Fuzzy Embedded Mobile Robot Systems Design through the Evolutionary PSO Learning Algorithm

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Abstract: - The evolutionary learning algorithm called particle swarm optimization (PSO) is developed in this paper. The image model of the embedded mobile robot is automatically generated with the omni-directional image concept to approach toward the behavior of the embedded mobile robot. The circumvolutory environment is dynamically captured from the head of the mobile robot, which will directly be transformed into the Cartesian coordinate system. The required parameters of fuzzy rules are automatically extracted with the guide of the flexible fitness function, which is efficiently approach toward the multiple objectives of avoiding obstacles, selecting favorable fuzzy rules to drive the desired targets at the same time. Three illustrated examples with various initial positions for the discussed environment map containing different blocks size and locations are illustrated the efficiency of the PSO leaning algorithm. Simulations demonstrate that the proposed mobile robot with the selected fuzzy rules can avoid the obstacles and achieve the targets as soon as possible.

Key-Words: - Particle swarm optimization; Fuzzy systems; Mobile robots, Evolutionary learning, Omnidirectional image.

1 Introduction

Several mobile robot systems have been widely used in different indoors and outdoors applications. The products of mobile robot systems are integrated with the advanced technologies in computer science & sensors measure, mechanism and electric engineering etc al... Because of the modern manufacture, precise image processing devices were rapidly developed in the last few years. The embedded vision-based reorganization software system contains the benefits of comfortable operations, lower power consumption, smaller size and higher portable ability, etc., are widely applied in human life applications. Due to the improvement in the production of image processing devices, several mobile robot systems are developed by the vision-based technology to identify the interesting objects in an unknown environment. The challenge in the design of the vision-based mobile robot algorithm is how to recognize the anomalous behavior, extract inspected features, analysis the treated patterns and start an appropriate control stratagem [5].

In this image processing stage, there are two mains standard-type image sensors to seize the observation of the objects in various environments. The omni-directional image sensor can obtain the wider scene in the complicated environment. These advantages of the omni-directional mobile robots are that it contains the higher capabilities to move toward arbitrary way without rotating the direction of wheels. In the other regarded point, it can nimbly attain any desired orientation and position in the traveling path-line. Williams et al. delivered the omni-directional wheels in 2002 [11], they established the dynamic model-based mobile robots with omni-directional wheels to form the slipping movement between the driving wheels and the motion surface. There are more and more research papers to successfully generate the vision-based robot system based on the extracted dynamics models in different environment [2, 4, 8]

Fuzzy logic theory was first introduced by Zadeh in 1965 [13]. Fuzzy systems are known the popular linguistic rules based knowledge acquisition machine, it is highly desirable to represent the human thinking to utilize the domain knowledge to create autonomous strategies for controlling the mobile robot plan. Fuzzy inference system embedded the control rules is applied successfully in several applications areas, especially in solving complex modeling, tracking or control mobile robot problems, while it is very hard to describe the robot plan's mathematic model. Fuzzy logic demonstrates their navigating ability to efficiently approach the desired targets due to the adaptable self-localization ability. Fuzzy logic is an adaptable theory to successfully solve the mobile robot model and control problems in the unknown environment [10, 121.

the great practical mobile Even robot applications are developed by the Fuzzy logic systems [2-3, 9], there are containing some parameters selection problems to approach the perfect the embedded fuzzy rules for efficiently achieving the available mobile robot traveling path. One of the main objectives in constructing the fuzzy control systems is to acquire the appropriate fuzzy rules parameters. In the traditional fuzzy rule generation, it derived from expert's experience and parameters tuning is gathered from the skilled operators by the trial-and-error ways. The abovementioned terms in obtaining the parameter value is a time-consuming task. It can be expected that selecting fuzzy rules form the high-dimensional search space is a difficult but crucial procedure.

