Comparison of Travel-time statistics of Backscattered Pulses from Gaussian and Non-Gaussian Rough Surfaces

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Abstract: - Travel-time statistics of backscattered pulses from Gaussian and non-Gaussian (Rayleigh and Exponential) surfaces are studied and compared in this paper. It is found that in both cases (the Gaussian and non-Gaussian surfaces), the probability distributions of the first and the second travel times are all only determined by a dimensionless parameter. Moreover, this study also found that the proportional relations between the dimensionless parameter and the median values of the first and the second travel times are remarkably different for Gaussian and non-Gaussian cases. Then the statistical parameters of the rough surface can be estimated from travel-time statistics of backscattered pulses above if the probability distributions of the surface elevation and slope (PDSES) are known in advance. However, PDSES are often unknown in most of practical applications, and little work has been applied to the problem of specifying the statistical characteristic of the rough surface in advance (Gaussian or non-Gaussian). To solve this problem, on the basis of the relations between the median values of travel times and the altitude of source, a new theoretical method is proposed to judge the rough surfaces with Gaussian, Rayleigh and Exponential distributions. It is believed that the method proposed in this paper will be helpful to improve the estimate accuracy the parameters of a rough surface.

Key-Words: probability distribution, travel time, backscatter, random surface.

1 Introduction

Travel times of backscattered waves from a random rough surface, especially the first arrivals of the backscattered signals play an important pole in a variety of applications to infer information about surface roughness and properties of the propagation medium [1]. For example, geophysical applications that employ the first arrivals include airborne laser altimetry [2], seismic and ocean acoustic tomography [3], satellite radar altimetry [4], etc.

In general, if a rough surface is random, travel time of a backscattered short pulse from the surface must be random too. Therefore, the statistics properties of travel-time have been the object of theoretical analyses by various authors. Frolov [5,6] studied the travel-time statistics of the signal backscattered from a rough surface in the limiting cases of large-scale roughness. Elfouhaily[7][8] solved the problem of the travel-time statistics in small-scale limit. Godin and Fuks [9] extended the results above to the three-dimensional problem. Recently on the basis of the mathematical theory of excursions of random functions[10,11], Fuks, Godin, *et. al* [12-14] obtained the probability distributions of travel times and intensity of the first and second arrivals of a short pulse backscattered by a rough surface with a Gaussian statistics, within the geometrical optics approximation. GAO and WANG [15] then investigated the corresponding inverse problem for the Gaussian case.

In the present paper, the problem of the first and the second travel times of a pulse backscattered by non-Gaussian rough surfaces (Rayleigh and Exponential distribution) is first studied and compared to the Gaussian case. The probability distributions, the median values of the travel times of the first and the second arrivals and the time delay between the first and the second arrivals are investigated theoretically and numerically. And the comparison of the results between Gaussian and non-Gaussian cases and many new, different conclusions are also shown in this article. Furthermore, to improve the estimation accuracy of the parameters of a rough surface, a new theoretical method based on travel-time statistics of backscattered pulses is presented to judge the rough surfaces with Gaussian, Rayleigh and Exponential distributions.

Arrangement of the paper is as follows. In section 2, we briefly recall some fundamental ingredients of the theory of Fuks and Godin [12] on probability distribution functions of the first and the second

