Some Mathematical Descriptions of Multi-Connected System of Fuzzy State Space Model via Number Theory Approach

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Abstract: - Some mathematical structures of multi-connected systems of Fuzzy State Space Model (FSSM) are proposed in this study by adapting some analogies in Number Theory. Some characteristics of the defined structures are also stated to easily identify these structures in some systems which are normally complex and made up by the combination of several structures. Multi-connected systems of FSSM are also divided into two parts to further describe each subsystem of the systems with their input and output parameters. In addition, these suggested structures are also applied to describe a real physical system which is a boiler's system of a combined cycle power plant. Some associations and relations between subsystems of the boiler's system can be understood deeper from the result which helps to increase the knowledge about the total system. In short, this study provides an alternative method to handsomely interpret connections of systems of FSSM in some interesting manner.

Key-Words: - Feeder, Receiver, Multi-Connected System, Boiler

1 Introduction

A multi-connected system of Fuzzy State Space Model (FSSM) is made up by several subsystems and each of the subsystems is needed to be studied carefully in order to identify the overall behavior of the whole system. For example, a boiler's system of a combined cycle power plant in [1] is multi-connected and built by several components namely furnace, superheater, reheater, economizer, riser and drum (see Appendix). Some traditional mathematical modelling of a boiler's system could be referred in [1] while modern techniques to study the same system were developed by using the state space model approach [2, 3].

While all studies proven to provide valuable outcomes in describing and understanding these multi-connected systems, a new approach in simplifying representation of the systems is still much needed. This is firstly because one will need to spend some times to identify each mathematical modelling of every subsystems which is normally either in ordinary or partial differential equations [1, 4]. Besides, the state space formulation of multi-connected systems normally is in the form of system of complicated state space equations which is difficult to be obtained.

Thus, a new method on how to easily represent the complexity of the structures of the system mathematically must be developed. By doing this, the nature of connections of multi-connected fuzzy state space system can be understood better and the knowledge about the total system will automatically increase. After that, more accurate model for the system can be developed by using relevant and useful information from that study in the future.

2 System of FSSM

Each component of system of FSSM performs some specific actions that determine the overall behaviour of the operation of the main system. Obviously, three elements are basically included in any multi-connected system of FSSM which are i) a system that deliver its output to a receiving system, ii) a system that receive input from a delivering system, and iii) the output that is delivered which becomes input when received. These entire three elements must be taken account when constructing mathematical structures of multi-connected system of FSSM so that total and compact findings are produced.

Previous studies divided the method on how to represent structures of state space system into two approaches which are by the geometric and algebraic approach. Most of the studies however focus the system as one system wholly such as in [5, 6, 7, 8] while researches on a multi-connected state space system are still less and can be found in [9, 10, 11, 12]. In this study, previous works which are initially done by Ismail [2] and further explored by Taufiq [10, 12] will be reestablished and extended with the objective to build a strong concept on algebraic structures of multi-connected state space systems.

3 Number Theory Approach to Describe System of FSSM

Algebra is a very wonderful language for describing and understanding the behaviour of mathematical objects while number theory is a branch of algebra that has interested mankind for thousands of years [13]. Nowadays, not only pure but applied mathematicians also found that algebra has been relevant for the problem that they were working on [14]. The fascinating background of algebra in mathematical problems and tremendous applications of number theory in many science areas is a main motivation for this study to model multi-connected fuzzy state space systems.

3.1 Some Mathematical Structures of Multi-Connected System of FSSM

Mathematical structures of multi-connected system of FSSM will be studied and presented in a more convenient way in this section compared to previous works in [2, 10, 12] since previous structures defined have their own weakness and does not really fit to a real actual system. However, some of previous structures will be used directly for the purposes of reproducing and introducing few new structures of multi-connected system of FSSM. The definition of FSSM of a multivariable dynamic single system proposed by Ismail [2] is firstly given since it is essential in building those structures.

