A Novel Method of Dynamic Balance Weighting for Single-Disk Rotor System

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Abstract: Reasons for rotating mechanical vibration are varied, while the situations on site show that the rotor mass imbalance is the main reason. A novel method of dynamic balance weighting for single-disk rotor system based on equivalent phase difference mapping is proposed. Firstly, the influence coefficient method and its characteristics are analyzed. Secondly, principle on how to measure phase by key pulse method and definition of phase are introduced, and physical meaning of phase by Discrete Fourier Transform (DFT) based on vibration signal triggered by key phase signal is analyzed in detail. Thirdly, the equivalent phase difference mapping relationship between incentive and vibration response for single-disk rotor system is proved by differential equations and Laplace transform theory. Finally, a specific application instance and procedure based on the proposed method are showed. The new proposed method is simple and easy to peel the phase coupling relationship between incentive and response, which can be used to guide dynamic balance weighting for single-disk rotor system on site.

Keywords: Dynamic Balance; Influence Coefficient; DFT; Laplace Transform; Equivalent Phase Difference Mapping; Single-Disk Rotor

1 Introduction

Rotating machinery, such as turbine, fans, pumps, electric motors, etc, may lead to vibration beyond standards, which bring the accidental shutdown and huge economy losses. Vibration causations for rotating machinery are numerous, such as rotor unbalance, oil film oscillation, rotor misalignment, random vibration, etc. While among all the reasons, rotor mass imbalance accounts for about 80%.

There are mainly two kinds of methods to do dynamic balance for rotor system: balancing method by measuring amplitude and balancing method by measuring phase. Balancing method by measuring amplitude is popular in the period without equipments to measure vibration phase, mainly including circular motion method by adding trials weight, three-point method, and two-point method, etc [1]-[2]. While the key points of balancing method by measuring phase are how to define the phase of vibration signal and how to find the phase of adding weight according to vibration signal obtained through vibration meter. On how to find weighting phase only by using the phase of vibration signal, W. C. Foiles proposed the method of only using phase information for the dynamic balancing of single-plane or multi-plane [3], which processed the vibration signals measured from the trial weight both at an arbitrary point and at its symmetric point only by sine theorem. L.Q. HUANG proposed a strategy to search phase on a circle based on constant amplitude trial force [4].

The traditional dynamic balancing technology is mainly based on influence coefficient method and modal balance method [5]-[10]. In theory, they have been very mature and widely used [13]-[21]. In these methods, influence coefficient, vibration, and weight are all vectors, initial phase zero point of which should be consistent. However, during dynamic balance realized on sites, amplitude and phase of vibrations are usually given directly by the vibration meter, while the initial phase of the vibration signal is usually combined by the rising time of key phase pulse signal, either the positive or negative peak, or positive or negative zero of vibration signal. There are many definition methods, the specific physical meanings of which are different. So far, there is no uniform definition for the initial phase of vibration signal in the world. As a result, there are some difficulties in doing dynamic balance, such as how to define the vibration phase of vibration meter and how to unify zero point of vibration and the adding weight, etc. These technical barriers have seriously affected the control and operation of the influence coefficient method and its improved algorithms in the specific implementation.

In this paper, the equivalent phase difference mapping relationship between incentive and vibration response for single-disk rotor system is proved by differential equations and Laplace transform theory. Furthermore, the detailed instance using the proposed method to perform dynamic balancing weight is given, which is compared with influence coefficient method. The analysis results show that the proposed method in this paper is simple and easy to peel the phase coupling relationship between incentive and response. There is great theoretical and practical significance to guide weight implementation of dynamic balance for single-disk rotor system on site.

2 Measuring phase of vibration signal by key pulse signal

Phase is very important for this paper, while there is no uniform definition for phase. In this section, the general definition of vibration signal phase is introduced firstly. Then the relationship between the general phase and the phase derived from the synchronous full period sampling by DFT is given.

2.1 The definition of the phase

There is special and explicit meaning of phase in rotating machinery, which is the phase difference between basic frequency vibration signal and the determined marker on the shaft. It is widely used to measure phase by key pulse when doing dynamic balance on site for its simple operation.