PSO algorithm is first introduced by the Eberhart and Kennedy in 1995 [6-7], which inspired by social behavior of bird flocking or fish schooling. The concept of the PSO learning stratagem is driven from the scenario of social behavior to search these ill-defined, complex and global optimization solutions. The simple but efficient evolutionary technique called Particle swarm optimization (PSO) is proposed to improve the training accuracy in this article. This PSO learning algorithm simulates natural creatures behave as a swarm and the individual particles are attracted stochastically toward the right positions of evaluated best performance. In a word, PSO learning algorithm is extracted the required parameter value among the best previous experience of particle's neighbors from the multidimensional search space. The learning algorithm of PSO through a metaphor of social interaction has been well-known to solve many global optimization mobile robot control problems [1-3].

2 Mobile Robot Structure and Image Model Design

The architectures of fuzzy embedded mobile robot systems are present in Figure 1. Robot plan contains the motor driver and mobile robot machine. Motor driver can directly control the moving speed and rotating angle of the discussed robot while receiving the fuzzy commands...An omnidirectional images are captured by the particular panorama sensor to represent the full views. The interesting targets and obstacles in the surrounding environment of the mobile robot are collected at once. Based on the image processing, the target and obstacle information will be mapped into the required Cartesian coordinate to create the image mathematical kinematics model of mobile robot. The evolutionary PSO learning algorithm with the evaluated fitness function value is applied to approach into the desired fuzzy rules. The purpose of PSO learning algorithm is delivered to minimize the traveling path between the initial robot positions and the desired targets. The fuzzy rule-based system is extracted by the PSO-based learning scheme with the highest performance to represent the prototype of the mobile robots.

The panoramic image is determined by the specific omni-directional image sensor. In this procedure, the required tracking objects are obtained with the proposed geometry pattern reorganization method. In general; the captured color image usually contains three main color channels, i.e. red, green and blue (RGB). The original color R. G and B is the main channel-values to represent the true real world image color. The HSV color space, denoted as the hue, the hue and bright values, is another popular way to recognize the image pattern. It is more perfected than that in RGB color space in the real experiment results. In the discussed HSV color space, H and S are determined to calculate the angle and length between these interesting objects. The determined HSV values are transformed from the RGB values to identify the image objects [3]. The tracking target and block objects will be identified by their (H, S) located position. To rebuild the true contour and retrieve the high quality image, the image morphology method is applied in this research. Two fundamental operations, openings and closings, are delivered to eliminate the small pellet and reconstruct the completed region for clearly extracting the target and obstacle objects in image training patterns. The erosion and dilation technologies are proposed to reduce the affect of image flat zones. The openings operation is used to reduce the narrow connection and clean the small outlier. In the other closings operation, the purpose of this function is that it can join narrow broken parts and collect thin gap area to mend the small holes in the black mark image zone. In addition, the small gap of the shape will be refilled out in this fixed image procedure. The destination and block size is recognized clearly while the one-openingone-closing function is completed.

Based on the simple image processing stage as mentioned above, the visual locations for the extracted objects, i.e. the mobile robot, destination and block, are illustrated in Figure. 2. These location values (x_T, y_T) , (x_B, y_T) and (x_R, y_R) are denoted as co-coordinates for the destination, block and robot, respectively. The mathematical robot model is defined by the following formulas: [3]

$$r_{T} = \sqrt{(x_{T} - x_{R})^{2} + (y_{T} - y_{R})^{2}}$$
(1)

$$r_{B} = \sqrt{(x_{B} - x_{R})^{2} + (y_{B} - y_{R})^{2}}$$
(2)

$$\theta_T = \cos\left(\frac{\vec{X}_{TR} \cdot \vec{Y}_{TR}}{\left|\vec{X}_{TR}\right| \cdot \left|\vec{Y}_{TR}\right|}\right)^{-1}$$
(3)

