Guaranteed Cost Control of Uncertain T-S Fuzzy Systems via Output Feedback Approach

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Abstract: This paper focuses on the problem of Guaranteed cost control of uncertain T-S fuzzy systems. Based on observer design, the guaranteed cost output feedback control law which guarantees that the closed-loop uncertain T-S fuzzy system is robustly asymptotically stable is proposed. By utilizing the Lyapunov function together with the linear matrix inequality (LMI) approach and free weighting matrix method, some sufficient conditions are obtained, which guarantee the asymptotic stability of uncertain T-S fuzzy systems. A numerical example is included to show the proposed method is effective and can provide less conservative results.

Keywords: Guaranteed cost control; Robustly asymptotically stable; Takagi-Sugeno (T-S) fuzzy system; Linear matrix inequality (LMI); Uncertain system

1. Introduction

For the last few decades, a fuzzy system has played an important role in nonlinear control and analysis because Takagi-Sugeno fuzzy system is a universal approximation for nonlinear systems. In face, nonlinear control design and analysis based on Takagi-Sugeno fuzzy systems have given successful results; see, for instance, [1-8] and the references therein. It is known that, by using the T–S fuzzy model, this class of nonlinear systems can be described as a weighted sum of some simple linear subsystems which are easily analyzed. The stability and stabilization problem for nonlinear systems in T-S fuzzy model has been studied extensively. In [9-12], some results on the stability analysis and stabilization synthesis for T–S fuzzy systems were obtained; while the H_∞ fuzzy control problem was investigated in [13-15]. When parametric uncertainties appear in a T–S fuzzy system, the robust stability problem was addressed in [16], where the stability conditions were expressed in terms of LMI. Sufficient conditions for the solvability of the robust H_∞ fuzzy control problem for uncertain T–S fuzzy systems were proposed in [17,18] by using the LMI-based approach, respectively.

In the domain of controller design, it is often of primary interest to synthesize a controller to satisfy certain performance function in addition to ensuring stability. Several control schemes were proposed based on this idea of performance function. Chang and Peng [19] introduced the idea of guaranteed cost control for systems with uncertain parameters. This approach provides a control law that guarantees an upper bound for the system performance functional. Chen and Liu [20] considered the fuzzy guaranteed cost control design problem for nonlinear systems with time-varying delay. They derived the sufficient conditions for construction of a guaranteed cost controller via state feedback and observer based output feedback. In [21], the polynomial matrix inequality (PMI) arising from static output feedback was solved but this approach must be extended for control problems which need to satisfy some performance criteria. The recent results related to fuzzy guaranteed cost control are based on LMI optimization method which can be handled efficiently with interior point methods [22]. Linear matrix inequality based conditions for guaranteed cost control of uncertain systems were presented which were further improved in [23]. Free weighting matrix method has been employed for linear time-delay systems in [24].

In this paper, we consider the robust guaranteed cost control for uncertain Takagi–Sugeno fuzzy systems. A guaranteed cost control makes a quadratic cost criterion less than a given finite scalar, and leads to stabilization with a satisfactory control performance. The key technique to obtain such generalized conditions is an appropriate selection of Lyapunov function, and design and analysis of the guaranteed cost controller. By using generalized inequality method and free weighting matrices technique, some sufficient conditions for the asymptotical stability and guaranteed cost performance of uncertain T-S fuzzy systems are given. We also provide a numerical example to demonstrate the effectiveness and applicability of the proposed results.

This paper is organized as follows. Section 2 reviews T–S fuzzy models and describes the tracking control problem and the formulation of the fuzzy guaranteed cost control approach. In Section 3, LMI based design procedure is proposed for the output feedback guaranteed cost control problem. The output feedback controller designs for robust stabilization of T–S fuzzy systems with parametric uncertainties also are presented. Simulation result is presented in Section 4. Section 5 contains our conclusion.

Notations. Throughout this paper, for real symmetric matrices X and Y, $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). R^n denotes the $n-$dimensional Euclidean space and $R^{m\times n}$ denotes the set of all $m \times n$ real matrices. I is an identity matrix with appropriate dimension. The superscript "T" represents the transpose of a matrix. The notation "*" is used as an ellipsis for terms that are induced by symmetry.

2. Preliminaries

In this section, we introduce Takagi–Sugeno fuzzy systems. Consider nonlinear system with parametric uncertainties represented by T-S fuzzy model as follow.

Plant Rule i:

IF $\theta_1(t)$ is F_{i1} and $\cdots$ $\theta_p(t)$ is F_{ip},

THEN

$$\begin{aligned} &\dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + w(t), \\ &y = C_i x(t) + v(t), \qquad i = 1, 2, 3, \cdots, r, \end{aligned} \tag{1}$$

where F_{ij} are fuzzy set and r is the number of IF-THEN rules, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $w(t)$ is bounded external disturbance, $v(t)$ is the measurement noise, $A_i \in R^{n\times n}$ and $B_i \in R^{n\times n}$ are system matrices, and $\theta_1(t), \cdots, \theta_p(t)$ are the premise variables. The matrices $\Delta A_i, \Delta B_i$ denote the uncertainties in system and they are of the form

$$\Delta A_i(t) = H_i F_i(t) E_{1i}, \quad \Delta B_i(t) = H_i F_i(t) E_{2i},$$

where H_i, E_{1i} and E_{2i} are known constant matrices and $F_i(t)$ is an unknown matrix function with the property

$$F^T{}_i(t) F_i(t) \leq I. \tag{2}$$

The final output of the fuzzy system is inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \lambda_i(\theta(t))\{(A_i + \Delta A_i)x(t) \\ &\quad + (B_i + \Delta B_i)u(t)\} + w(t), \\ y(t) &= \sum_{i}^{r} \lambda_i(\theta(t)) C_i x(t) + v(t), \end{aligned} \tag{3}$$

where

$$\lambda_i(\theta(t)) = \frac{\beta_i(\theta(t))}{\sum_{i=1}^{r}\beta_i(\theta(t))}, \beta_i(\theta(t)) = \prod_{j=1}^{p} F_{ij}(\theta_j(t)),$$

in which $F_{ij}(\theta(t))$ is the degree of the membership of θ_j in F_{ij}.

