check matrices.

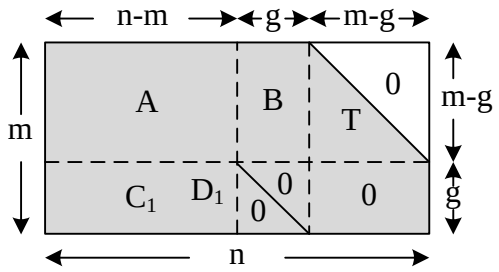

Fig. 6. Parity Check Matrix, HH, in systematic approximate lower triangular (SALT) form

Due to the structure of the SALT form Fig. 6 we can conveniently pick the first $n-m$ bit positions (from the left) in the codeword to be the data bit positions, i.e., the columns corresponding to the matrices A and C_1 are those of the data bits. Hence, the codewords have the following structure: $v = (\mathbf{u}, \mathbf{p_1}, \mathbf{p_2})$ with u the $n-m$ data bits, p_1 the first g parity bits and $\mathbf{p_2}$ the remaining $(m-g)$ parity bits.

The first g parity bits $\mathbf{p_1}$ can be directly determined from the sub-matrices C_1 and D_1 according to $\mathbf{p_1} = \mathbf{u}\,.\,C_1^T$. Further, from the parity-check condition $\mathbf{H}\,.\,\mathbf{v}^T = \mathbf{0}_{n\times 1}$ for any codeword $\mathbf{v}$, we obtain $A.\mathbf{u^T} + B.\mathbf{p_1^T} + T.\mathbf{p_2^T} = \mathbf{0}_{m\times 1}$. As the matrix T has lower triangular form, we obtain the second set $\mathbf{p_2} = \{p_2(1),\ p_2(2), \cdots,\ p_2(m-g)\}$ of parity bits by back-substitution.

2.5 Efficient encoding approach for generalized low density parity check codes

Inspired by the work in [9], the authors in [33] investigated a similar efficient encoding scheme for $(N, 2, n)$ generalized low-density (GLD) parity check codes. In [9] the greedy algorithms are used to construct approximate upper/lower triangular LDPC parity check matrix. Different with that approach, based on the structure of GLD parity check matrix, the authors proposed a systematic approach to construct approximate upper triangular $(N, 2, n)$ GLD parity check matrix H under the condition that no two constituent submatrices have more than one overlapping nonzero column.

Construction of H

Let the constituent code C_0 be an (n, k) code and its parity check matrix H_0 have systematic form $[I, P]$, where I is an $(n-k)$ by $(n-k)$ identity matrix. They defined N/n as s and $s\cdot(n-k)$ as L, respectively. The systematic construction approach of H can be shown in two steps:

1. Construct a matrix $\widehat{H} = \left[\widehat{H}^{1^T},\ \widehat{H}^{2^T}\right]^T$ where both $\widehat{H}^1$ and $\widehat{H}^2$ are L by N dimensional and contain s constituent submatrices.

2. Obtain H by reordering certain columns of $\widehat{H}$.

First, $\widehat{H}^1$ is constructed as $[I, SP]$, where I is an L by L identity matrix and SP is a block diagonal matrix containing s copies
of submatrix P as shown in Fig. 7. It is noted that $\widehat{H}^1$ can be seen as the parity check matrix of a super-code which consists of s constituent codes.

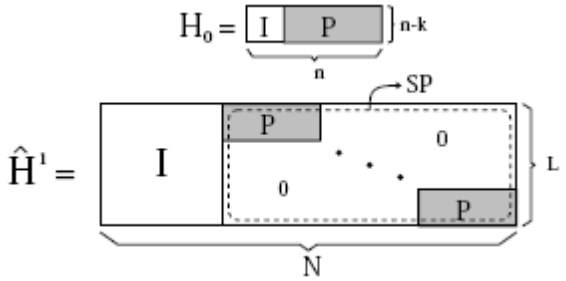

Fig. 7. Structure of matrix $\widehat{H}^1$

$\widehat{H}^2$ is constructed by permuting columns of matrix Q as shown in Fig. 8. They wrote matrix Q in block matrix form as $[Q_1, Q_2]$, where Q_1 and Q_2 are $L \times (N-L)$ and $L \times L$, respectively. By introducing two column permutations, π_1 and π_2, we construct $\widehat{H}^2$ as $[\pi_1(Q_2), \pi_2(Q_1)]$. $\widehat{H}^2$ also defines a super-code consisting of s constituent codes. Combining $\widehat{H}^1$ and $\widehat{H}^2$ together, a $(N;\ 2;\ n)$ GLD parity check matrix $\widehat{H} = \left[\widehat{H}^{1^T},\ \widehat{H}^{2^T}\right]^T$ is developed. Here π_1 and π_2 are chosen at random with the condition that no two constituent submatrices in $\widehat{H}$ have more than one overlapping nonzero column. Based on the prerequisite that $N/n \geq n$ and the structure of $\widehat{H}^1$ and Q, it can be proved that such two permutations always exist.