Measuring the phase of unbalance vibration by pulse method is to set a fixed referencing marker of phase on rotor, and install a reference sensor on the still pedestal. Therefore, the reference sensor outputs a series of pulse signals related with rotor velocity when the phase marker turns over the reference phase sensor. These pulse signals are named key phase signals (reference signals), while the reference sensor is called key phase sensor. Phase signal is obtained by comparing unbalance vibration signal and key phase signal in time domain, while the unbalance vibration signal is harmonic vibration signal which is the same frequency with the rotor. Compare vibration signal and pulse signal through certain rules and obtain the time difference between the two reference points, and then compare the time difference and the period of vibration (or pulse), with 360° as its dividing scale, turning into phase angle, and get the unbalance vibration phase [23]. It is shown in Fig.1

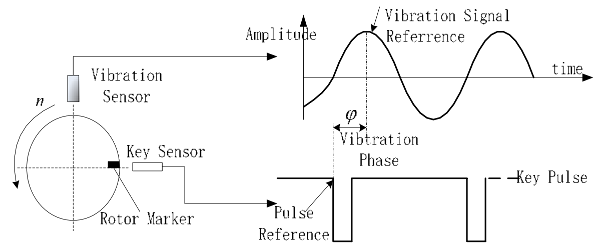


Fig. 1 The principle diagram of pulse method to measure the phase

There are many methods to set the reference points of vibration signal and key pulse signal. For key pulse signal, reference point could be defined as frontier or lagging edge. While for vibration signal, there are 4 kinds of methods: zero-crossing (the positive slope is zero or the negative slope is zero) and peak point (positive peak point or negative peak point). So there exist 8 kinds of definition methods for phase, one of which is shown as Fig. 1. There are no unified standards to set the reference points. Therefore, phase readings measured by different vibration meters could be different even if it is under the situation with the same vibration signal and the same key pulse signal. As a result, it is a hard work for operators to do dynamic balance because they need to know the phase definition of these vibration meters before using them.

2.2 The physical meaning of the phase derived the synchronous full period sampling by DFT

After the vibration signal gotten by the synchronous full period sampling, the spectrum and phase of each harmonic is available by discrete Fourier transform formula. The expansion of fourier series is shown as follows:

$$X(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t) \quad (1)$$

As to discrete period vibration signal $x(t_k)$, $k = 0,1,\cdots,n-1$, the calculating formula of a_n and b_n is shown as follows:

$$a_n = \frac{2}{N}\sum_{k=0}^{n-1} x(t_k)\cos\frac{2\pi nk}{N} \quad (2)$$

$$b_n = \frac{2}{N}\sum_{k=0}^{n-1} x(t_k)\sin\frac{2\pi nk}{N}, n = 1, 2, \cdots, N/2 \quad (3)$$

Phase of any harmonic component:

$$\varphi_n = tg^{-1}\frac{a_n}{b_n} \quad (4)$$

The meaning of the phase is that the frontier of the synchronous key pulse lags behind the first positive zero point of vibration signal in forward direction [25], as φ_n shown in Fig.2. In general engineering applications, vibration phase is usually defined as the frontier of the synchronous key pulse leading the positive peak of vibration signal, as α shown in Fig. 2, so $\alpha = \frac{\pi}{2} - \varphi_1$.

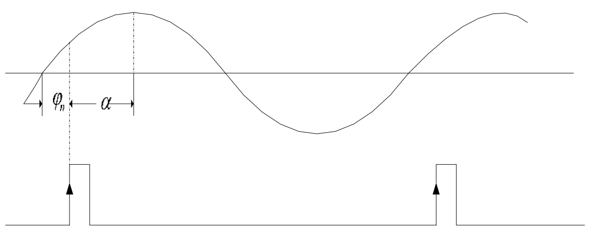


Fig. 2 Phase diagram of vibration signal triggered by key phase signal

In summary, there is difference between the phase meaning of base frequency vibration signal derived from DFT transform and that of measured by key pulse method in theory, so it needs mathematical conversion in the specific application.