In this paper, we assure that:

$$\beta_i(\theta(t)) \geq 0, \qquad \sum_{i=1}^{r}\beta_i(\theta(t)) > 0,$$
$$\lambda_i(\theta(t)) \geq 0, \qquad \sum_{i=1}^{r}\lambda_i(\theta(t)) = 1, \qquad i = 1, 2, \cdots, r.$$

Consider a reference model as follows:

$$\dot{x}_r = A_r x_r(t) + r(t),$$

where $x_r(t)$ is the reference state, A_r is specific asymptotically stable matrix, and $r(t)$ is a bounded reference input.

We define the cost function

$$J = \int_{t_0}^{t_f} \{(x(t) - x_r(t))^T Q(x(t) - x_r(t)) + u^T(t) R u(t)\} dt, \tag{4}$$

where t_0 and t_f are the initial and terminal time of control respectively, and $Q > 0$ $R > 0$. Associated with the cost (4), the fuzzy guaranteed cost control is defined as follows.

If there exists a fuzzy control law $u(t)$ and a scalar $J_0 > 0$ such that the closed-loop system is asymptotically stable and the closed-loop value of the cost function (4) satisfies $J \leq J_0$, then J_0 is said to be a guaranteed cost and the control law $u(t)$ is said to be a guaranteed cost control law for system (3).

Suppose the following fuzzy observer is proposed to deal with the state estimation of nonlinear system.

Observer Rule i:

IF $\theta_1(t)$ is F_{i1} and, $\cdots$, $\theta_p(t)$ is F_{ip},

THEN

$$\begin{aligned} \dot{\hat{x}}(t) &= A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t)), \\ \hat{y}(t) &= C_i\hat{x}(t), \qquad i = 1, 2, \cdots, r, \end{aligned} \tag{5}$$

where L_i denotes the designed observer gain. Note that premise variables do not depend on the state variables estimated by a fuzzy observer.

The overall fuzzy observer is represented as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{r}\lambda_i(\theta(t))\{A_i\hat{x}(t) + B_i u(t) \\ &\quad + L_i(y(t) - \hat{y}(t))\}, \\ \hat{y} &= \sum_{i=1}^{r}\lambda(\theta(t))C_i\hat{x}(t). \end{aligned} \tag{6}$$

To stabilize this class of systems, we define the observer-based controller as follows:

Controller Rule i:

IF $\theta_1(t)$ is F_{i1} and, $\cdots$, $\theta_p(t)$ is F_{ip} ,

THEN

$$u(t) = K_j(\hat{x}(t) - x_r(t)), \qquad i = 1, 2, \cdots, r, \tag{7}$$

where K_j denotes the designed controller gain. The overall fuzzy controller is given by

$$u(t) = \sum_{i=1}^{r}\lambda_j(\theta(t))\{K_j(\hat{x}(t) - x_r(t))\}. \tag{8}$$

Let us denote the estimation error by:

$$e(t) = x(t) - \hat{x}(t). \tag{9}$$

Differentiation of $e(t)$ with respect to t yields

$$\begin{aligned} \dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= \sum_{i=1}^{r}\lambda_i(\theta(t))\{(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)\} \\ &\quad - \sum_{i=1}^{r}\lambda_i(\theta(t))\{A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\} \\ &\quad + w(t) \\ &= \sum_{i=1}^{r}\lambda_i(\theta(t))\{(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \\ &\quad - A_i\hat{x}(t) - B_i u(t)\} + v(t) - \sum_{j=1}^{r}\lambda(\theta(t))C_j\hat{x}(t)) \end{aligned}$$

$$
\begin{aligned}
&-\sum_{i=1}^{r}\lambda_i(\theta(t))L_i(\sum_{j=1}^{r}\lambda_j(\theta(t))C_j x(t)+w(t)\\
&=\sum_{i=1}^{r}\lambda_i(\theta(t))\{A_i e(t)+\Delta A_i x(t)+\Delta B_i u(t)\}\\
&\quad-\sum_{i=1}^{r}\lambda_i(\theta(t))L_i(\sum_{j=1}^{r}\lambda_j(\theta(t))C_j e(t)+v(t))\\
&\quad+w(t)\\
&=\sum_{i=1}^{r}\lambda_i(\theta(t))\{A_i e(t)+\Delta A_i x(t)\\
&\quad+\Delta B_i\sum_{j=1}^{r}\lambda_j(\theta(t))\{K_j(\hat{x}(t)-x_r(t))\}\\
&\quad-\sum_{i=1}^{r}\lambda_i(\theta(t))L_i(\sum_{j=1}^{r}\lambda_j(\theta(t))C_j e(t)+v(t))\\
&\quad+w(t)\\
&=\sum_{i=1}^{r}\lambda_i(\theta(t))\sum_{j=1}^{r}\lambda_j(\theta(t))\{A_i e(t)+\Delta A_i x(t)\\
&\quad+\Delta B_i(K_j(x(t)-e(t)-x_r(t)))-L_i(C_j e(t)+v(t))\}\\
&\quad+w(t)\\
&=\sum_{i=1}^{r}\lambda_i(\theta(t))\sum_{j=1}^{r}\lambda_j(\theta(t))\{(A_i-\Delta B_i K_j-L_i C_j)e(t)\\
&\quad+(\Delta A_i+\Delta B_i K_j)x(t)-\Delta B_i K_j x_r(t)-L_i v(t)\}\\
&\quad+w(t).
\end{aligned}
$$

By combining (3), (6), (8) and (9), the resulting multiple systems can be rewritten in the following form:

$$\dot{\tilde{x}}=\sum_{i=1}^{r}\lambda_i(\theta(t))\sum_{j=1}^{r}\lambda_j(\theta(t))\{\tilde{A}_{ij}\tilde{x}(t)+\tilde{B}_i\tilde{w}(t)\},\qquad (10)$$

where

$$\tilde{A}_{ij}=\begin{pmatrix}\tilde{a}_{ij} & \Delta A_i+\Delta B_i K_j & -\Delta B_i K_j\\ \tilde{c}_{ij} & \tilde{b}_{ij} & \tilde{c}_{ij}\\ 0 & 0 & A_r\end{pmatrix},\qquad (11)$$

$$\tilde{B}_i=\begin{pmatrix}-L_i & I & 0\\ 0 & I & 0\\ 0 & 0 & I\end{pmatrix},\qquad \tilde{x}(t)=\begin{pmatrix}e(t)\\ x(t)\\ x_r(t)\end{pmatrix},$$

$$\tilde{w}(t)=\begin{pmatrix}v(t)\\ w(t)\\ r(t)\end{pmatrix},\qquad (12)$$

and

$$
\begin{aligned}
\tilde{a}_{ij}&=A_i-\Delta B_i K_j-L_i C_j,\\
\tilde{b}_{ij}&=(A_i+\Delta A_i)+(B_i+\Delta B_i)K_j,\\
\tilde{c}_{ij}&=-(B_i+\Delta B_i)K_j.
\end{aligned}
$$

Before stating our main results, the following lemma is first presented, which will be used in the proofs of our results.