Since $\widehat{H}_2^2 = \pi_2(\mathrm{Q}_1)$ and Q_1 contains $\left\lfloor\frac{N-L}{n}\right\rfloor = \left\lfloor\frac{s\cdot n - s\cdot(n-k)}{n}\right\rfloor = \left\lfloor\frac{s\cdot k}{n}\right\rfloor$ complete copies of systematic parity check matrix H_0, A column permutation π_3 can always be found which makes $H = \pi_3(\widehat{H})$. The matrix has the approximate upper triangular

form as shown in Fig. 9, in which each $P_i, i = 1, \cdots, s$, is obtained by removing some columns from matrix P. As shown in Fig. 9, H can be written as

$$H = \begin{bmatrix} T & B & D \\ A & C & E \end{bmatrix}$$

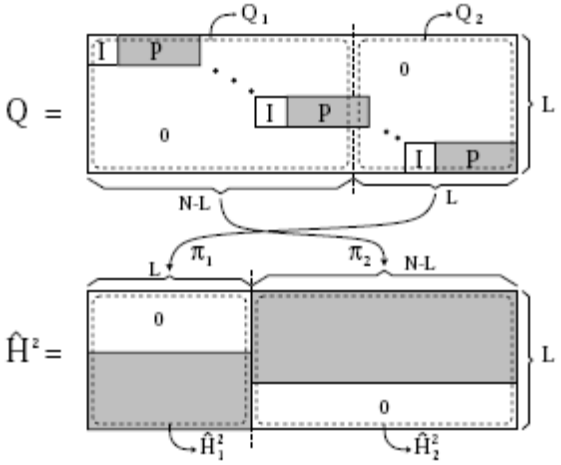

Fig. 8. Structure of matrix $\widehat{H}^2$

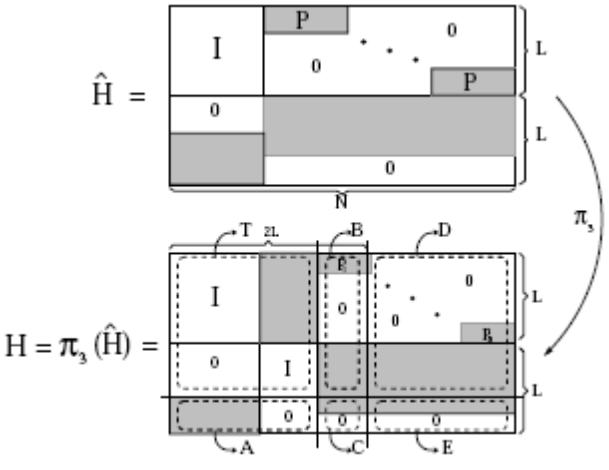

Fig. 9. Structure of matrix H

Encoding Process

i. Compute $y_C = D.x_c$ and $z_C = E.x_c$ which is efficient because both D and E are sparse;

ii. Solve $T.\hat{x}_a = y_C$. Since T has the form as shown in Fig 8, it can be proved that $T^{-1} = T$. Therefore, it can be written $\hat{x}_a = T.y_C$ which can be easily computed since T T is sparse;

iii. Evaluate $\hat{s} = A.\hat{x}_a + z_C$. which is also efficient since A A is sparse;

iv. Compute $x_b = \varphi \cdot \hat{s}$, where $\varphi = (A \cdot T \cdot B + C)^{-1}$. In this step, the complexity is scaled by $[(L - k \cdot Floor\left\{\frac{s-k}{n}\right\})]^2$.

v. Finally x_a can be obtained by solving $T \cdot x_a = B \cdot x_b + y_C$. Since $T^{-1} = T, x_a = T \cdot (B \cdot x_b + y_C)$. This is efficient since both T and B are sparse.