<u>Definition 1</u>

A multivariable dynamic system of Fuzzy State Space Model (FSSM) is defined as :

$$S_{GF} \quad : \quad \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\tilde{\mathbf{u}}(t)$$

$$\tilde{\mathbf{y}}(t) = \mathbf{C}\mathbf{x}(t)$$

where $\tilde{\mathbf{u}}$ denotes the fuzzified input vector $[u_1, u_2, ..., u_n]^T$ and $\tilde{\mathbf{y}}$ denotes the fuzzified output vector $[y_1, y_2, ..., y_m]^T$ with initial conditions as $t_0 = 0$ and $x_0 = x(t_0) = 0$. The elements of state matrix $\mathbf{A}$ ($p \times p$), input matrix $\mathbf{B}$ ($p \times n$) and output matrix $\mathbf{C}$ ($m \times p$) are known to a specified accuracy.

The block diagram in Figure 1 represents a single system of FSSM with n input and m output as defined in Definition 1. For a multi-connected system of FSSM, a single system of FSSM might receive inputs from and deliver outputs to many other FSSM (see Figure 2). Thus, mathematical structures of multi-connected system of FSSM are redeveloped in the next definitions based on three important elements in the system namely (i) a delivering system, (ii) a receiving system and (iii) the output from a delivering system that becomes input of a receiving system.

<u>Definition 2 (Feeder)</u>

For $S_{gF1}, S_{gF2} \in S_{GF}$, S_{gF1} is a **feeder** of S_{gF2} (written as $S_{gF1} | S_{gF2}$) if and only if any output of S_{gF1} is/are the inputs of S_{gF2}.

<u>Definition 3 (Receiver)</u>

For $S_{gF1}, S_{gF2} \in S_{GF}$, S_{gF2} is a **receiver** of S_{gF1} if and only if any input of S_{gF2} is/are the outputs of S_{gF1}.

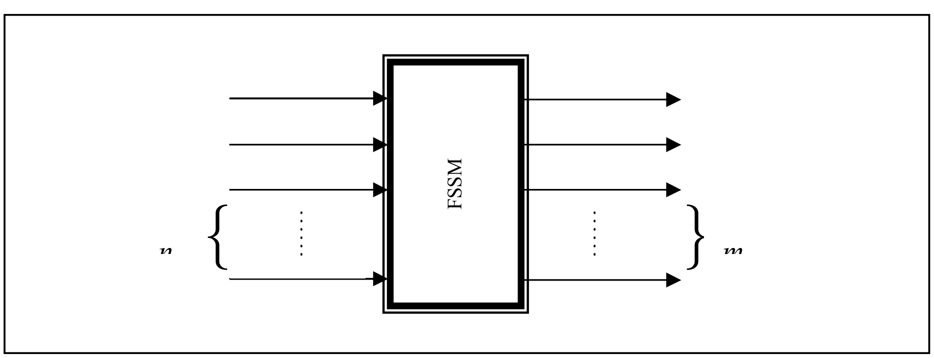

Figure 1 A System of FSSM with n inputs and m outputs

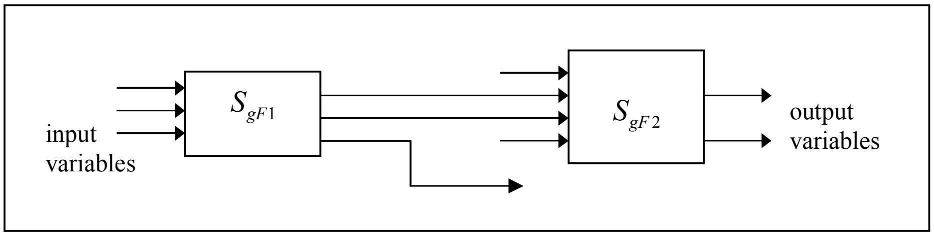

Figure 2 A Multi-Connected System of FSSM

Definition 4 (Connector)
For $S_{gF1}, S_{gF2} \in S_{GF}$, a connector between S_{gF1} and S_{gF2} is an output of S_{gF1} that becomes an input of S_{gF2}.