3 Establishment of differential equation for vibration system with unbalanced rotor

To set up differential equation of vibration system is to establish the mathematical expression of incentive, response and characteristics. A mechanical model based on single freedom system with single-disk rotor is set up, shown as Fig.3 [23].

As shown in Fig.3, the forces of vibration system with single-disk rotor are as follows:

Spring reaction force: $F_k = -\kappa s$ the direction between spring force and vibration displacement is opposite, the size of which is proportional.

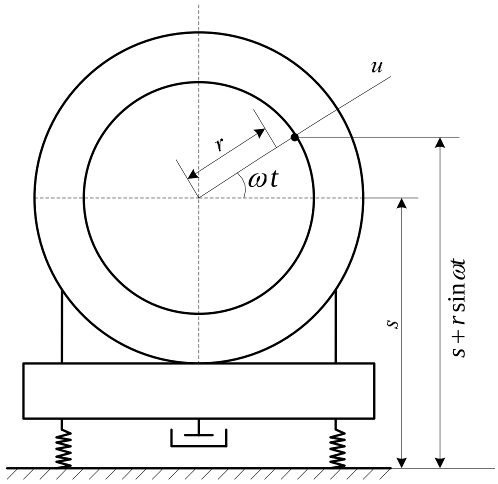


Fig. 3 Diagram of single freedom vibration system with single-disk rotor

Viscous damping force: $F_c = -Cds/dt$ the direction between damping force and velocity is opposite, the size of which is proportional.

Gravity: $W = mg$

Unbalanced harmonic excitation force:

$$P = F_o \sin(\omega t + \phi).$$

Force synthesis in the system:

$$F = F_k + F_c + W + P$$
$$= mg + F_o\sin(\omega t + \phi) - ks - Cds/dt.$$

According to basic laws of dynamics (inertia force equals mass times acceleration) $F = ma$, then:

$$mg + F_o\sin(\omega t + \phi) - ks - C\,ds/dt = ma = m\,d^2s/dt^2 \quad (5)$$

Where, k is the system supporting static stiffness; C is damping coefficient; m is mass of rotor system; S is the displacement; ds/dt is velocity; d^2s/dt^2 is acceleration; g is the acceleration of gravity; ω is the angular frequency of the driving forces; ϕ is the imbalance phase angle (the initial phase of excitation force); F_o is the amplitude of unbalanced centrifugal force, $F_o = ur\omega^2$, u is the unbalanced mass; r is the radius of unbalanced mass.

The differential equation of vibration system with unbalanced rotor is come from Formula (5):

$$m\ddot{s} + C\dot{s} + Ks = F_o\sin(\omega t + \phi) \quad (6)$$

Where, $\ddot{s} = (d^2s/dt^2 - g)$ is acceleration;

$\dot{s} = ds/dt$ is velocity.

The differential equation of vibration system with unbalanced rotor is as to a single freedom damping forced vibration system, which is the basic equation to study unbalanced vibration.

4 Relationship of equivalent phase difference mapping

In this section, on the grounds of the equation described in the previous section, an analytical general solution is given. Then the relationship of equivalent phase difference mapping is proved.

4.1 Analyzing differential equations of vibration system with unbalanced rotor

In order to analyze differential equations of vibration system with unbalanced rotor, Formula (6) is rewritten as follows:

$$m\ddot{s}(t) + C\dot{s}(t) + Ks(t) = F_o \sin(\omega t + \phi) \quad (7)$$

According to Laplace transform, regard z as independent variable of image function: [the initial value of $s(t)$ is 0, and non-zero initial value is corresponding to transient solution]

$$mz^2 X(z) + CzX(z) + KX(z) = F_o \frac{\omega\cos\phi + z\sin\phi}{z^2+\omega^2}.$$

Then $X(z) = F_o \dfrac{(\omega\cos\phi + z\sin\phi)}{(z^2+\omega^2)(mz^2+Cz+K)}$ (8)

Take the inverse transformation of Laplace:

$$s(t) = \mathrm{Res}\left[X(z)e^{zt}, \omega i\right] + \mathrm{Res}\left[X(z)e^{zt}, -\omega i\right].$$