2.6 Two stage encoding with Triangular Factorization

Two stage encoding with Triangular Factorization (TSTF) algorithm shown in [34] explains the encoding in two steps.

1. Pre-computation step: Permute row vectors and column vectors of the parity check matrix H so that the H_2 part of H satisfies the LP condition, and the triangular matrices L and U with $H_2 = LU$ mod 2 are sparse.
2. Encoding step: Given an information vector $\boldsymbol{s}$ and parity check vector $\boldsymbol{p}$, the encoding stages are

   i. Compute $\boldsymbol{u}^T = H_1 \boldsymbol{s}^T$.
   ii. Solve $H_2 \boldsymbol{p}^T = \boldsymbol{u}^T$ after computing $\boldsymbol{v}^T = L^{-1}\boldsymbol{u}^T$ by back substitution for L and computing $\boldsymbol{p}^T = U^{-1}\boldsymbol{v}^T$ by back substitution for U.

3 Families of Algebraic Construction of QC-LDPC codes

3.1 Algebraic Construction of QC-LDPC codes: Bresnan Code

The paper in [10] discusses an algebraic construction for the regular and irregular QC-LDPC codes. The regular LDPC codes have the same number of ones in every row and column. The irregular LDPC codes have a different number of ones in columns and rows. The QC-LDPC codes consist of horizontally concatenated circulant sub-matrices. Each circulant sub-matrix is a square matrix for which every row is

the cyclic shift of the previous row, and the first row is obtained by the cyclic shift of the last row. In this way, every column of each circulant sub-matrix is automatically the cyclic shift of the previous column, and the first column is obtained by the cyclic shift of the last column. The H matrix of dimension $(m \times L_m)$ for the QC-LDPC can be written as

$$H = [H_1\ H_2\ H_3\ \cdots\ H_L] \tag{8}$$

where H_i is the i-th circulant sub-matrix of dimension $(m \times m)$, $i = 1, \cdots, L$. For the circulant matrices, the row weight and column weight are the same and fixed. Once the parity check matrix $\mathbf{H}$ is defined, the generator matrix is obtained. The matrices are created such that they should satisfy the constraint $GH^T = 0$. All the bits to be encoded are run through the generator matrix, and, therefore, all valid code words obey the property $CH^T = 0$ where C is the codeword.

The Quasi-Cyclic Generator matrix of rate $R = (L-1)/L$ has the following structure:

$$G = \begin{bmatrix} P_2^T & I_m & 0 & 0 & \cdots & 0 \\ P_3^T & 0 & I_m & 0 & \cdots & 0 \\ P_4^T & 0 & 0 & I_m & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ P_L^T & 0 & 0 & 0 & 0 & I_m \end{bmatrix}$$

As one of the requirements is $GH^T = 0$, we can write

$$GH^T = \begin{bmatrix} P_2^T & I_m & 0 & 0 & \cdots & 0 \\ P_3^T & 0 & I_m & 0 & \cdots & 0 \\ P_4^T & 0 & 0 & I_m & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ P_L^T & 0 & 0 & 0 & 0 & I_m \end{bmatrix} \times \begin{bmatrix} H_1^T \\ H_2^T \\ H_3^T \\ \vdots \\ H_L^T \end{bmatrix} = 0 \tag{9}$$

From the above relation, we can get $P_i = H_1^{-1} H_i$, where $i = 1 \cdots L$. The inverse of a circulant matrix is a circulant, and the product of two circulant matrices is also a circulant matrix.

Therefore, the QC-LDPC of different rates $(L-1)/L$ can be produced from the above-defined generator matrix G. By using this construction, the quasi-cyclic nature of generator matrix is preserved. Since the generator matrix is quasi-cyclic, the first row of each circulant sub-matrix is stored, and successive rows can be generated by a shift register generator. This greatly simplifies the encoder design. It is crucial that the circulant sub-matrix H_1 must be a nonsingular matrix. In order to maintain the non singularity of the circulant sub matrix H_1, polynomial representation of its first row should not be a factor of $x^m - 1$.