Based on Definition 2, 3, 4 and Figure 2, S_{gF1} is said to be a feeder of a receiver S_{gF2} ($S_{gF1} | S_{gF2}$) with two connectors between them. Notice that S_{gF1} and S_{gF2} are just symbols to denote the system which is in S_{GF} which can be changed or replaced with other suitable symbol. The main concerned of this study is to describe and represent the connection between some FSSMs without concerning the inside behavior of each single system.

Definition 2 which is adapted based on divisor is then extended to further describe mathematical structures of multi-connected system of FSSM. The terminologies common feeder and the greatest common feeder discussed by Ismail [2] and Taufiq [10, 12] which are used to represent specific case of multi-connected system of FSSM are reconstructed in definition 5 and 6 to describe a more general multi-connected system of FSSM. These structures are then shown in Figure 3 and Figure 4.

Definition 5 (Common Feeder)
For $S_{gFa}, S_{gFb}, S_{gf} \in S_{GF}$, S_{gf} is called a **common feeder** of S_{gFa} and S_{gFb} written as $\mathrm{cf}(S_{gFa}, S_{gFb}) = S_{gf}$ if $S_{gf} | S_{gFa}$ and $S_{gf} | S_{gFb}$.

Definition 6 (Greatest Common Feeder)
For $S_{gFa}, S_{gFb}, S_{FF} \in S_{GF}$, S_{FF} is called **the greatest common feeder** of S_{gFa} and S_{gFb} written as $\mathrm{gcf}(S_{gFa}, S_{gFb}) = S_{FF}$ where

(i). $S_{FF} | S_{gFa}$ and $S_{FF} | S_{gFb}$, and

(ii). if exist $S_{gf} \in S_{GF}$ such that $S_{gf} | S_{gFa}$ and $S_{gf} | S_{gFb}$ then $S_{FF} | S_{gf}$.

Notice that condition (ii) in Definition 6 implicitly mentions that a common feeder of a multi-connected system is the greatest common feeder if it is the only (unique) common feeder in that system. For example, S_{gf} is the greatest common feeder of the system in Figure 3 if it is the only and the unique common feeder of S_{gFa} and S_{gFb} in that system. Thus, the greatest common feeder is defined to actually represent the dominant source in a multi-connected system.