Lemma 1. [25] Given matrices A, H, E and $R>0$ with appropriate dimensions, as well as for any scalar $\varepsilon>0$. Let $F(t)$ be of appropriate dimensions, and satisfying the property $F^T(t)F(t)\le I$. Then, the following hold:

$$HF(t)E+E^T F^T(t)E^T\le\varepsilon HH^T+\varepsilon^{-1}E^T E.$$

3. Main results

In this section, the problem of robust stability and the guaranteed cost control is first investigated for T–S fuzzy systems with uncertainties. The stabilization conditions are proposed.

3.1. Robust stability analysis

In this subsection, to present a stability criterion for system (3) we assure that the feedback gain matrices $K_j(i,j=1,2,...,r)$ and the observer gain matrices $L_i(i,j=1,2,...,r)$ have been well defined.

Set $J_0=\tilde{x}^T(0)P\tilde{x}(0)+\gamma^2\int_0^{t_f}\tilde{w}^T(t)\tilde{w}(t)\,dt.$

For the stability analysis of (3) we have the following result.

Theorem 1. Given matrices $Q>0$ and $R>0$ with appropriate dimensions, the closed-loop fuzzy system (3) is robustly asymptotically stable with guaranteed cost bound J_0 if there exist positive definite symmetric matrices $P>0$, $P_1>0$ with appropriate dimensions, and scalars $\varepsilon_i>0$ and $\gamma>0$ such that the following *LMI* hold for $i, j = 1, 2, ..., r$:

$$\begin{pmatrix} \Theta_{ij} + \tilde{Q} & \Gamma_{ij} & \tilde{K}_j^T \\ * & \Psi_i & 0 \\ * & * & -R^{-1} \end{pmatrix} < 0, \tag{13}$$

where

$$\Theta_{ij} = \begin{pmatrix} \Omega_{11ij} & \tilde{P}\tilde{B}_i & \Omega_{13ij} \\ * & -\frac{1}{2}\gamma^2 I & \tilde{B}_i^T \tilde{P}_1 \\ * & * & \Omega_{33} \end{pmatrix},$$

$$\Gamma_{ij} = \begin{pmatrix} \Gamma_{11ij} & 0 & 0 \\ 0 & 0 & 0 \\ \Gamma_{31ij} & 0 & 0 \end{pmatrix}, \quad \tilde{Q} = \begin{pmatrix} \tilde{Q}_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Psi_i = diag(-\frac{\varepsilon_i^{-1}}{2} I \quad -\frac{\varepsilon_i^{-1}}{2} I \quad -I \quad -I \quad -I \quad -I \quad -I \quad -I \quad -I),$$

$$\tilde{K}_j = \begin{pmatrix} K_j & -K_j & K_j & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma_{11ij} = \begin{pmatrix} PH_i & PH_i & 0 \\ PH_i & PH_i & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma_{31ij} = \begin{pmatrix} P_1 H_i & P_1 H_i & 0 \\ P_1 H_i & P_1 H_i & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Omega_{11ij} = \begin{pmatrix} \phi_{11ij} & \phi_{12ij} & \phi_{13ij} \\ * & \phi_{22ij} & \phi_{23ij} \\ * & * & \phi_{33ij} \end{pmatrix},$$

$$\Omega_{13ij} = \begin{pmatrix} (A_i - L_i C_j)^T P_1 & -(B_i K_j)^T P_1 & 0 \\ 0 & (A_i + B_i K_j)^T P_1 & 0 \\ 0 & -(B_i K_j)^T P_1 & A_r^T P_1 \end{pmatrix},$$

$$\Omega_{33} = \begin{pmatrix} -2P_1 & 0 & 0 \\ 0 & -2P_1 & 0 \\ 0 & 0 & -2P_1 \end{pmatrix},$$

$$Q_{11} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & Q & -Q \\ 0 & -Q & Q \end{pmatrix},$$

$$\phi_{11ij} = P(A_i - L_i C_j) + (A_i - L_i C_j)^T P + \varepsilon_i^{-1} K_j^T E_{2i}^T E_{2i} K_j,$$

$$\phi_{12ij} = -(B_i K_j)^T P - \varepsilon_i^{-1} K_j^T E_{2i}^T (E_{1i} + E_{2i} K_j),$$

$$\phi_{13ij} = \varepsilon_i^{-1} K_j^T E_{2i}^T E_{2i} K_j,$$

$$\phi_{22ij} = P(A_i + B_i K_j) + (A_i + B_i K_j)^T P + \varepsilon_i^{-1} (E_{1i} + E_{2i} K_j)^T (E_{1i} + E_{2i} K_j),$$

$$\phi_{23ij} = -PB_i K_j - \varepsilon_i^{-1} (E_{1i} + E_{2i} K_j)^T E_{2i} K_j,$$

$$\phi_{33ij} = PA_r + A_r^T P + \varepsilon_i^{-1} K_j^T E_{2i}^T E_{2i} K_j,$$

and the guaranteed cost bound is described by:

$$J_0 = \tilde{x}^T(0) P \tilde{x}(0) + \gamma^2 \int_0^{t_f} \tilde{w}^T(t) \tilde{w}(t)\, dt.$$