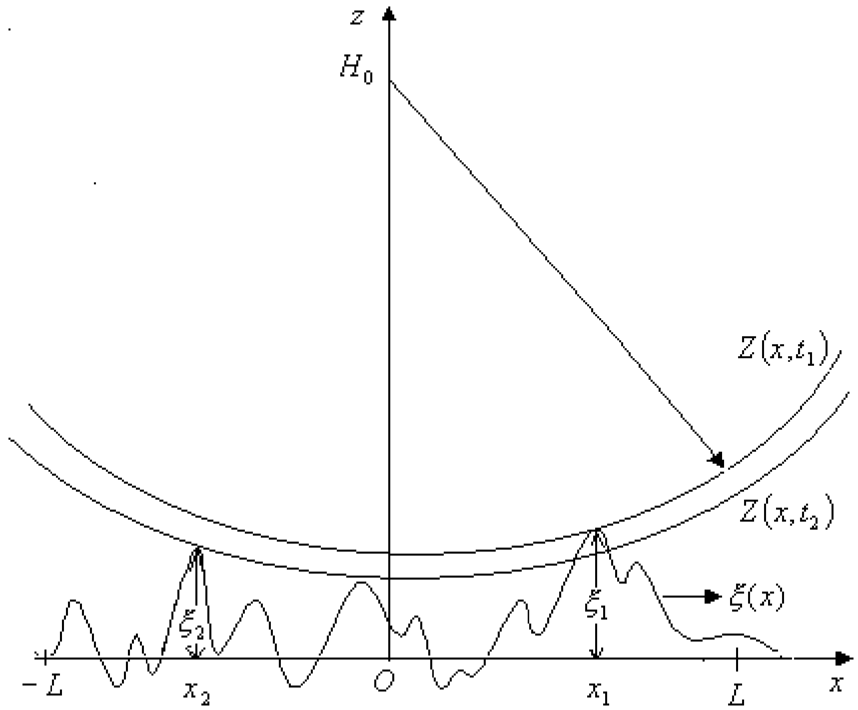


Fig. 1. Problem geometry and notation. A point source located at $(0, H)$. (x_1, t_1) and (x_2, t_2) are the first and the second points of contact between the wave front $Z(x,t)$ and the rough surface $\xi(x)$, respectively.

travel times of backscattered pulses from a rough surface. In section 3.1, these equations are applied to the case of rough surfaces with non-Gaussian (Rayleigh and Exponential) statistics. In section 3.2, the median values of travel times of the first and the second arrivals are discussed. In section 3.3, we investigate the probability distributions of time delay between the first and the second arrivals. Our summaries are presented in section 4.

2 Fundamental equations

Let us consider two-dimensional problem of a backscattered short pulse from a random rough surface, and do not concern the propagation effects of the medium between the source and the surface. The random surface is characterized by the equation $z = \xi(x)$, (see Figure 1). A point source is located in the Cartesian coordinates $(0, H_0)$ and emits a short pulse at the moment of time $t = 0$. The wave front makes the first contact with the surface at some point (x_1, t_1). The first point of the contact must be a specularly reflecting point, namely the tangent plane to the surface at (x_1, t_1) is perpendicular to the ray connecting the source and the reflecting point. According to [12], within the interval $x \in (-L, L)$ the probability of the absence of crossings between the random surface and the wave front satisfied the equation:

$$\mathrm{P}\{t_1 > t\} = \exp[-\int_{-L}^{L} \lambda(x,t)dx] \tag{1}$$

where

$$\lambda(x) = \int_{\dot{Z}(x)}^{\infty} d\dot{\xi}[\dot{\xi} - \dot{Z}(x)]\omega[Z(x), \dot{\xi}] \tag{2a}$$

$$\lambda(x) = \int_{\dot{Z}(x)}^{\infty} d\dot{\xi}[\dot{\xi} - \dot{Z}(x)]\omega[Z(x), \dot{\xi}]\left[\int_{-\infty}^{Z(x)} \omega(\xi)d\xi\right]^{-1} \tag{2b}$$

where $Z(x)$ denoting the equation of the wave front, $\omega(\xi, \dot{\xi})$ is the joint probability density function (PDF) of the surface elevation $\xi(x)$ and slopes $\dot{\xi}(x) = d\xi(x)/dx$. Namely, Equation (1) represents the probability of the event (there is not a specularly reflecting point at the interval $x \in (-L, L)$). It is noted that Equation (2a) is the approximation given by Fuks and Godin [12] and

Equation (2b) is another different approximation result obtained by Smith [16].

For the reader's convenience, we introduce the dimensionless travel time difference $\tau = \frac{H-R}{\sigma}$, (where $R = ct$, c is the signal propagation speed, σ is the root of mean square (rms) of the elevation, H is the distance between the source and the mean surface) and investigate the statistical properties of τ. It is obvious that the relation between H and H_0 satisfies the following form:

$$H_0 = H + \langle \zeta \rangle \quad (3)$$

where, $\langle \zeta \rangle$ is the mean of the elevation ξ of the surface.