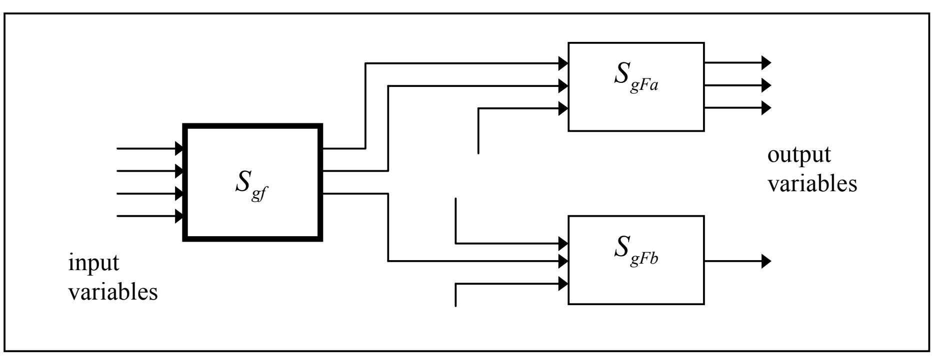

Figure 3 S_{gf} is a Common Feeder of S_{gFa} and S_{gFb}

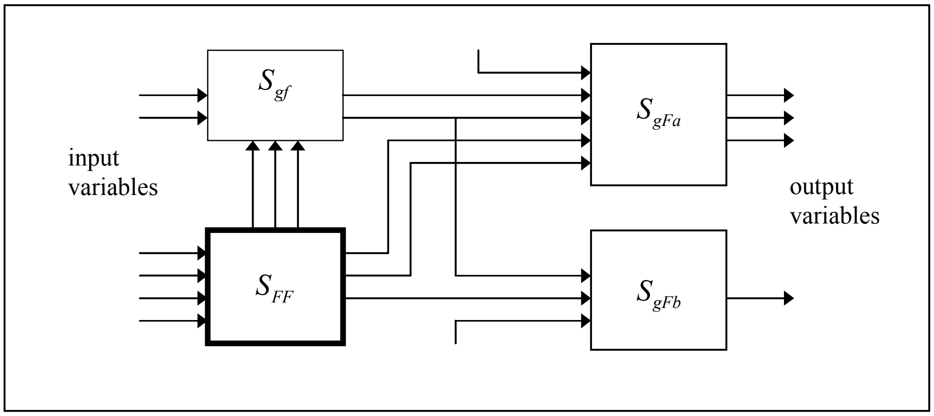

Figure 4 S_{FF} is the Greatest Common Feeder of S_{gFa} and S_{gFb}

Furthermore, two new terminologies to further describe multi-connected systems of FSSM which are adapted based on the idea of common and the least common multiple in number theory are introduced in this study. These structures which are extension from receiver are named as common and the least common receiver and are defined in Definition 7 and 8 respectively. Figure 5 and Figure 6 then describe both defined structures.

Definition 7 (Common Receiver)
For $S_{gF1}, S_{gF2}, S_{gfc} \in S_{GF}$, S_{gfc} is called a **common receiver** of S_{gF1} and S_{gF2} (written as $\mathrm{cr}(S_{gF1}, S_{gF2}) = S_{gfc}$) if $S_{gF1} \mid S_{gfc}$ and $S_{gF2} \mid S_{gfc}$.

Definition 8 (Least Common Receiver)
For $S_{gFa}, S_{gFb}, S_{gfC} \in S_{GF}$, S_{gfC} is called **the least common receiver** of S_{gFa} and S_{gFb} (written as $\mathrm{lcr}(S_{gFa}, S_{gFb}) = S_{gfC}$) where

(i). $S_{gFa} \mid S_{gfC}$ and $S_{gFb} \mid S_{gfC}$, and
(ii). if exist $S_{gf} \in S_{GF}$ such that $S_{gFa} \mid S_{gf}$ and $S_{gFb} \mid S_{gf}$ then $S_{gfC} \mid S_{gf}$.

Similarly, condition (ii) in Definition 8 mentions that the only (unique) common receiver of a multi-connected system will become the least common receiver for the system. Therefore, S_{gfc} is also the least common receiver of the multi-connected system in Figure 5. Contrary to the greatest common divisor, the least common receiver denotes the nominal receiving system in a multi-connected system of FSSM.

3.2 Some Additional Characteristics of Mathematical Structures of Multi-Connected System of FSSM

In a real world situation, system of FSSM can be very large and perhaps is joined together by several mathematical structures defined. Using every definition to identify each mathematical structure in a large and complex system might be very tedious and cumbersome. In order to increase the ability to recognize existing structures, information about each mathematical structure proposed in previous section and its minimum number of delivering and receiving system which is stated in Table 1 can possibly be used.

Table 1 Minimum Number of Delivering and Receiving System for Each Structure

Structure	Minimum Number	
	Delivering System	Receiving System
Feeder	1	1
Common Feeder (cf)	1	2
Greatest Common Feeder (gcf)		
i. a unique cf	1	2
ii. non-unique cf	2	3
Receiver	1	1
Common Receiver (cr)	1	2
Least Common Receiver (lcr)		
i. a unique cr	1	2
ii. non-unique cr	3	2