Proof. Consider a Lyapunov function candidate as $V = \tilde{x}^T(t) \tilde{P} \tilde{x}(t)$, where

$$\tilde{P} = diag\begin{pmatrix} P & P & P \end{pmatrix}.$$

The time derivative of V is expressed as

$$\begin{aligned} \dot{V}(t) &= \dot{\tilde{x}}^T(t) \tilde{P} \tilde{x}(t) + \tilde{x}^T(t) \tilde{P} \dot{\tilde{x}}(t) \\ &= \sum_{i=1}^r \lambda_i(\theta(t)) \sum_{j=1}^r \lambda_j(\theta(t)) \{\tilde{A}_{ij} \tilde{x}(t) + \tilde{B}_i \tilde{w}(t)\}^T \tilde{P} \tilde{x}(t) \\ &\quad + \tilde{x}^T(t) \tilde{P} \sum_{i=1}^r \lambda_i(\theta(t)) \sum_{j=1}^r \lambda_j(\theta(t)) \{\tilde{A}_{ij} \tilde{x}(t) + \tilde{B}_i \tilde{w}(t)\} \\ &= \sum_{i=1}^r \lambda_i(\theta(t)) \sum_{j=1}^r \lambda_j(\theta(t)) \{\tilde{x}^T(t)(\tilde{A}_{ij}^T \tilde{P} + \tilde{P} \tilde{A}_{ij}) \tilde{x}(t) \\ &\quad + (\tilde{B}_i \tilde{w}(t))^T \tilde{P} \tilde{x}(t) + \tilde{x}^T(t) \tilde{P} \tilde{B}_i \tilde{w}(t)\} \\ &\le \sum_{i=1}^r \lambda_i(\theta(t)) \sum_{j=1}^r \lambda_j(\theta(t)) \{\tilde{x}^T(t)(\tilde{A}_{ij}^T \tilde{P} + \tilde{P} \tilde{A}_{ij}) \tilde{x}(t) \\ &\quad + \tilde{w}^T(t) \tilde{B}_i^T \tilde{P} \tilde{x}(t) + (\tilde{B}_i^T \tilde{P} \tilde{x}(t))^T \tilde{w}(t) \\ &\quad + \frac{1}{2} \gamma^2 \tilde{w}^T(t) \tilde{w}(t)\}. \end{aligned}$$

It can also be verified that

$$\dot{V}(t) \le \sum_{i=1}^r \lambda_i(\theta(t)) \sum_{j=1}^r \lambda_j(\theta(t)) \tilde{\xi}^T(t) \Lambda_{ij} \tilde{\xi}(t) + \gamma^2 \tilde{w}^T(t) \tilde{w}(t), \tag{14}$$

where

$$\Lambda_{ij} = \begin{pmatrix} \tilde{A}_{ij}^T \tilde{P} + \tilde{P}\tilde{A}_{ij} & \tilde{P}\tilde{B}_i & 0 \\ * & -\frac{1}{2}\gamma^2 I & 0 \\ * & * & 0 \end{pmatrix},$$

$$\tilde{\xi}(t) = (\tilde{x}^T(t), \tilde{w}^T(t), \dot{\tilde{x}}^T(t))^T.$$

From (14), it is clearly seen that in order to get $\dot{V} < 0$ the free-weighting matrix will be used. It follows from (10) that for any matrix

$$\tilde{P}_1 = diag\begin{pmatrix} P_1 & P_1 & P_1 \end{pmatrix} > 0,$$

with appropriate dimensions the following equality is true

$$\sum_{i=1}^{r} \lambda_i(\theta(t)) \sum_{j=1}^{r} \lambda_j(\theta(t)) 2(\tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_i \tilde{w}(t))^T \tilde{P}_1 \dot{\tilde{x}} - 2\dot{\tilde{x}}^T \tilde{P}_1 \dot{\tilde{x}} = 0. \tag{15}$$

Furthermore, putting (12) and (13) together yields

$$\dot{V}(t) \le \sum_{i=1}^{r} \lambda_i(\theta(t)) \sum_{j=1}^{r} \lambda_j(\theta(t)) \tilde{\xi}^T(t) \bar{\Lambda}_{ij} \tilde{\xi}(t) + \gamma^2 \tilde{w}^T(t)\tilde{w}(t), \tag{16}$$

where

$$\bar{\Lambda}_{ij} = \begin{pmatrix} \tilde{A}_{ij}^T \tilde{P} + \tilde{P}\tilde{A}_{ij} & \tilde{P}\tilde{B}_i & \tilde{A}_{ij}^T \tilde{P}_1 \\ * & -\frac{1}{2}\gamma^2 I & \tilde{B}_i^T \tilde{P}_1 \\ * & * & -2\tilde{P}_1 \end{pmatrix}.$$

From (11) and (12), (16) can be expressed follows:

$$\dot{V}(t) \le \sum_{i=1}^{r} \lambda_i(\theta(t)) \sum_{j=1}^{r} \lambda_j(\theta(t)) \{\tilde{\xi}^T(t)[\Pi + \Delta\Pi]\tilde{\xi}(t)\} + \gamma^2 \tilde{w}^T(t)\tilde{w}(t), \tag{17}$$

where

$$\Pi = \begin{pmatrix} \Phi_{11ij} & \tilde{P}\tilde{B}_i & \Phi_{13ij} \\ * & -\frac{\gamma^2}{2} I & \tilde{B}_i^T \tilde{P}_1 \\ * & * & -2\tilde{P}_1 \end{pmatrix},$$

$$\Delta\Pi = \begin{pmatrix} \Delta\Phi_{11ij} & 0 & \Delta\Phi_{13ij} \\ * & 0 & 0 \\ * & * & 0 \end{pmatrix},$$

$$\Phi_{11ij} = \begin{pmatrix} \alpha_{ij} & -(B_i K_j)^T P & 0 \\ * & \beta_{ij} & -PB_i K_j \\ * & * & PA_r + A_r^T P \end{pmatrix},$$

$$\Phi_{13ij} = \begin{pmatrix} (A_i - L_i C_j)^T P_1 & -(B_i K_j)^T P_1 & 0 \\ 0 & (A_i + B_i K_j)^T P_1 & 0 \\ 0 & -(B_i K_j)^T P_1 & A_r^T P_1 \end{pmatrix},$$

$$\Delta\Phi_{11ij} = \begin{pmatrix} \Delta\alpha_{ij} & \Delta\chi_{ij} & -P\Delta B_i K_j \\ * & \Delta\beta_{ij} & -P\Delta B_i K_j \\ * & * & 0 \end{pmatrix},$$