$$\theta_B = \cos\left(\frac{\vec{X}_{BR} \cdot \vec{Y}_{BR}}{\left|\vec{X}_{BR}\right| \cdot \left|\vec{Y}_{BR}\right|}\right)^{-1} \tag{4}$$

$$\theta_{T-B} = \theta_T - \theta_B \tag{5}$$

Were r_T and r_B are the distances length between robot (R) and target (T); robot and block (B), respectively. θ_T and θ_B are denoted as the corresponding angle of the robot for the target and block in y-axis, respectively. \vec{X}_{TR} and \vec{Y}_{TR} are denoted as vectors which is calculated from the robot to destinations with respective to the x-axis and y-axis, respectively. \vec{X}_{BR} and \vec{Y}_{BR} are also the vectors between the robot to blocks for the x-axis and y-axis, respectively . Symbol || means the vector length. The proposed mobile robot model .by previous mathematical formulas can be described as a visual image workspace; it is shown in Figure 2.

In the designed fuzzy image mobile robot system, three input variables (r_T, r_B, θ_{T-B}) and two output variables (θ_w, v_w) are applied to construct the fuzzy inference structure. Where θ_w and v_w are the turn-rate in angle to the motor wheels driver and robot's traveling line-speed, respectively.

The mobile robot speed in the current time step k is denoted as v_w^k and its turn-rate angle is setting as θ_w^k , the location of the mobile robot with the very small increasing time interval (Δk) at next time step (k+1) is given by

$$x_R^{k+1} = x_R^k + v_W^k \bullet \Delta k \bullet \cos(\theta_W^k)$$
(6)

$$y_R^{k+1} = y_R^k + v_W^k \bullet \Delta k \bullet \sin(\theta_W^k)$$
(7)

Based on the equations (1)-(7), the whole kinematics of mobile robot in the coordinate x-y space is finished. The novel fuzzy system design by the evolutionary PSO learning algorithm will be discussed in the next section.

3 Evolutionary Fuzzy Rule-based System Generation

These variables (r_T, r_B, θ_{T-B}) considered as the input vector X= $(x_1, x_2, ..., x_n)$ is collected in the whole n-dimensional pattern, which is regarded as the premise part of the fuzzy inference system. The constructed fuzzy rules can be illustrated as follows:

$$\mathbf{R}^{(i)}$$
: IF X is ME_i THEN Y is y_i, $i = 1, 2, ..., m$, (8)

Where m means the total number of fuzzy rules and MEi is denoted as a fuzzy membership function with respective to the input vector (X). The yi is denoted as a real value in the consequent part. In our research, the defined fuzzy membership function set is represented by the following formula:

$$ME_{i}(X) = \exp(-(\frac{(x_{1} - a_{i1})^{2}}{b_{i1}^{2}} + ... + \frac{(x_{n} - a_{in})^{2}}{b_{in}^{2}})).$$
(9)

It is clear that there is a hyper-ellipsoid type function in an n-dimensional search space. Where parameter set $(a_{i1}, a_{i2}, ..., a_{in})$ is the center value and (b_{ij}) is the length of the j-th principal axis of the hyper-ellipsoid, respectively. In this article, the consequent part (y_i) is denoted as a single real number. An appropriate combination of parameters set $(a_{ij}, b_{ij} \text{ and } y_i)$ is retrieved by the efficient PSO learning algorithm.

In this study, the weighted average operation is used as a defuzzifier to convert the fuzzy domain value into the real number. While the firing process is determined in the premise part, the i-th rule of the fuzzy system will be deserved. The output value of fuzzy system (y°) can be determined by (10)

$$y^{o} = \frac{\sum_{i=1}^{m} ME_{i}(\mathbf{x}) \bullet y_{i}}{\sum_{i=1}^{m} ME_{i}(\mathbf{x})}.$$
(10)

According to the above description, the contour of the membership function $ME_i(X)$ is extracted to generate the suitable traveling path of the mobile robot. To provide the optimal estimates of the parameters in the kinematics model of mobile robot, parameters selection problems can be solved by the evolutional PSO learning algorithm to derive the appropriate fuzzy embed mobile robot system