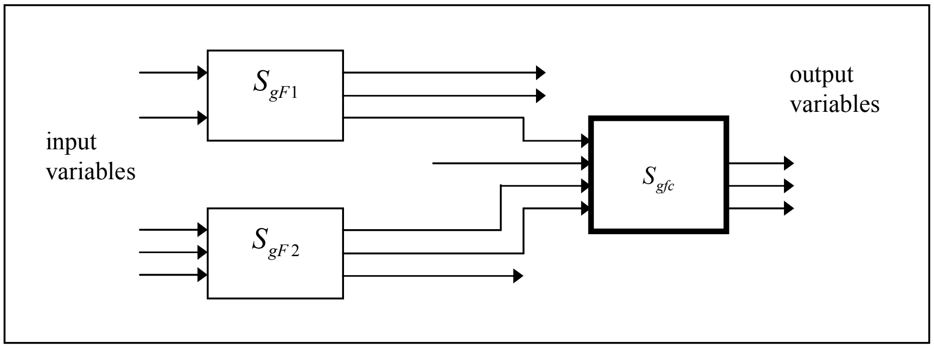

Figure 5 S_{gfc} is a Common Receiver of S_{gF1} and S_{gF2}

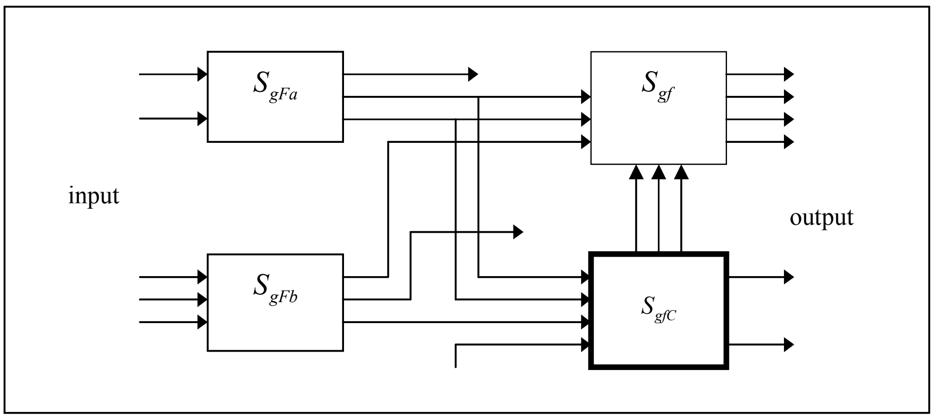

Figure 6 S_{gfC} is the Least Common Receiver of S_{gFa} and S_{gFb}

3.3 Classifications of Multi-Connected System of FSSM

Due to the complex structures of multi-connected system of FSSM, it is actually difficult to include the connectors between a feeder and a receiver in each structure. Therefore, an attempt to reduce these complexities in this study is made by classifying multi-connected system of FSSM into two types as stated in Definition 9 and Definition 10. These classifications which are actually based on the previous work done by Ismail [2] are redefined by including the concept of feeder and receiver discussed in the previous section.

Definition 9 (Multi-Connected System of FSSM of type A)

Let S_{gF} be a system of FSSM from specified in Definition 1. A multi-connected system of S_{gF} of type A is defined as $S_{gFa} \mid S_{gFb}$ where output(s) of S_{gFa} is/are the only input(s) of S_{gFb} for $S_{gFa}, S_{gFb} \in S_{GF}$.

Definition 10 (Multi-Connected System of FSSM of type B)

Let S_{gF} be a system of FSSM from specified in Definition 1. A multi-connected system of S_{gF} of type B is defined as $S_{gFi} \mid S_{gFc}$ where input of S_{gFc} come from the combination of other systems S_{gFi} where $S_{gFc}, S_{gFi} \in S_{gF}$ for some $i = 1, 2, ...n$.

Figure 7 shows an example of structure of Type A such that S_{gF1} is the only input source for both S_{gF2} and S_{gF3} while inputs of S_{gFc} in Figure 8 come from combination of S_{gF1} and S_{gF2} which means they are structured in the form of Type B. Based on this two classifications, the number of connector between feeder and receiver can now be included in the structures. This is explained without lost of generality in actual systems by the next two theorems.

Theorem 1
Let $S_{gFa}, S_{gFi} \in S_{GF}$ for some $i = 1,2,...n$. If $S_{gFa} \mid S_{gFi}$ by connection of Type A then the number of connector between S_{gFa} and S_{gFi} for any *i* is always less than or equal to the number of output of S_{gFa} .