Because their real parts are negative, the remaining poles are corresponding to the transient solutions.

$$s(t) = F_o e^{i\omega t}\frac{(\omega\cos\phi + i\omega\sin\phi)}{2\omega i\left(K - m\omega^2 + Ci\omega\right)} + F_o e^{-i\omega t}\frac{(\omega\cos\phi - i\omega\sin\phi)}{-2\omega i\left(K - m\omega^2 - Ci\omega\right)}$$

$$= F_o \,\mathrm{Im}\left[e^{i\omega t}\frac{(\cos\phi + i\sin\phi)}{\left(K - m\omega^2 + Ci\omega\right)}\right]$$

When $K - m\omega^2 > 0$, or $\omega < \sqrt{\dfrac{K}{m}}$:

$$K - m\omega^2 + Ci\omega = \sqrt{\left(K-m\omega^2\right)^2 + (C\omega)^2}\; e^{i\arctan\frac{C\omega}{K-m\omega^2}},$$

Then $s(t) = \dfrac{F_o \sin\left(\omega t + \phi - \arctan\dfrac{C\omega}{K-m\omega^2}\right)}{\sqrt{\left(K-m\omega^2\right)^2 + (C\omega)^2}}$.

When $K - m\omega^2 < 0$, or $\omega > \sqrt{\dfrac{K}{m}}$:

$$K - m\omega^2 + Ci\omega = \sqrt{\left(K-m\omega^2\right)^2 + (C\omega)^2}\; e^{i\left(\pi + \arctan\frac{C\omega}{K-m\omega^2}\right)},$$

Then

$$s(t) = \frac{F_o \sin\left(\omega t + \phi - \arctan\dfrac{C\omega}{K-m\omega^2} - \pi\right)}{\sqrt{\left(K-m\omega^2\right)^2 + (C\omega)^2}}.$$

Here, $-\pi$ can also be changed into $+\pi$, namely:

When $K - m\omega^2 > 0$, or $\omega < \sqrt{\dfrac{K}{m}}$.:

$$s(t) = \frac{F_o \sin\left(\omega t + \phi - \arctan\dfrac{C\omega}{K-m\omega^2}\right)}{\sqrt{\left(K-m\omega^2\right)^2 + (C\omega)^2}}. \quad (9)$$

When $K - m\omega^2 < 0$, or $\omega > \sqrt{\dfrac{K}{m}}$.:

$$s(t) = \frac{F_o \sin\left(\omega t + \phi - \arctan\dfrac{C\omega}{K-m\omega^2} + \pi\right)}{\sqrt{\left(K-m\omega^2\right)^2 + (C\omega)^2}} \quad (10)$$

If $\ddot{S} = (d^2s/dt^2 - g)$, simply add a special solution $\dfrac{mg}{K}$ in the above $s(t)$.

4.2 Proposal of equivalent phase difference mapping

When any two driving forces $F_o \sin(\omega t + \phi_a)$ and $F_o \sin(\omega t + \phi_b)$ applied, the corresponding vibration response is obtained:

When $K - m\omega^2 > 0$, or $\omega < \sqrt{\frac{K}{m}}$:

$$s_a(t) = \frac{F_o \sin\left(\omega t + \phi_a - \arctan\frac{C\omega}{K - m\omega^2}\right)}{\sqrt{\left(K - m\omega^2\right)^2 + \left(C\omega\right)^2}}, \quad (11)$$

$$s_b(t) = \frac{F_o \sin\left(\omega t + \phi_b - \arctan\frac{C\omega}{K - m\omega^2}\right)}{\sqrt{\left(K - m\omega^2\right)^2 + \left(C\omega\right)^2}}. \quad (12)$$

Define $\alpha_a = \phi_a - \arctan\frac{C\omega}{K - m\omega^2}$, $\alpha_b = \phi_b - \arctan\frac{C\omega}{K - m\omega^2}$.

Then $\phi_a - \alpha_a = \phi_b - \alpha_b = \arctan\frac{C\omega}{K - m\omega^2}$ (13)

Therefore: $\phi_a - \phi_b = \alpha_a - \alpha_b$ (14)

I.e.: when velocity of the rotor system is lower than the first-order critical velocity, initial phase difference of any two driving forces is equal to the phase angle difference of vibration displacement caused by the two driving forces respectively, and vice versa.