3.2 Algebraic Construction of QC-LDPC codes by Dispersion

In this section, a dispersion method for constructing QC-LDPC codes is presented for correcting erasure bursts [31]. The codes constructed by this method also perform well over the AWGN and binary random erasure channels. Consider a $4 \times 4k$ $\mathbf{H}_{\mathrm{EG}}(4,4k)$ subarray of an array $\mathbf{H}_{\mathrm{EG}}$ of circulant permutation matrices given by

$$\mathbf{H}_{\mathrm{EG}} = \mathbf{M}_{\mathrm{EG}}^{\mathrm{T}}$$

Where $\mathbf{H}_{\mathrm{EG}}$ is the transpose of $\mathbf{M}_{\mathrm{EG}}$ and $\mathbf{H}_{\mathrm{EG}}$ is a $q \times K$ array of $(q^{m-1}-1) \times (q^{m-1}-1)$ circulant permutation matrices and is a $q(q^{m-1}-1) \times K(q^{m-1}-1)$ matrix over $GF(2)$ with column and row weights q and K, respectively. Here $4 \leq q$ and $1 \leq k \leq \left\lfloor \frac{K}{4} \right\rfloor$. Dividing $\mathbf{H}_{\mathrm{EG}}(4,4k)$ into k 4×4 sub arrays as follows: $\mathbf{H}_{\mathrm{EG}}(4,4k) = [\mathbf{M}_0\ \mathbf{M}_1 \cdots\ \mathbf{M}_{k-1}]$, where for $0 \leq j < k$

$$\mathbf{M}_j = \begin{bmatrix} \mathbf{A}_{0,4j} & \mathbf{A}_{0,4j+1} & \mathbf{A}_{0,4j+2} & \mathbf{A}_{0,4j+3} \\ \mathbf{A}_{1,4j} & \mathbf{A}_{1,4j+1} & \mathbf{A}_{1,4j+2} & \mathbf{A}_{1,4j+3} \\ \mathbf{A}_{2,4j} & \mathbf{A}_{2,4j+1} & \mathbf{A}_{2,4j+2} & \mathbf{A}_{2,4j+3} \\ \mathbf{A}_{3,4j} & \mathbf{A}_{3,4j+1} & \mathbf{A}_{3,4j+2} & \mathbf{A}_{3,4j+3} \end{bmatrix}$$

Since $\mathbf{H}_{\mathrm{EG}}(4,4k)$ satisfies the RC constraint, each subarray $\mathbf{M}_j$ also satisfies the RC constraint. From $\mathbf{M}_j$, we form an 8×8 array of circulant permutation and zero matrices, as shown below

$$\mathbf{D}_j = \begin{bmatrix} \mathbf{A}_{0,4j} & 0 & 0 & 0 & 0 & \mathbf{A}_{0,4j+1} & \mathbf{A}_{0,4j+2} & \mathbf{A}_{0,4j+3} \\ \mathbf{A}_{1,4j} & \mathbf{A}_{1,4j+1} & 0 & 0 & 0 & 0 & \mathbf{A}_{1,4j+2} & \mathbf{A}_{1,4j+3} \\ \mathbf{A}_{2,4j} & \mathbf{A}_{2,4j+1} & \mathbf{A}_{2,4j+2} & 0 & 0 & 0 & 0 & \mathbf{A}_{2,4j+3} \\ \mathbf{A}_{3,4j} & \mathbf{A}_{3,4j+1} & \mathbf{A}_{3,4j+2} & \mathbf{A}_{3,4j+3} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{A}_{0,4j+1} & \mathbf{A}_{0,4j+2} & \mathbf{A}_{0,4j+3} & \mathbf{A}_{0,4j} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{A}_{1,4j+2} & \mathbf{A}_{1,4j+3} & \mathbf{A}_{1,4j} & \mathbf{A}_{1,4j+1} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{A}_{2,4j+3} & \mathbf{A}_{2,4j} & \mathbf{A}_{2,4j+1} & \mathbf{A}_{2,4j+2} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{A}_{3,4j} & \mathbf{A}_{3,4j+1} & \mathbf{A}_{3,4j+2} & \mathbf{A}_{3,4j+3} \end{bmatrix}$$