Natural creatures behave like swarm intelligence. The main objective of artificial intelligent is to observe how natural creatures act as a swarm. The evolutionary PSO learning scheme simulate the swarm models inside a computer computation to approach the best of the characters among the comprehensive old population. At the evolutionary training cycle, each particle's position and velocity are regulated by the current best selected positions value. The personal best one called P_best is every particle's best solution, which has been achieved so far. The second best one called G best is obtained by choosing the overall best value from all particles of populations. At the learning step of the iteration, the velocity of the particle is modified by the relative locations of P best and G best. The next new velocity for each particle is regulated by the following formulas:

$$v_{n} \beta^{\nu}(k + k) = \cdot_{np} ()$$

$$+ \beta_{\Gamma} and () \mathcal{P} (best_{n} \beta_{\Gamma} (-k) - \beta_{\Gamma} ())$$

$$+ \beta_{\Gamma} and () \mathcal{P} (best_{n} \beta_{\Gamma} (-k) - \beta_{\Gamma} ())$$

$$(11)$$

Therefore, the new particle position will be regulated by

$$Y_{ni}(k+1) = Y_{ni}(k) + v_{ni}(k+1)$$
(12)

Here, n and p are selected as the number of dimension and particle, respectively. k employs current state, k+1 descript the next time step,
$$\beta_1$$
 and

β_2 are constant learning rate by the designer.

The fitness function FIT(.) is defined in (13), which is determined to find the near optimal solutions in the evaluation of the highest fitness value. If Ri presents the parameter set of fuzzy system in the search space PR, $FIT(R_i)$ is denoted as the evaluated fitness value with the input parameter R_i .

$$FIT(R_i) = \exp(-\frac{RMSE}{5}) * OB.$$
 (13)

Where RMSE is considered as the mean square errors of the distance length between the desired target (T) and the robot (R). The OB means the robot tracking state which is selected as 1 when the moving robot not touches the defined block areas. Otherwise, OB is chosen as 0 when the mobile robot successfully goes through the obstacle in the traveling path. From the definition of fitness function, the goal of the evolutionary PSO is to maximize the fitness function vale, i.e. minimize the root mean square error (RMSE) and successfully move to the desired target point. The evolutionary PSO learning algorithm is taken to generate the fuzzy system as following steps,

- Step1) Set the maximal iteration number (G) and initialize g=0. Select appropriate fuzzy rules number and the PSO learning rate (β_1, β_2) in advance. Randomly construct the initial populations.
- Step2) Determine the best parameters values by the proposed evolutionary PSO learning formulas (11)-(12) to achieve the corresponding fuzzy system.
- Step3) Evaluate the fitness value for the related individual particle according by the defined fitness function in (13).
- Step4) Compare each individual's evaluation fitness value with the personal best value (P_best) to select the new P_best. The best evaluation value among the P_best is setting to G_best.
- Step5) g=g+1.
- Step6) If g=G, then go to exit, otherwise go to step 2.
- Step7) The best particle's value will be selected as the finial parameter set to develop the desired fuzzy system.

4 Case Studies and Conclusions

In this experiment study, the developed fuzzy system as an identifier to describe the behavior of the mobile robot. The near-optimal parameters of fuzzy system are automatically built by the evolutional PSO learning algorithm. The PSO swarm sizes =50; generation=50 and $\beta_1 = \beta_2 = 0.75$. If the fuzzy system is first given 6 rules in the initial condition, then 50 particles are randomly generated with the computer simulations to approach the desired fuzzy systems.