Proof :

Assume that $S_{gFa} \mid S_{gFi}$ by connection of Type A where $S_{gFa}, S_{gFi} \in S_{GF}$ for some $i = 1,2,...n$.
Since $S_{gFa} \mid S_{gFi}$ by connection of Type A then by Definition 9, S_{gFi} will only receive output from S_{gFa} as its input for each *i*.
Therefore, by Definition 4, the number of connector between S_{gFa} and S_{gFi} must be less than or equal to the number of output of S_{gFa} for any *i*.

□

Theorem 2
Let $S_{gFi}, S_{gFc} \in S_{GF}$ for each $i = 1,2,...n$. If $S_{gFi} \mid S_{gFc}$ by connection of Type B and k_i is the number of connector between S_{gFi} and S_{gFc} for each *i* than the number of input of S_{gFc} is $\sum k_i$ for each *i*.

Proof :

Assume that $S_{gFi} \mid S_{gFc}$ by connection of Type B where $S_{gFi}, S_{gFc} \in S_{GF}$ for each $i = 1,2,...n$ and k_i is the number of connector between S_{gFi} and S_{gFc} .
Since $S_{gFi} \mid S_{gFc}$ by connection of Type B, Definition 10 implies that inputs of S_{gFc} will come from the outputs of S_{gFi} for each *i*.
Hence, by Definition 4, the total input of S_{gFc} will equal to $\sum k_i$ for each *i*.

□

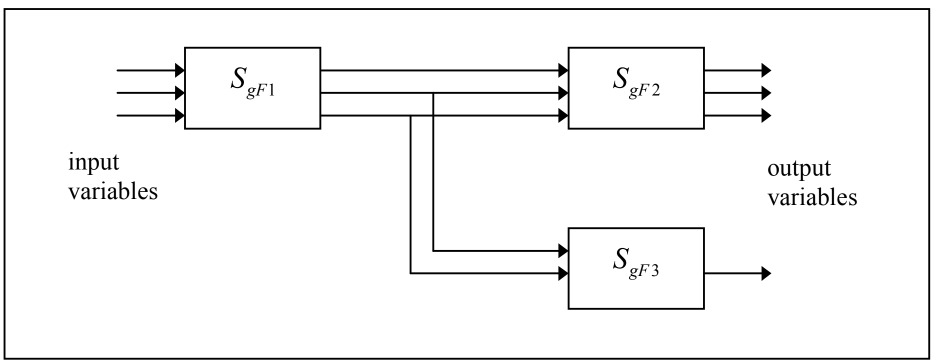

Figure 7 Multi Connected System of FSSM of Type A

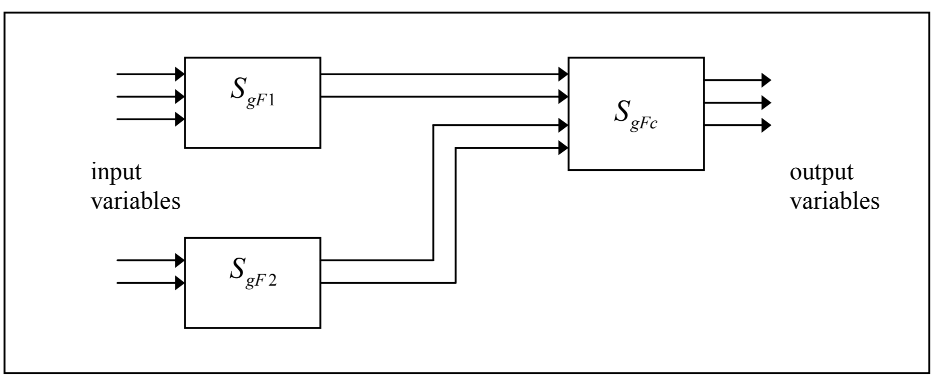

Figure 8 Multi Connected System of FSSM of Type B

4 Describing a Boiler's System Algebraically

A boiler of a combined cycle power plant in [1] is a multi-connected system of FSSM which operates by the combination of some functional subsystems of FSSM. Since the boiler itself is a subsystem of a combined cycle power plant such that it receives input from and delivers output to other subsystems of power plant, mathematical structure of the boiler's system in this section is built up by only considering subsystems of the boiler and inputs or outputs recycled inside the boiler's system. This restriction is made in order to focus to the complexity of boiler's system without including the whole power plant system. Based on mathematical modelling of each subsystem in [1], each subsystem of the boiler's system is presented with its feeders and receivers in Table 2.

