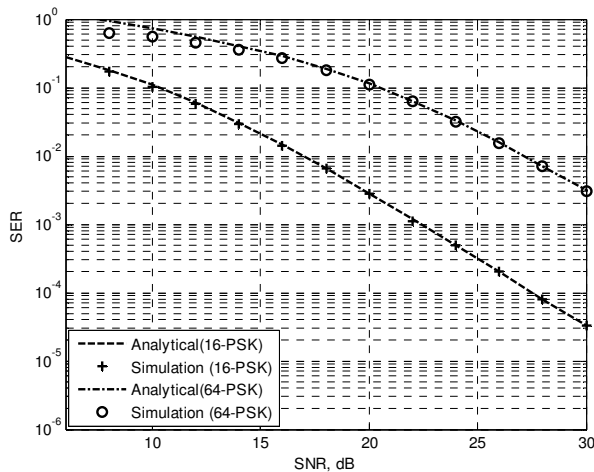


Fig. 13: SER over Rayleigh fading channels ($\alpha=2, \mu=1$) with dual-SC.

6 Conclusion

In this paper, we derived approximate closed-form expressions for the average BER of MQAM and MPSK modulation schemes over independent and identically distributed α - μ fading channels employing MRC diversity scheme at the receiver side. Under the same channel conditions, we also derived closed-form expression for the average SER for both MQAM and MPSK modulation schemes considering dual branch selection combining as combining technique for the diversity system. These expressions are new, and have not been reported in literature before. Also, these derived expressions are in closed-form and valid for both integer and non-integer values of the fading parameters. Moreover, the obtained expressions can reduce to other expressions

considering other channel models as special cases. Numerical results for the expressions obtained for MRC showed that the approximation for MQAM is in very good agreement with simulation results. Numerical results also showed that the approximation for MPSK is in close agreement with simulation results. For dual branch SC, numerical results showed that the obtained expressions are almost matched with simulation results for both MQAM and MPSK and these expressions are valid for large values of M .

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