4.1 Case 1 Study

In the first case, the robot is starting at (20, 20), the desired target location is to approach toward (80, -60) and the block position is located at (50, -30) for the x-axis and y-axis, respectively. The selected diameter of this block size is denoted as 20. Based on the evaluated objective of the fitness function value, the selected fuzzy embedded control rules can drive the mobile robot from the initial position to the tracking target and avoid the defined obstacle. The near-optimal parameters in solving the proposed mobile robot problem is deserved in the training cycle. The parameter set are selected by the PSO learning algorithm, here it is listed in Table1. Software simulations for the embedded mobile robot problems are illustrated in the plot of Fig. 3. In this experimental case, Fig. 3(a) and Fig. 3(b) are separately shown the line speed and the turning angle of the mobile robot. Fig. 3(c) shows the embedded vision-based tracking results for the mobile robot in the dynamic environment. From this simulation results, mobile robot is gradually and smoothly moving into the target. The appropriate fuzzy rules are applied in this example to simultaneously achieve the target within a shorter time and avoid hitting on the defined obstacle. The best fitness value with respective to the iteration number is displayed in Fig. 3(d). The traced fitness value shows that the selected parameters of the fuzzy system are the near best solutions with the highest fitness value.

4.2 Case 2 Study

In the illustrated second example, the mobile robot is controlled from starting position (20, 20) to the desired target position (80, -60), and the block positions is also defined at (50, -30). The diameter of this block size is changed from 20 cm to 25 cm which is different from the previous example. The selected parameters by the PSO are listed in Table2. From the simulation results of Figure 4, Fig. 4(a) is the trend of moving speed and the turning angle of the mobile robot is illustrated in Fig. 4(b). The moving trace of the mobile robot is shown in Fig. 4(c), it can demonstrate the ability of the embedded vision-based reorganization methods for the mobile robot. This simulations show that the mobile robot can dodge the bigger block area and move into the desired target. Fig. 4(d) shows the best fitness value trace while the iteration number is from 1 to 50. It shows that the best fitness value is gradually toward into the .bigger one. Finally, the best parameters selection is applied to obtain the best solutions.

4.3 Case 3 Study

The case of the initial mobile robot position is selected at the different position (-50, -20), the desired target is defined in the position (80, 50). The block's size is 15 and its position is setting form (50, -30) to (45, 0). The simulation is illustrated in the Figure 5. Here, the trend of moving speed and its related rotating angle are sequentially plotted in Fig. 5(a) and Fig. 5(b). Due to the evolutionary learning PSO machine, the mobile robot can go pass through the define block position. The simulation result of the mobile robot in the dynamic image tracking surroundings is illustrated in Fig. 5(c). The indicated global best solution will generate the related best fitness value in every training cycle. Fig. 5(d) shows the select fitness value in every nitration.

From the illustrated examples, the constructed fuzzy embedded mobile robot system demonstrates that it conations the omni-directional vision-scanned ability to widely detect the environed objects and automatically generate the great fuzzy system with the evolutional PSO learning method for successfully avoiding the obstacles and achieving the desired targets in the traveling path.

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Figure 1. Fuzzy Embedded Mobile Robot System structure.



Figure 2. The mobile robot mathematical plane coordinate system



Figure3. Simulation results of the proposed fuzzy robot system in the first example. Robot initial position (20, 20), target position (80, -60), Block position (50, -30) and block size=20. (a) Time response of the line speed. (b) Time response of the turning angle. (c) Path tracking representation of the mobile robot in the searching space 9c) Fitness-value against the generation.



Figure 4. Simulation results of the proposed fuzzy robot system in the first example. Robot initial position (20, 20), target position (80, -60), Block position (50, -30) and block size=25. (a) Time response of the line speed. (b) Time response of the turning angle. (c) Path tracking representation of the mobile robot in the searching space 9c) Fitness-value against the generation.