Table 2 Subsystem and Its Feeders and Receivers

Subsystem	Subsystem's Feeder	Subsystem's Receiver
Furnace	1. Reheater 2. Superheater 3. Economizer	1. Economizer 2. Riser 3. Superheater 4. Reheater
Superheater	1. Furnace 2. Drum 3. Economizer	1. Furnace 2. Drum
Reheater	1. Furnace	1. Furnace
Economizer	1. Furnace	1. Superheater 2. Furnace 3. Drum
Riser	1. Furnace 2. Drum	1. Drum
Drum	1. Superheater 2. Economizer 3. Riser	1. Riser 2. Superheater

By examining carefully column Subsystem and Its Receivers in Table 2, the following other structures can be identified :-

- $cf(\text{Superheater, Riser}) = \text{Furnace}$
- $cf(\text{Superheater, Riser}) = \text{Drum}$
- $gcf(\text{Economizer, Riser, Superheater, Reheater}) = \text{Furnace}$
- $gcf(\text{Superheater, Furnace, Drum}) = \text{Economizer}$
- $gcf(\text{Furnace, Drum}) = \text{Economizer}$

Similarly, the following structures are obtained when the column Subsystem and Its Feeders in Table 2 are examined :-

- $cr(\text{Superheater, Economizer}) = \text{Furnace}$
- $cr(\text{Superheater, Economizer}) = \text{Drum}$
- $cr(\text{Furnace, Drum}) = \text{Superheater}$
- $cr(\text{Furnace, Drum}) = \text{Riser}$
- $lcr(\text{Reheater, Superheater, Economizer}) = \text{Furnace}$
- $lcr(\text{Furnace, Drum, Economizer}) = \text{Superheater}$
- $lcr(\text{Superheater, Economizer, Riser}) = \text{Drum}$

Furthermore, second column of Table 2 obviously indicates that each of Reheater and Economizer is connected to its feeder namely Furnace by connection of Type A while other subsystems are connected to their feeders by connection of Type B. Theorem 1 and Theorem 2 can now be used to calculate the total input for each subsystem of Boiler's system. The letters TC used in the column Total Input of Table 3 stands for total connectors between subsystem and its receiver. The number of the connector between subsystem and a receiver is given in the bracket after the receivers in each row of the table. All of the values used in the calculation are based on mathematical modelling of each subsystem which are given in [1].

All of the results obtained are then combined and transformed into block diagrams to represent a boiler's system which is built up by several subsystems as shown in Figure 9 below. In this figure, what actually inputs and outputs parameters involved are not mentioned so that the nature of connections of the system can be totally focused. These complexities are needed to be understood carefully because of their big influence to the overall behavior of the whole systems.

5 Discussion and Conclusion

In this research, structures of multi-connected systems of FSSM which are usually complex are mathematically interpreted by adapting some terminologies in number theory. The concepts of divisor and multiple are adapted and extended in order to reestablish and introduce connections between FSSMs. This yields to the proposition of feeder, receiver, connector, common feeder, common receiver, the greatest common feeder and the least common receiver to describe these multi-connected systems in some interesting manner.