$$\Delta\Phi_{13ij} = \begin{pmatrix} -(\Delta B_i K_j)^T P_1 & -(\Delta B_i K_j)^T P_1 & 0 \\ \Delta\delta_{ij} & \Delta\delta_{ij} & 0 \\ -(\Delta B_i K_j)^T P_1 & -(\Delta B_i K_j)^T P_1 & 0 \end{pmatrix},$$

$$\alpha_{ij} = P(A_i - L_i C_j) + (A_i - L_i C_j)^T P,$$
$$\beta_{ij} = (A_i + B_i K_j)^T P + P(A_i + B_i K_j),$$

$$\Delta\alpha_{ij} = -P\Delta B_i K_j + (\Delta B_i K_j)^T P,$$
$$\Delta\beta_{ij} = P(\Delta A_i + \Delta B_i K_j) + (\Delta A_i + \Delta B_i K_j)^T P,$$
$$\Delta\chi_{ij} = P(\Delta A_i + \Delta B_i K_j) - (\Delta B_i K_j)^T P,$$

$$\Delta\delta_{ij} = (\Delta A_i + \Delta B_i K_j)^T P_1.$$

Note that

$$\Delta\Pi = M^T H_i F_i(t) N + N^T F_i^T(t) H_i^T M,$$

where

$$M = \begin{pmatrix} P & P & 0 & 0 & 0 & 0 & P_1 & P_1 & 0 \end{pmatrix},$$

$$N_{ij} = \begin{pmatrix} -E_{2i}K_j & E_{1i} + E_{2i}K_j & -E_{2i}K_j & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Using Lemma 1, we have

$$\Delta\Pi \le \varepsilon_i M^T H_i H_i^T M + \varepsilon_i^{-1} N_{ij}^T N_{ij}$$

$$= \begin{pmatrix} \eta_{ij} & \eta_{ij} & 0 & 0 & 0 & 0 & \vartheta_{ij} & \vartheta_{ij} & 0 \\ * & \eta_{ij} & 0 & 0 & 0 & 0 & \vartheta_{ij} & \vartheta_{ij} & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \upsilon_{ij} & \upsilon_{ij} & 0 \\ * & * & * & * & * & * & * & \upsilon_{ij} & 0 \\ * & * & * & * & * & * & * & * & 0 \end{pmatrix} + \begin{pmatrix} \rho_{ij} & \varsigma_{ij} & \rho_{ij} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \sigma_{ij} & \varsigma_{ij}^T & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \rho_{ij} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 \end{pmatrix}, \quad (18)$$

where

$$\eta_{ij} = \varepsilon_i P H_i H_i^T P,$$
$$\vartheta_{ij} = \varepsilon_i P H_i H_i^T P_1,$$
$$\upsilon_{ij} = \varepsilon_i P_1 H_i H_i^T P_1,$$
$$\rho_{ij} = \varepsilon_i^{-1} K_j^T E_{2i}^T E_{2i} K_j,$$
$$\varsigma_{ij} = -\varepsilon_i^{-1} K_j^T E_{2i}^T (E_{1i} + E_{2i} K_j),$$
$$\sigma_{ij} = \varepsilon_i^{-1} (E_{1i} + E_{2i} K_j)^T (E_{1i} + E_{2i} K_j),$$

where $\varepsilon_i > 0$ is constant.

Now, substituting (18) into (17) produces

$$\dot{V}(t) \le \sum_{i=1}^{r} \lambda_i(\theta(t)) \sum_{j=1}^{r} \lambda_j(\theta(t)) \tilde{\xi}^T(t) \Xi_{ij} \tilde{\xi}(t) + \gamma^2 \tilde{w}^T(t) \tilde{w}(t), \quad (19)$$

where

$$\Xi_{ij} = \begin{pmatrix} \bar{\Omega}_{11ij} & \tilde{P}\tilde{B}_i & \bar{\Omega}_{13ij} \\ * & -\frac{1}{2}\gamma^2 I & \tilde{B}_i^T \tilde{P}_1 \\ * & * & \bar{\Omega}_{33ij} \end{pmatrix}, \quad (20)$$

and

$$\bar{\Omega}_{11ij} = \begin{pmatrix} \bar{\phi}_{11ij} & \bar{\phi}_{12ij} & \bar{\phi}_{13ij} \\ * & \bar{\phi}_{22ij} & \bar{\phi}_{23ij} \\ * & * & \bar{\phi}_{33ij} \end{pmatrix},$$

$$\bar{\Omega}_{13ij} = \begin{pmatrix} \kappa_{ij} + \vartheta_{ij} & -(B_i K_j)^T P_1 + \vartheta_{ij} & 0 \\ \vartheta_{ij} & (A_i + B_i K_j)^T P_1 + \vartheta_{ij} & 0 \\ 0 & -(B_i K_j)^T P_1 & A_r^T P_1 \end{pmatrix},$$

$$\bar{\Omega}_{33ij} = \begin{pmatrix} -2P_1 + \upsilon_{ij} & \upsilon_{ij} & 0 \\ * & -2P_1 + \upsilon_{ij} & 0 \\ * & * & -2P_1 \end{pmatrix},$$

$$\kappa_{ij} = (A_i - L_i C_j)^T P_1,$$

$$\bar{\phi}_{11ij} = P(A_i - L_i C_j) + (A_i - L_i C_j)^T P + \varepsilon_i P H_i H_i^T P + \varepsilon_i^{-1} K_j^T E_{2i}^T E_{2i} K_j,$$

$$\bar{\phi}_{12ij} = -(B_i K_j)^T P + \varepsilon_i P_1 H_i H_i^T P - \varepsilon_i^{-1} K_j^T E_{2i}^T (E_{1i} + E_{2i} K_j),$$

$$\bar{\phi}_{13ij} = \varepsilon_i^{-1} K_j^T E_{2i}^T E_{2i} K_j,$$

$$\bar{\phi}_{22ij} = P(A_i + B_i K_j) + (A_i + B_i K_j)^T P + \varepsilon_i P H_i H_i^T P + \varepsilon_i^{-1} (E_{1i} + E_{2i} K_j)^T (E_{1i} + E_{2i} K_j),$$

$$\bar{\phi}_{23ij} = -P B_i K_j - \varepsilon_i^{-1} (E_{1i} + E_{2i} K_j)^T E_{2i} K_j,$$

$$\bar{\phi}_{33ij} = P A_r + A_r^T P + \varepsilon_i^{-1} K_j^T E_{2i}^T E_{2i} K_j.$$