When $K - m\omega^2 < 0$, or $\omega > \sqrt{\frac{K}{m}}$:

$$s_a(t) = \frac{F_o \sin\left(\omega t + \phi_a - \arctan\frac{C\omega}{K - m\omega^2} + \pi\right)}{\sqrt{\left(K - m\omega^2\right)^2 + \left(C\omega\right)^2}},$$

$$s_b(t) = \frac{F_o \sin\left(\omega t + \phi_b - \arctan\frac{C\omega}{K - m\omega^2} + \pi\right)}{\sqrt{\left(K - m\omega^2\right)^2 + \left(C\omega\right)^2}}$$

Define $\alpha_a = \phi_a - \arctan\frac{C\omega}{K - m\omega^2} + \pi$, $\alpha_b = \phi_b - \arctan\frac{C\omega}{K - m\omega^2} + \pi$.

Then $\phi_a - \alpha_a = \phi_b - \alpha_b = \arctan\frac{C\omega}{K - m\omega^2} + \pi$.(15)

Therefore: $\phi_a - \phi_b = \alpha_a - \alpha_b$

I.e.: when velocity of the rotor system is higher than the first-order critical velocity, initial phase difference of any two driving forces is equal to the phase angle difference of vibration displacement caused by the two driving forces respectively, and vice versa.

To sum up, single freedom vibration system with single rotor-disc, whether under the first-order critical velocity (corresponding to the rigid rotor) or above the first-order critical velocity (corresponding to the flexible rotor), initial phase difference of any two driving forces is equal to the phase difference of vibration displacement caused by the two driving forces respectively, and vice versa.

5 Applications and analysis of equivalent phase difference mapping

In contrast with influence coefficient method, basic principle of equivalent phase difference mapping method is introduced in detail, the application of which is described

5.1 Basic principle of influence coefficient method

The basic principle of influence coefficient method [26] is as follows: given selected balance velocity, by adding trial weight experiments, influence coefficient of weight to vibration is obtained. Then according to the influence coefficient, the weight to eliminate the initial vibration should be calculated. In the dynamic balance process, velocity of rotor must be constant. The specific steps are as followed:

(1) Measure the vibration $\overrightarrow{A_0}$ in the original state.

(2) Measure the vibration $\overrightarrow{A_1}$ after adding trial weight $\vec{P}$ in rotor.

(3) Calculate the influence coefficient of vibration

$$\vec{\alpha} = \frac{\overrightarrow{A_1} - \overrightarrow{A_0}}{\vec{P}} \quad (16)$$

Where, $\overrightarrow{A_1} - \overrightarrow{A_0}$ ------ vibration variance whether the weigh $\vec{P}$ is added or not.

$\vec{\alpha}$ ------Influence coefficient, represents the variation of vibration caused by adding per unit mass (e.g. 1kg) at the zero direction of the rotor, and in the area of radius being 1m. For example, if using vibration displacement to do dynamic balance, its unit should be $(\mu m\angle^\circ)/(kg\angle^\circ)$.

(4) Calculate balance weight $\vec{Q}$ which should be added in rotor

$$\vec{Q}\vec{\alpha}+\vec{A_0}=\vec{0} \tag{17}$$

The physical meaning of the above equation: the vibration variance $\vec{\alpha}\vec{Q}$ caused by balance weight $\vec{Q}$ that should eliminate original vibration $\vec{A_0}$ .Then we can derive:

$$\vec{Q}=-\frac{\vec{A_0}}{\vec{\alpha}} \tag{18}$$

5.2 Basic principle of equivalent phase difference mapping method

When using equivalent phase difference mapping to do dynamic balancing for single-disk rotor system, the basic principle is as follows: on the selected balance velocity, by adding trial weight experiments, phase variation of weight to vibration is obtained. Then according to equivalent phase difference mapping, the weight of balance original vibration to be added is calculated. In dynamic balance process, velocity of rotor must be constant. The specific steps are as followed:

(1) Measure the vibration $\vec{A_0}$ .in the original state

(2) Measure the vibration $\vec{A_1}$ after adding trial weight $\vec{P}$ in rotor.