Figure 5. Simulation results of the proposed fuzzy robot system in the first example. Robot initial position (-50, -20), target position (80, 50), Block position (45, 0) and block size=15. (a) Time response of the line speed. (b) Time response of the turning angle. (c) Path tracking representation of the mobile robot in the searching space 9c) Fitness-value against the generation.

| - | | | 5 | | 00 | | | |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------------|----------------|
| • | a _{i1} | a _{i2} | a _{i3} | b _{i1} | b _{i2} | b _{i3} | y ₂ | y _i |
| i=1 | 5.0000 | 63.8999 | 58.3316 | -50.0000 | 34.4876 | 49.7966 | 5.3433 | 47.4889 |
| i=2 | -0.0357 | 5.0845 | 91.5847 | -42.4167 | 50.0000 | 48.2880 | 6.1949 | -14.6472 |
| i=3 | -4.4988 | -4.4988 | 16.8733 | 49.3105 | 3.8130 | 3.8130 | 0.6843 | 0.6843 |
| i=4 | 5.0000 | 69.1941 | 71.8848 | -50.0000 | 50.0000 | 10.4840 | 8.4863 | 35.0714 |
| i=5 | 0.0417 | 120.0000 | 120.0000 | 50.0000 | 35.9760 | 34.0224 | 0.0100 | 46.4851 |
| i=6 | 4.9211 | 45.6678 | 110.7400 | -47.4813 | 6.2067 | 36.2036 | 10.0000 | 49.3449 |

Table1. Parameters selection by the PSO learning algorithm for the illustrated case 1

 Table2. Parameters selection by the PSO method for the case 2

| | a :1 | a :2 | a:2 | ba | ha | h:2 | V ₂ | V: |
|-----|-------------|---|-------------|----------|---------|---------|----------------|------------|
| | ••11 | ang | u 13 | | 012 | 013 | J2 | <i>J</i> 1 |
| i=1 | 5.0000 | 5.0000 | 72.7218 | -50.0000 | 30.0020 | 31.3533 | 5.4697 | 50.0000 |
| i=2 | 4.8042 | 0.1000 | 94.5027 | 13.7892 | 50.0000 | 49.5807 | 2.9968 | -24.3098 |
| i=3 | -4.7811 | 64.4812 | 46.6093 | 39.8709 | 47.9294 | 26.6063 | 0.2898 | 25.9457 |
| i=4 | -1.9517 | 55.9016 | 25.2580 | -50.0000 | 50.0000 | 12.7851 | 9.9617 | 38.8438 |
| i=5 | 2.3448 | 119.7996 | 119.9999 | 33.9722 | 27.2709 | 27.2709 | 0.5019 | 30.3399 |
| i=6 | 30.3399 | 50.3113 | 95.5806 | -7.2437 | 6.2194 | 47.1851 | 3.0135 | 50.0000 |

 Table 3. Parameters selection by the PSO learning algorithm is for case 3

| | a _{i1} | a _{i2} | a _{i3} | b _{i1} | b _{i2} | b _{i3} | y ₂ | y _i |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------------|----------------|
| • 1 | 5 0000 | 0 1000 | 14 5022 | 22.20.40 | 26,6002 | 11 (074 | 10,0000 | 06.2200 |
| 1=1 | 5.0000 | 0.1000 | 14.5033 | 32.2948 | 30.0882 | 11.6974 | 10.0000 | 86.3299 |
| i=2 | 1.5230 | 12.7531 | 0.0100 | -50.0000 | 4.8292 | 24.8850 | 92.4642 | 9.4558 |
| i=3 | 47.4682 | 16.6912 | 0.8686 | -39.9726 | 23.4572 | -44.2988 | 21.8797 | 45.9611 |
| i=4 | -2.1266 | 54.9712 | 54.9712 | 44.4830 | 0.0100 | 14.5282 | 10.0000 | -88.934 |
| i=5 | 17.8598 | 42.0857 | 11.1932 | -37.6687 | 4.9983 | 5.5492 | 78.5923 | 23.4501 |
| i=6 | 0.0100 | 49.4292 | 1.5949 | 93.5782 | 31.9320 | -16.7225 | 37.8075 | -36.6087 |