By the Schur complement formula and (13), we have

$$\begin{pmatrix} \Xi_{ij} + \tilde{Q} & \tilde{K}_j^T \\ * & -R^{-1} \end{pmatrix} < 0.$$

Then by the Schur complement formula, the above inequality is equivalent to

$$\Xi_{ij} + \tilde{Q} + \tilde{K}_j^T R \tilde{K}_j < 0. \quad (21)$$

By substituting (21) into (19), we obtain

$$\dot{V}(t) \le \sum_{i=1}^{r} \lambda_i(\theta(t)) \sum_{j=1}^{r} \lambda_j(\theta(t)) \tilde{\xi}^T \{-\tilde{Q} - \tilde{K}_j^T R \tilde{K}_j\} \tilde{\xi} + \gamma^2 \tilde{w}^T(t)\tilde{w}(t)$$
$$\le -\sum_{j=1}^{r} \lambda_j(\theta(t)) \{(x(t)-x_r(t))^T Q (x(t)-x_r(t)) + (\hat{x}(t)-x_r(t))^T K_j^T R K_j (\hat{x}(t)-x_r(t))\} + \gamma^2 \tilde{w}^T(t)\tilde{w}(t)$$
$$\le -\{(x(t)-x_r(t))^T Q (x(t)-x_r(t)) + u^T(t) R u(t)\} + \gamma^2 \tilde{w}^T(t)\tilde{w}(t).$$

By integrating from 0 to t_f, we obtain

$$V(t_f) - V(0) = \int_0^{t_f} \gamma^2 \tilde{w}^T(t)\tilde{w}(t)\,dt - \int_0^{t_f} \{(x(t)-x_r(t))^T Q (x(t)-x_r(t)) + u^T(t) R u(t)\}\,dt.$$

Since $V(t_f) \ge 0$, we have

$$\int_0^{t_f} \{(x(t)-x_r(t))^T Q (x(t)-x_r(t)) + u^T(t) R u(t)\}\,dt \le \tilde{x}^T(0) P \tilde{x}(0) + \gamma^2 \int_0^{t_f} \tilde{w}^T(t)\tilde{w}(t)\,dt.$$

Therefore, $J \le J_0$.

From the above analysis, it is easily shown that $\dot{V} \le \gamma^2 \tilde{w}^T(t)\tilde{w}(t)$. This inequality satisfies the input-to-state stability condition [26] and then the closed-loop fuzzy system (10) is robustly asymptotically stable. Therefore the closed-loop fuzzy system (3) is robustly asymptotically stable. This completes the proof of Theorem 1.

3.2 Robust stabilization

In this subsection, we assume that the feedback gain matrices $L_i (i = 1,2,...,r)$ have been well defined. The main object of this subsection is to design the feedback gain matrices $K_j (j = 1,2,...,r)$ such that the resulting closed-loop fuzzy system to be robustly asymptotically stable. For this problem we have the following result.

Theorem2. Given scalar $d > 0$, the closed-loop fuzzy system (3) is robustly asymptotically stable with guaranteed cost bound J_0 if there exist positive definite symmetric matrices $X > 0$ and matrices $R > 0, Q > 0, F_j \ (j = 1,2,...,r)$ with appropriate dimensions, as well as constants $\varepsilon_i > 0$ and $\gamma > 0$ such that the following *LMI* hold for $i, j = 1,2,...,r$:

$$\begin{pmatrix} \hat{\Theta}_{ij} & \hat{\Gamma}_{ij} \\ * & \hat{\Psi}_i \end{pmatrix} < 0, \tag{22}$$

where

$$\hat{\Theta}_{ij} = \begin{pmatrix} \hat{\Phi}_{11ij} & \hat{\Phi}_{12ij} & \hat{\Phi}_{13ij} \\ * & 0 & \hat{\Phi}_{23ij} \\ * & * & \hat{\Phi}_{33ij} \end{pmatrix},$$

$$\hat{\Gamma}_{ij} = \begin{pmatrix} \hat{\Gamma}_{11ij} & 0 & \hat{\Gamma}_{13ij} \\ 0 & \hat{\Gamma}_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{\Psi} = diag\left(-\varepsilon_i I \quad -\varepsilon_i I \quad -Q^{-1} \quad -2\gamma^{-2} I \quad -2\gamma^{-2} I \quad -2\gamma^{-2} I \quad -R^{-1} \quad -I \quad -I\right),$$

$$\hat{\Gamma}_{11ij} = \begin{pmatrix} F_j^T E_{2i}^T & 0 & 0 \\ X E_{1i}^T + F_j^T E_{2i}^T & 0 & X \\ F_j^T E_{2i}^T & 0 & -X \end{pmatrix},$$

$$\hat{\Gamma}_{13ij} = \begin{pmatrix} F_j^T & 0 & 0 \\ -F_j^T & 0 & 0 \\ F_j^T & 0 & 0 \end{pmatrix}, \hat{\Gamma}_{22} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix},$$

$$\hat{\Phi}_{11ij} = \begin{pmatrix} \hat{\psi}_{11ij} & -(B_i F_j)^T + \varepsilon_i H_i H_i^T & 0 \\ * & \hat{\psi}_{22ij} & -B_i F_j \\ * & * & \hat{\psi}_{33ij} \end{pmatrix},$$

$$\hat{\Phi}_{12ij} = \begin{pmatrix} -L_i X & X & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix}, \hat{\Phi}_{23ij} = \begin{pmatrix} -dXL_i^T & 0 & 0 \\ dX & dX & 0 \\ 0 & 0 & dX \end{pmatrix},$$

$$\hat{\Phi}_{13ij} = \begin{pmatrix} \tilde{\alpha}_{ij} & d(B_i F_j)^T + d\varepsilon_i H_i H_i^T & 0 \\ d\varepsilon_i H_i H_i^T & \tilde{\beta}_{ij} & 0 \\ 0 & -d(B_i F_j)^T & dXA_r^T \end{pmatrix},$$

$$\hat{\Phi}_{33ij}=\begin{pmatrix} -2dX+\tau_{ij} & d\varepsilon_i H_i H_i^T & 0 \\ * & -2dX+\tau_{ij} & 0 \\ * & * & -2dX \end{pmatrix},$$

$$\tilde{\alpha}_{ij}=dXA_i^T-dL_iC_jX+d\varepsilon_iH_iH_i^T,$$
$$\tilde{\beta}_{ij}=dXA_i^T+d(B_iF_j)^T+d\varepsilon_iH_iH_i^T,$$
$$\tau_{ij}=d^2\varepsilon_iH_iH_i^T,$$
$$\hat{\psi}_{11ij}=A_iX+XA_i^T-L_iC_jX-XC_j^TL_i^T+\varepsilon_iH_iH_i^T,$$
$$\hat{\psi}_{22ij}=A_iX+XA_i^T+B_iF_j+F_j^TB_i^T+\varepsilon_iH_iH_i^T,$$
$$\hat{\psi}_{33ij}=A_rX+XA_r^T.$$