$\mathbf{D}_j$ is called a *dispersion* of $\mathbf{M}_j$, and we can readily see that $\mathbf{D}_j$ also satisfies the RC constraint. Each submatrix in $\mathbf{D}_j$ is either a $(q^m-1)\times(q^m-1)$ circulant permutation matrix or a $(q^m-1)\times(q^m-1)$ zero matrix. $\mathbf{D}_j$ is an $8(q^m-1)\times 8(q^m-1)$ matrix over $GF(2)$ with both column and row weights four. Since each circulant permutation matrix in $\mathbf{D}_j$ is followed by four $(q^m-1)\times(q^m-1)$ zero matrices (including the end-around case), the zero span of $\mathbf{D}_j$ is at least $4(q^m-1)$. Replacing each subarray $\mathbf{M}_j$ in $\mathbf{H}_{\mathrm{EG}}(4,4k)$ by its dispersion $\mathbf{D}_j$, we obtain an $8\times 8k$ array of $(q^m-1)\times(q^m-1)$ circulant permutation and zero matrices, $\mathbf{H}_{\mathrm{EG,d}}(8,8k)=[\mathbf{D}_0\ \mathbf{D}_1\cdots\ \mathbf{D}_{k-1}]$. $\mathbf{H}_{\mathrm{EG,d}}(8,8k)$ is an $8(q^m-1)\times 8k(q^m-1)$matrix over GF(2) with column and row weights 4 and , respectively, that satisfies the RC constraint and has a zero-covering span of length at least $4(q^m-1)$ bits. $\mathbf{H}_{\mathrm{EG,d}}(8,8k)$ is called the dispersion of $\mathbf{H}_{\mathrm{EG}}(4,4k)$. The null space of $\mathbf{H}_{\mathrm{EG,d}}(8,8k)$ gives a QC-LDPC code $C_{qc,d}$ whose Tanner graph has a girth of at least six. The code is capable of correcting any erasure burst of length at least up to $4(q^m-1)+1$ bits.

3.3 Algebraic Construction of QC-LDPC codes: Rakibul Code

The *H* matrix for the QC-LDPC code proposed in [13] is written as

$$H = [H_{L-1}\ \cdots\ H_2\ H_1\ H_2\ \cdots\ H_L] \tag{10}$$

The Quasi-Cyclic Generator matrix of rate $R = 1/2$ has the following structure:

$$G = \begin{bmatrix} 0 & \cdots & 0 & P_2^T & I_m & 0 & \cdots & 0 \\ 0 & \cdots & P_3^T & 0 & 0 & I_m & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ P_L^T & \cdots & 0 & 0 & 0 & 0 & \cdots & I_m \end{bmatrix}$$

As one of the requirements is $GH^T = 0$, we can write

$$\begin{bmatrix} 0 & \cdots & 0 & P_2^T & I_m & 0 & \cdots & 0 \\ 0 & \cdots & P_3^T & 0 & 0 & I_m & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ P_L^T & \cdots & 0 & 0 & 0 & 0 & \cdots & I_m \end{bmatrix} \times \begin{bmatrix} H_{L-1}^T \\ \vdots \\ H_2^T \\ H_1^T \\ H_2^T \\ \vdots \\ H_L^T \end{bmatrix} = 0 \tag{11}$$

From the above equation, several relations can be written

$$\begin{aligned} P_2^T H_1^T &= H_2^T \\ P_3^T H_2^T &= H_3^T \\ &\vdots \\ P_L^T H_{L-1}^T &= H_L^T \end{aligned} \tag{12}$$

The previous equation concludes $P_i = H_{i-1}^{-1} H_i$, where $i = 2\cdots L$. The inverse of a circulant matrix is circulant, and the product of two circulant matrices is also a circulant matrix. By using this construction, the quasi-cyclic nature of generator matrix is preserved. Since the generator matrix is quasi-cyclic, the first row of each circulant sub-matrix is stored, and successive rows can be generated by a shift register generator. The Bresnan code and Rakibul code can be compared and shown in Fig. 10.