(3) Compute system vibration caused by adding $\vec{P}$ itself.

$$\vec{A}=\vec{A_1}-\vec{A_0} \tag{19}$$

Where, $\vec{A}$ is the vibration variation caused only by adding $\vec{P}$.

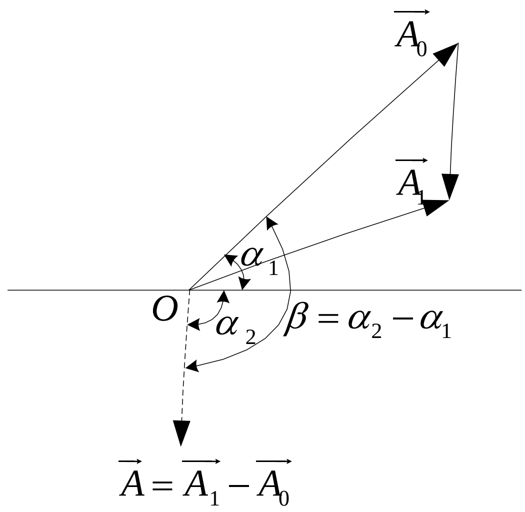
(4) Calculate the relative phase relationship between $\vec{A_0}$ and $\vec{A}$ to get the phase difference β that can show the two vibration signals being relatively leading or lagging.

(5) To balance the original vibration $\vec{A_0}$, taking the trial weight $\vec{P}$ as original point, we can move the weight with $180^\circ+\beta$.

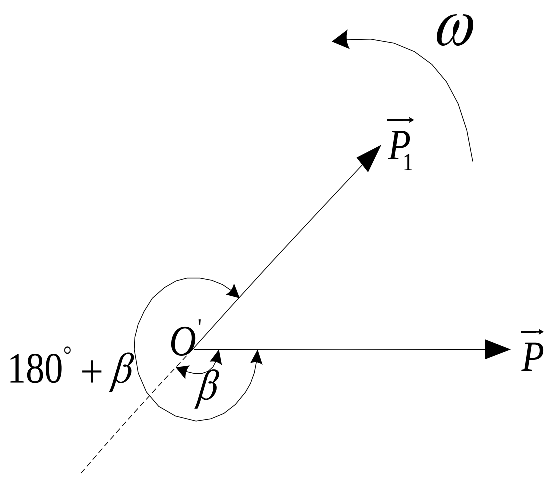
(6) According to the vibration amplitude, change weight at the point which is determined in step (5), until meet requirement of the system vibration.

5.3 Analysis

As Fig. 4 (a) and Fig. 5 (a) show, when doing dynamic balance on site, firstly, we can measure amplitude and phase of the original vibration signal $\vec{A_0}$ by any vibration meter, denoting the phase as α_1 . Secondly, under the same system conditions, amplitude and phase of the vibration signal $\vec{A_1}$ is measured after adding trial weight $\vec{P}$. Thirdly, using parallelogram law of vectors, we can calculate the amplitude and phase of system response $\vec{A}=\vec{A_1}-\vec{A_0}$ caused only by adding trial weight $\vec{P}$, and denote the phase of $\vec{A}$ as α_2 .



(a) Diagram of vibration signal



(b) Diagram of weight exerted method

Fig. 4 Application diagram of equivalent phase difference mapping when $\beta<0$

Finally, calculate phase difference $\beta = \alpha_2 - \alpha_1$ caused by $\vec{A}$ and $\vec{A_0}$. So far, it is conveniently to get the phase difference value between trial weight $\vec{P}$ and original system unbalance mass without considering how to define the initial phase of vibration signal which measured by vibration meter.

After getting the phase difference $\beta = \alpha_2 - \alpha_1$ between vibration $\vec{A}$ and $\vec{A_0}$, shown as Fig.4 (b) and Fig.5 (b), we can determine special location of weight $\vec{P_1}$ that should be added to balance the original vibration $\vec{A_0}$ according to equivalent phase difference mapping. Firstly, we regard the location of trial weight $\vec{P}$ as relative zero phase of weight $\vec{P_1}$.