Moreover, the feedback gain matrices K_j $(j=1,2,...,r)$ are given by

$$K_j=F_jX^{-1}.$$

Proof. Let $X=P^{-1}$, $F_j=K_jX$, $P_1=dP$. pre- and post- multiplying (21) by

$$diag(X \quad X \quad X \quad X \quad X \quad X \quad X \quad X \quad X)$$

yields

$$\bar{\hat{\Xi}}_{ij}=\begin{pmatrix} \bar{\hat{\Omega}}_{11ij} & \bar{\hat{\Omega}}_{12ij} & \bar{\hat{\Omega}}_{13ij} \\ * & \bar{\hat{\Omega}}_{22} & \bar{\hat{\Omega}}_{23ij} \\ * & * & \bar{\hat{\Omega}}_{33ij} \end{pmatrix}<0, \qquad (23)$$

where

$$\bar{\hat{\Omega}}_{11ij}=\begin{pmatrix} \bar{\hat{\phi}}_{11ij} & \bar{\hat{\phi}}_{12ij} & \bar{\hat{\phi}}_{13ij} \\ * & \bar{\hat{\phi}}_{22ij} & \bar{\hat{\phi}}_{23ij} \\ * & * & \bar{\hat{\phi}}_{33ij} \end{pmatrix}, \quad \bar{\hat{\Omega}}_{12ij}=\begin{pmatrix} -L_iX & X & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix},$$

$$\bar{\hat{\Omega}}_{13ij}=\begin{pmatrix} \bar{\tilde{\alpha}}_{ij} & d(B_iF_j)^T+d\varepsilon_iH_iH_i^T & 0 \\ d\varepsilon_iH_iH_i^T & \bar{\tilde{\beta}}_{ij} & 0 \\ 0 & -d(B_iF_j)^T & dXA_r^T \end{pmatrix},$$

$$\bar{\hat{\Omega}}_{22}=\begin{pmatrix} -\frac{1}{2}\gamma^2XIX & 0 & 0 \\ 0 & -\frac{1}{2}\gamma^2XIX & 0 \\ 0 & 0 & -\frac{1}{2}\gamma^2XIX \end{pmatrix},$$

$$\bar{\hat{\Omega}}_{23ij}=\begin{pmatrix} -dXL_i^T & 0 & 0 \\ dX & dX & 0 \\ 0 & 0 & dX \end{pmatrix},$$

$$\bar{\hat{\Omega}}_{33ij}=\begin{pmatrix} -2dX+\tau_{ij} & d\varepsilon_iH_iH_i^T & 0 \\ * & -2dX+\tau_{ij} & 0 \\ * & * & -2dX \end{pmatrix},$$

$$\bar{\tilde{\alpha}}_{ij}=dXA_i^T-dXC_j^TL_i^T+d\varepsilon_iH_iH_i^T,$$
$$\bar{\tilde{\beta}}_{ij}=dXA_i^T+d(B_iF_j)^T+d\varepsilon_iH_i^TH_i,$$
$$\bar{\hat{\phi}}_{11ij}=A_iX+XA_i^T-L_iC_jX-XC_j^TL_i^T+\varepsilon_iH_iH_i^T+\varepsilon_i^{-1}F_j^TE_{2i}^TE_{2i}F_j+F_j^TRF_j,$$
$$\bar{\hat{\phi}}_{12ij}=-(B_iF_j)^T-\varepsilon_i^{-1}F_j^TE_{2i}^T(E_{1i}X+E_{2i}K_j)-F_j^TRF_j,$$
$$\bar{\hat{\phi}}_{13ij}=\varepsilon_i^{-1}F_j^TE_{2i}^TE_{2i}F_j+F_j^TRF_j,$$
$$\bar{\hat{\phi}}_{22ij}=A_iX+XA_i^T+B_iF_j+F_j^TB_i^T+\varepsilon_iH_iH_i^T+\varepsilon_i^{-1}(E_{1i}X+E_{2i}K_j)^T(E_{1i}X+E_{2i}K_j)+XQX+F_j^TRF_j,$$
$$\bar{\hat{\phi}}_{23ij}=-B_iF_j-\varepsilon_i^{-1}(E_{1i}X+E_{2i}K_j)^TE_{2i}F_j-XQX-F_j^TRF_j,$$
$$\bar{\hat{\phi}}_{33ij}=A_rX+XA_r^T+\varepsilon_i^{-1}F_j^TE_{2i}^TE_{2i}F_j+XQX+F_j^TRF_j.$$

Obviously (21) is equivalent to (23), on the other hand, by the Schur complement formula, (22) holds if and only if (23) holds. So (22) is equivalent to (21). Consequently, (22) implies (13). Hence, according to Theorem 1, the closed-loop fuzzy system (3) is robustly asymptotically stable. This completes the proof.

If there is no parameter uncertainty in system (3), i.e., $\Delta A_i=0$, $\Delta B_i=0$, Theorem 2 reduces to the following result.