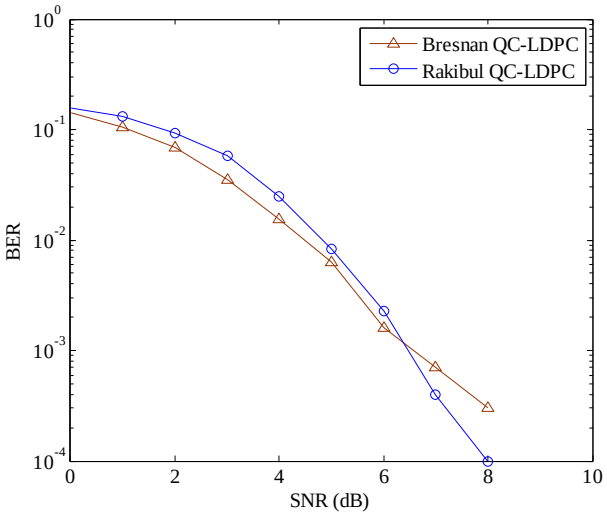

Fig. 10 Comparison between Bresnan and Rakibul QC-LDPC codes

4 Other Encoding Schemes

4.1 Encoding of QC-LDPC Codes Related to Cyclic MDS Codes

The authors in [33] presented an efficient systematic encoding algorithm for QC-LDPC codes that are related to cyclic maximum-distance separable (MDS) codes. They showed that the algebraic nature of the QC-LDPC codes related to cyclic MDS codes makes it possible to design a systematic encoding algorithm with linear time complexity. The algorithm can be easily implemented by using polynomial multiplication and division circuits. The division polynomials can be completely characterized by their zeros, and the sum of the respective numbers of their zeros will be equal to the parity-length of the codes.

The encoding procedure is shown below.

1) Input $c^{(0)}, c^{(1)}, \cdots, c^{(q)}$.

2) For $i = 0, 1, \cdots, r-1$, compute $p^{(i)}$ as $p(i) \equiv \sum_{j=0}^{q-r} c^{(j)} \cdot \psi_{q-j}^{(i)} \; mod \; (x^{q-1} - 1)$, where $\deg(p^{(i)}) < q - 1$.

3) For $i = r-1, r-2, \cdots, 1, 0$, update $c^{(q-i)}$ by $c^{(q-i)} \leftarrow c^{(q-i)} + ((c^{(q-i)} + p^{(i)}) \; mod \; \psi_I^{(i)})$, and then update $p^{(0)}, p^{(1)}, \cdots, p^{(i-1)}$ by $p^{(j)} \leftarrow \left(p^{(j)} + c^{(q-i)} \psi_I^{(i)}\right) \bmod (x^{q-1} - 1)$, where $j = 0, 1 \cdots, i-1$.

4) Output $c^{(0)}, c^{(1)}, \cdots, c^{(q)}$.

4.2 Efficient Encoding of IEEE 802.11n LDPC Codes

Given a (sparse) parity check matrix H, the goal of encoding is to compute the systematic codeword $\mathbf{c}$ from the input sequence m. Owing to the special structure of the IEEE 802.11n LDPC parity check matrices, the encoding process can be done very efficiently. The IEEE 802.11n LDPC codes are based on block-structured LDPC codes with circular block matrices [28], i.e. the entire parity check matrix can be partitioned into an array of block matrices; each block matrix is either a zero matrix or a right cyclic shift of an identity matrix. The parity check matrix designed in this way can be conveniently represented by a base (block) matrix. The base matrix $\mathrm{H_b}$ for an IEEE 802.11n LDPC code with code length $N = 1944$ bits and $rate = 1/2$ can be seen from the standard. The block size is $Z = 81$ bits with $m_b = 12$ and $n_b = 24$. In this matrix, each entry represents a circular right shift of the identity matrix $\mathrm{I_Z}$. For example if $Z = 3$ and the entry is 1, then the corresponding block is [0 1 0; 0 0 1; 1 0 0]. The -1 entry means a null (all zero) block. In this way the above matrix is a compact expression of a binary $2D[M = 12 \times 81, N = 24 \times 81]$ matrix. Note that in the above matrix there are always three non -1 elements at the $\mathrm{k_b}$th column (usually they are 1 0 1). This property holds for all 12 LDPC codes defined in IEEE 802.11n. This observation, together with the (dual) diagonal parity check sub-matrix (the right-hand side of $\mathrm{H_b}$), can be explored to encode IEEE 802.11n LDPC codes efficiently.

The input information sequence is denoted as $\mathbf{m}$ and it is divided into $k_b = n_b - m_b$ groups of $\mathbb{Z}$ bits, i.e. $m = [m_0, m_1, \ldots, m_{k_b-1}]$, where each element of $\mathbf{m}$ is a vector of length $\mathbb{Z}$. The parity sequence can also be grouped as length of $\mathbb{Z}$ bits. The codeword is denoted as

$$c_b = [mp] = \left[m_0, m_1, \ldots, m_{k_b-1}, p_0, p_1, \ldots, p_{m_b-1}\right]$$