Table 3 Subsystem of a Boiler and Its Total Input

Subsystem	Type of Connection	Total Input
Furnace	**B**	TC-Economizer (1) + TC-Superheater (1) + TC-Reheater (1) = **3**
Superheater	**B**	TC-Furnace (1) + TC-Drum (1) + TC-Economizer (2) = **4**
Reheater	**A**	TC-Furnace (1) = **1** < Total Output of Furnace (4)
Economizer	**A**	TC-Furnace (1) = **1** < Total Output of Furnace (4)
Riser	**B**	TC-Furnace (1) + TC-Drum (6) = **7**
Drum	**B**	TC-Economizer (1) + TC-Superheater (1) + TC-Riser (2) = **4**

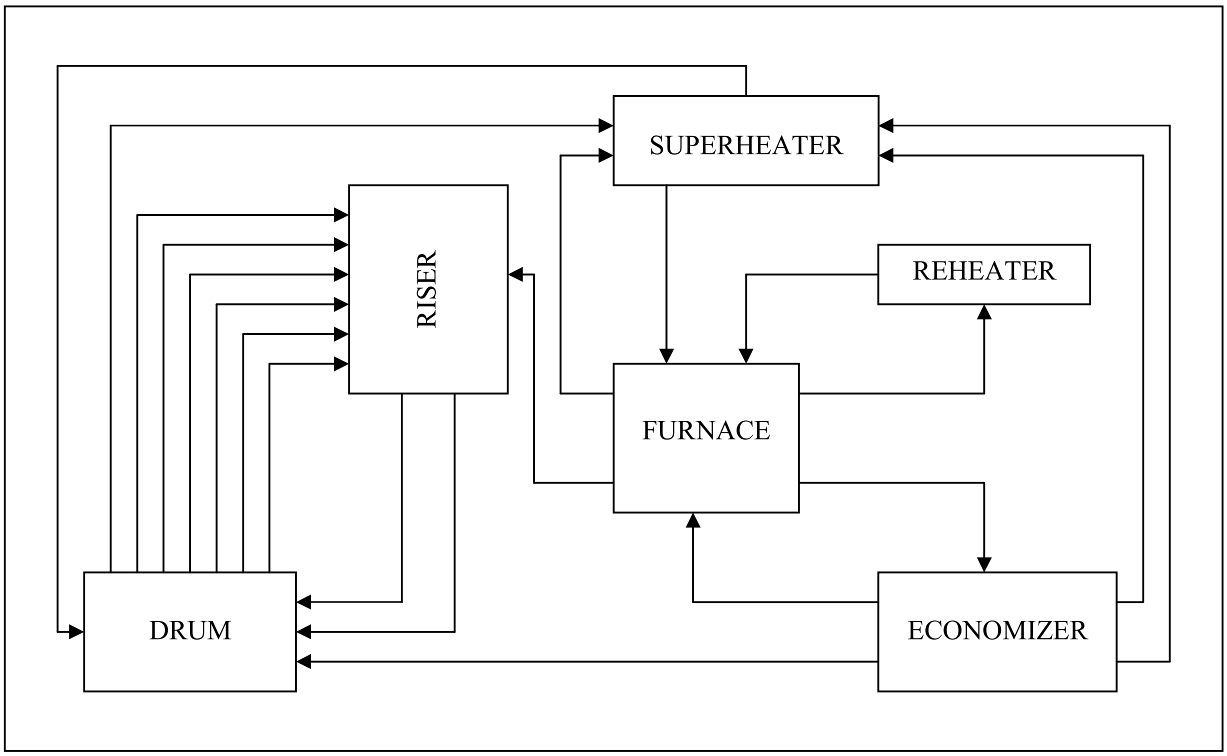

Figure 9 Mathematical Structures of a Boiler's System

After that, these structures are also discussed in terms of their total minimum number of delivering and receiving system to increase the ability in interpreting them in a real physical system. Furthermore, the systems are also divided into two parts to focus and increase the understanding on the behavior of the connector between each system. Some related characteristics about the connector between systems are also highlighted in this paper.

In addition, a boiler's system of a combined cycle power plant is used when describing a real-physical system by applying definitions and theorems proposed. This will provide an alternative way to understand relations between subsystems of the boiler algebraically if compared in the previous literatures. By doing so, all relevant and additional information and knowledge obtained in this study can be used to develop more precise study about boiler system in the future.

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APPENDIX

BLOCK DIAGRAM OF A BOILER SYSTEM