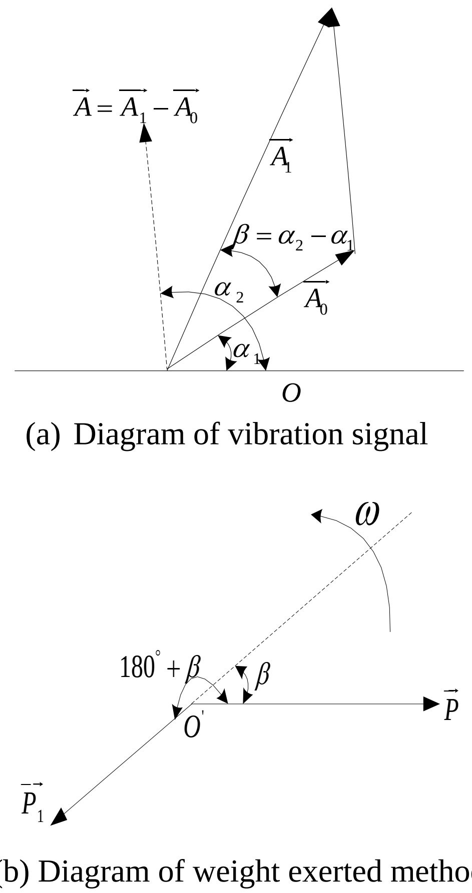


Fig. 5 Application diagram of equivalent phase difference mapping when $\beta > 0$

Secondly, if $\beta < 0$, it indicates the location of trial weight $\vec{P}$ leads over the need add weight $\vec{P_1}$. Therefore, in order to reduce or balance the original system unbalance, we need add weight $\vec{P_1}$ with $180° + \beta$ in the opposite direction of rotor rotation, as shown in Fig. 4(b). Finally, if $\beta > 0$, it indicates the location of weight $\vec{P}$ lags behind the original system unbalance. Therefore, in order to reduce or balance the original system unbalance, we need add weight $\vec{P_1}$ with $180° + \beta$ in the same direction of rotor rotation, as shown in Fig. 5(b).

Above all, we can know explicitly that dynamic balance weight method for single-disc rotor system based on equivalent phase difference mapping is simple and easy to operate. It shields effectively technical barriers required in the influence coefficient method that the initial phase zero point of influence coefficient, vibration, and weight must be the same point.

6 Verification experiments based on incentive to response

In order to further verify equivalent phase difference mapping relationship, we did relevant verification experiments on the Bently vibration system. The experimental system is shown in Fig. 6. We selected bearing pedestal close to driving motor as measuring point 1#, the other one away from the driving motor as measuring point 2#, Disc rotor close to the motor as rotor 1 #, the other one away from the motor as rotor 2 #. The first order critical speed of system is about 2100rpm. Using Bently displacement sensor, measure key phase signal and vibration displacement signal of measuring point 1# and 2# in the horizontal and vertical direction respectively. From the non-drive end, shaft rotation direction is clockwise. All the adding weights were on rotor 1 #, and the weight was a 2.5g standard screw. And from the position 180°, the trial mass was added on six weight holes by even distribution of which the interval is 22.5°. Two groups of vibration data were recorded at 1813.09rpm and 2801.71rpm, shown as Table 1 and Table 2.

As shown in Table 1 and Table 2, Synthetic vibration is caused by resultant force of the original system unbalance and each adding weight, and net value is the response of system vibration only caused by adding weight each time. Difference value is the phase difference of the system vibrations caused by two adjacent adding weights. Error $\zeta = \frac{22.5 - Phase\ difference}{22.5} \cdot \frac{100}{100}$. we can clearly draw a conclusion that when we adding balance weights on the single-plane of vibration system , initial phase difference of any two driving forces is equal to the phase difference of vibration displacement caused by the two driving forces respectively, and vice versa.