Recall that a codeword has to satisfy $\mathrm{H_b c_b} = 0$. Expanding the above equation, the following equations hold:

$$\begin{aligned}
&\sum_{j=0}^{k_b-1} h_{0,j} m_j + \pi_1 p_0 + p_1 = 0 \ (0^{th}\ row)\\
&\sum_{j} h_{i,j} m_j + p_i + p_{i+1} = 0 \ (i \neq 0, x, m_b - 1)\\
&\sum_{j} h_{x,j} m_j + p_0 + p_x + p_{x+1} = 0 \ (x^{th}\ row)\\
&\sum_{j} h_{m_b-1,j} m_j + \pi_1 p_0 + p_{m_b-1} = 0 \ ((m_b-1)^{th}\ row)
\end{aligned} \tag{13}$$

where $\pi_1 \mathrm{p}_0$ makes p_0 circular shift 1-cycle. Adding up all the above equations, we have

$$p_0 = \sum_{i=0}^{m_b-1} \sum_{j=0}^{k_b-1} h_{i,j} m_j$$

$\lambda_i = \sum_{j=0}^{k_b-1} h_{i,j} m_j$ for $i = 0, \cdots, m_b - 1$, the above equation becomes $p_0 = \sum_{i=0}^{m_b-1} \lambda_i$. With p_0 in hand, p_1 and p_{m_b-1} can be easily obtained from (13)

$$p_1 = \lambda_0 + \pi_1 p_0$$

$$p_{m_b-1} = \lambda_{m_b-1} + \pi_1 p_0$$

Other parity sub-vectors can be solved by upward and downward recursions, according to (13). In summary, $\lambda_i = \sum_{j=0}^{k_b-1} h_{i,j} m_j$ and $\sum_{i=0}^{m_b-1} \lambda_i$ are needed to get the codeword c. Since $h_{i,j} m_j$ is nothing but a circular shift of m_j, the resource requirement is trivial.

4.3 Encoding of Array LDPC Codes

The authors in [30] started with a code that satisfies the condition $Hv^T = 0$ and therefore defined

$$C_A := \{v = (v_0, v_1, \cdots, v_{k-1}) \in F_2^{pk} \,|\, Hv^T = 0\}. \tag{14}$$

They called the code C_A an *array LDPC code*. The code C_A is a (j, k)-regular LDPC code.

The array LDPC code C_A has an algebraic characterization similar to that of RS codes. Although an RS code is defined over a finite field, the array LDPC code can be defined over a ring. Then they defined a subcode $C_A' \subset CA$ as follows:

$C_A' := \{A(z)G(z) \,|A(z) \in Rp[z]$ s.t. $\deg A(z) < k - j\}$.

$G(z)$ can be considered as a "generator polynomial" of the sub-code C_A'. Note that C_A' has length $N = pk$, dimension $K1 := p(k - j)$ and rate $R := K1/N = 1 - j/k$, which is the so-called design rate of (j, k)-regular LDPC codes. The dimension of C_A' is smaller than that of C_A by $j - 1$, but in practice j is small, e.g., $3 \leq j \leq 6$, so that the loss of the information rate is negligible.

Let $\boldsymbol{u} := (\boldsymbol{u_0}, \boldsymbol{u_1}, \ldots, \boldsymbol{u}_{k-j-1})$ be an information vector, where $\boldsymbol{u_i} = (u_{i,0}, u_{i,1}, \ldots, u_{i,p-1}) \in F_p^2, i = 0, 1, \ldots, k - j - 1$. For each $\boldsymbol{u_i}$, we define $u_i(\alpha) := \sum_{s=0}^{p-1} u_{i,s}\, \alpha s \in Rp$. First, construct the information polynomial $U(z)$ as follows:

$$U(z) = \sum_{i=0}^{k-j-1} u_i(\alpha) z^i.$$

Next, compute the residue $R(z)$ of $z^j U(z)$ modulo $G(z)$, i.e.,

$$R(z) \equiv z^j\, U(z) \bmod G(z).$$

Finally, set $V(z) := z^j\, U(z) - R(z)$. Then $V(z) \in C_A'$. Note that since the leading coefficient of $G(z)$ is 1, no divisions in the ring Rp are required. This encoding algorithm can be implemented on digital circuits.

5 Conclusion

QC-LDPC code has been the focus of interest for the last few years. Being the low complexity counterpart of the LDPC code, QC-LDPC has successfully drawn the attention of the potential researchers. Encoding in the LDPC code has been the most critical part in low complexity applications. Decoding can be performed at the fixed node and encoding is crucial for multi-hop transmission. This paper discusses several encoding techniques which may be considered for energy aware low complexity applications, such as in wireless sensor network. The future work of this paper is to develop an energy efficient encoding scheme for energy constraint wireless sensor network.

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