Corollary 1. Consider system (3) with $\Delta A_i = 0,$ $\Delta B_i = 0.$ Given scalar $d > 0$, the closed-loop fuzzy system (3) is robustly asymptotically stable with guaranteed cost bound J_0 if there exist positive definite symmetric matrices $X > 0$ and matrices $R > 0,\ Q > 0,\ F_j\ (j = 1, 2, ..., r)$ with appropriate dimensions such that the following LMI hold for $i, j = 1, 2, ..., r$:

$$\begin{pmatrix} \hat{\Phi}_{ij} & \hat{\Gamma}_{ij} \\ * & \hat{\Psi}_i \end{pmatrix} < 0, \tag{24}$$

where

$$\hat{\Phi}_{ij} = \begin{pmatrix} \hat{\Phi}_{11ij} & \hat{\Phi}_{12ij} & \Phi_{13ij} \\ * & 0 & \hat{\Phi}_{23ij} \\ * & * & \hat{\Phi}_{33} \end{pmatrix}, \hat{\Gamma}_{ij} = \begin{pmatrix} \hat{\Gamma}_{11} & 0 & \hat{\Gamma}_{13ij} \\ 0 & \hat{\Gamma}_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{\Gamma}_{11} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & X \\ 0 & 0 & -X \end{pmatrix}, \quad \hat{\Gamma}_{13ij} = \begin{pmatrix} F_j^T & 0 & 0 \\ -F_j^T & 0 & 0 \\ F_j^T & 0 & 0 \end{pmatrix},$$

$$\hat{\Gamma}_{22} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix},$$

$$\hat{\Psi}_i = diag\big(-I \quad -I \quad -Q^{-1} \quad -2\gamma^{-2} I \quad -2\gamma^{-2} I \quad -2\gamma^{-2} I \quad -R^{-1} \quad -I \quad -I\big)$$

$$\hat{\Phi}_{11ij} = \begin{pmatrix} \varphi_{11ij} & -(B_i F_j)^T & 0 \\ * & \varphi_{22ij} & -B_i F_j \\ * & * & \varphi_{33ij} \end{pmatrix},$$

$$\hat{\Phi}_{12ij} = \begin{pmatrix} -L_i X & X & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix}, \quad \hat{\Phi}_{23ij} = \begin{pmatrix} -dXL_i^T & 0 & 0 \\ dX & dX & 0 \\ 0 & 0 & dX \end{pmatrix},$$

$$\hat{\Phi}_{13ij} = \begin{pmatrix} dXA_i^T - dXC_j^T L_i^T & d(B_i F_j)^T & 0 \\ 0 & dXA_i^T + d(B_i F_j)^T & 0 \\ 0 & -d(B_i F_j)^T & dXA_r^T \end{pmatrix},$$

$$\hat{\Phi}_{33} = \begin{pmatrix} -2dX & 0 & 0 \\ 0 & -2dX & 0 \\ 0 & 0 & -2dX \end{pmatrix},$$

$$\varphi_{11ij} = A_i X + XA_i^T - L_i C_j X - XC_j^T L_i^T,$$

$$\varphi_{22ij} = A_i X + XA_i^T + B_i F_j + F_j^T B_i^T,$$

$$\varphi_{33ij} = A_r X + XA_r^T.$$

Moreover, the feedback gain matrices $K_j\ (j = 1, 2, ..., r)$ are given by

$$K_j = F_j X^{-1}.$$

4. Simulation example

In this section, a numerical example is used to illustrate the proposed results.

Consider the following T-S fuzzy system:

$$\dot{x}(t) = \sum_{i=1}^{2} \lambda_i(\theta(t))\{(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)\} + w(t),$$

where

$$A_1 = \begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 1 & -1 \end{pmatrix}^T, \quad C_1 = \begin{pmatrix} 2 & 2 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & -2 \end{pmatrix}^T, \quad C_2 = \begin{pmatrix} 2 & 1 \end{pmatrix},$$

$$E_{11} = E_{12} = I, \quad E_{21} = E_{22} = \begin{pmatrix} 1 & 1 \end{pmatrix}^T,$$

$$H_1 = H_2 = I, \quad F_1(t) = \begin{pmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{pmatrix},$$

$$w(t) = 0.005 \times \big(\sin(2t) \quad \cos(2t)\big)^T.$$

The reference model is given as:

$$A_r = \begin{pmatrix} 0 & 1 \\ -3 & -2 \end{pmatrix}, \quad r(t) = 0.03 \times \begin{pmatrix} 0 \\ \sin(t) \end{pmatrix}.$$

The purpose of this example is to find that the LMI-based conditions P, P_1 in Theorem 1 such that the resulting closed-loop system is robustly asymptotically stable. If we fixed

$$K_1 = (1.454 \quad 1.276), \quad K_2 = (1.546 \quad 2.724)^T,$$

$$L_1 = (5.364 \quad 3.897), \quad L_2 = (3.636 \quad 4.103)^T,$$

and $\gamma = 0.6,$ then, by using the Matlab to solve the LMI in (13),and choosing $Q = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}, R = I$ and $\varepsilon_1 = \varepsilon_2 = 1,$ we obtain a set of feasible solutions as follow:

$$P = \begin{pmatrix} 2.6099 & 2.7485 \\ 2.7485 & 4.3784 \end{pmatrix}, \quad P_1 = \begin{pmatrix} 0.4936 & 0.0371 \\ 0.0371 & 0.2073 \end{pmatrix}.$$

With the initial condition given by $X(0) = (-3 \quad 2)^T$, Figure 1 shows the asymptotically stable state of $x(t)$. Note that the measurement noise $v(t)$ is assumed to be zero mean white noise with variance equals to 0.01%.

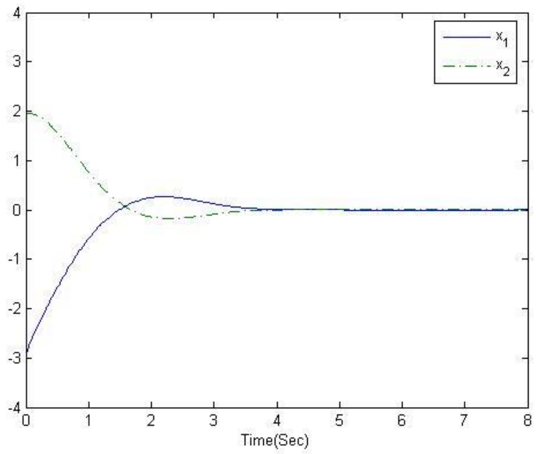

Fig.1. State response of the closed-loop system

5. Conclusions

In this paper, the guaranteed cost control problem has been studied. We have provided some sufficient conditions for the solvability of the problems of robust stabilization and guaranteed cost controller design for a class of uncertain fuzzy T–S model systems via output feedback approach. These conditions are expressed in terms of LMI, which can be easily tested by using commercially available software. Finally, a numerical example has been given to show the effectiveness of the proposed method.

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