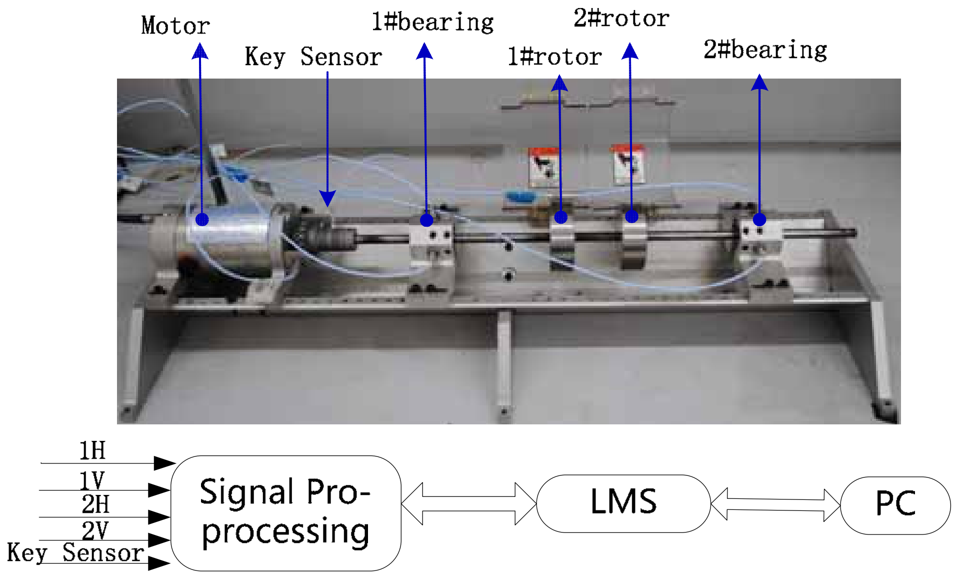


Fig. 6 Bently vibration system

Table 1 Vibration data in horizontal direction under 1813.09rpm

Phase	2H Comprehensive		2H Net value		Difference value	error %
	amplitude (um)	phase(°)	amplitude (um)	phase(°)		
Initial state	5.6000	-1.2700				
180	5.6900	163.1700	11.1861	170.8876		
202.5	5.6100	-154.0300	10.8948	-167.6376	21.4748	4.5564
225	6.9100	-117.2800	10.6323	-145.5320	22.1056	1.7529
247.5	8.9500	-90.2800	10.4752	-122.5910	22.9410	-1.9600
270	11.1400	-69.5500	10.454	-99.3945	23.1965	-3.0751
292.5	13.0600	-52.3100	10.4858	-76.8467	22.5478	-0.2124

Table 2 Vibration data in vertical direction under 2801.71rpm

Phase	2V Comprehensive		2V Net value		Difference value	error %
	amplitude (um)	phase(°)	amplitude (um)	phase(°)		
Initial state	3.5800	108.57				
180	18.9000	-55.59	22.3654	-58.0941		
202.5	19.4600	-29.82	22.2640	-35.9497	22.1444	1.5804
225	20.4000	-6.13	22.1362	-14.5790	21.3707	5.0191
247.5	20.6300	17.04	21.0323	7.2432	21.8222	3.0124
270	21.0500	38.83	20.0930	29.2079	21.9647	2.3791
292.5	21.2800	59.60	19.1216	51.4807	22.2728	1.0098

7 Conclusion

Shaft balancing method is directly related to its start-up, shutdown and balancing effect. Influence coefficient method requires that the starting phase zero point of influence coefficient, vibration and weight should be the same point, which makes it difficult to grasp and operate in the practical engineering applications.

In this paper, the equivalent phase difference mapping relationship between incentive and vibration response for single-disk rotor system is proved by differential equations of single-disk rotor and Laplace transform theory. The example for doing dynamic balance by influence coefficient method and equivalent phase difference mapping respectively shows that the latter divides the vibration and weight with consistent leading or lagging relationship into two different coordinate systems. The proposed method successfully overcomes technical barriers in the influence coefficient method that initial phase zero point of influence coefficient, vibration, and weight must be the same point. There is great theoretical and practical significance to guide weight implementation of dynamic balance for single-disk rotor system on site.

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