Then the probability distribution functions $F_1(\tau)$ of the normalized time τ_1 of the first arrival and $F_2(\tau)$ of the second arrival satisfy:

$$F_1(\tau) = \mathrm{P}\{\tau_1 < \tau\} = \mathrm{P}\{t_1 > t\} \quad (4)$$

$$F_2(\tau) = \mathrm{P}\{\tau_2 < \tau\} = 1 - \mathrm{P}\{t_2 < t\} \quad (5)$$

3 Comparison between Gaussian and non-Gaussian surfaces

Refs.[12] and [15] have studied the case of the backscattering surface with Gaussian distribution. In this section, we should apply the mathematical theory of excursions of random functions to investigate the problems of two specific random surfaces satisfied the non-Gaussian distributions: Rayleigh or Exponential distribution, and give the comparison between Gaussian and non-Gaussion cases in the following paper.

3.1 The first and second travel time

Firstly, we study the case of Rayleigh surfaces. When the surface is a Rayleigh distribution (**RD)**, the PDFs of surface elevation ξ and slope $\dot{\xi}$ can be written:

$$\omega(\xi) = \begin{cases} \frac{\xi}{\sigma_1^{\ 2}} \exp(-\frac{\xi^2}{2\sigma_1^{\ 2}}), \xi > 0 \\ 0, \xi \le 0 \end{cases} \quad (6)$$

$$\omega(\dot{\xi}) = \begin{cases} \frac{\dot{\xi}}{\gamma_1^{\ 2}} \exp(-\frac{\dot{\xi}^2}{2\gamma_1^{\ 2}}), \dot{\xi} > 0 \\ 0, \dot{\xi} \le 0 \end{cases} \quad (7)$$

It is noted that the variances of the elevations and slopes are $\frac{4-\pi}{2}\sigma_1^{\ 2}$ and $\frac{4-\pi}{2}\gamma_1^2$, respectively. And the means of ξ and $\dot{\xi}$ equal to $\sqrt{\frac{\pi}{2}}\sigma_1$ and $\sqrt{\frac{\pi}{2}}\gamma_1$.

For simplifying analysis, we consider the surface elevation and slope are independent statistically at the same point as considered by Fuks and Godin [12]. Then the cumulative PDF of the surface elevation and slope can be written:

$$\omega(\xi, \dot{\xi}) = \omega(\xi) \times \omega(\dot{\xi}) \quad (8)$$

When the inequality $x \le L << R(t)$ holds, the wave front equation can be represented approximately in the following form:

$$Z(x,t) = H_0 - \sqrt{R^2(t) - x^2} \cong H_0 - R(t) + \frac{x^2}{2R(t)} \quad (9)$$

Substituting (6)-(9) into (1), (2) and (4), we obtain the probability function $F_1(\tau')$:

$$F_1(\tau') = \exp\left\{-\pi\sqrt{2T'}\int_{-L_1'}^{L_1'} \Phi(\tau')dy'\right\} \quad (10a)$$

$$F_1(\tau') = \exp\left\{-\pi\sqrt{2T'}\int_{-L_1'}^{L_1'} \frac{\Phi(\tau')}{1-\exp[-\frac{(\tau'+y'2)^2}{2}]}dy'\right\} \quad (10b)$$

where

$$\tau' = \sqrt{\frac{4-\pi}{2}}\tau + \sqrt{\frac{\pi}{2}},$$

$\tau = \frac{H-R}{\sigma}$,
$T' = \sqrt{\frac{2}{4-\pi}}T$,
$T = \frac{\gamma_0^2 H}{2\pi\sigma}$,
$y' = \frac{x}{\sqrt{2H\sigma_1}}$,
$L_1{}' = \frac{L}{\sqrt{2H\sigma_1}}$,
$\Phi(\tau') =$
$\left[(\tau'+y'^2)\exp[(-\frac{1}{2})(\tau'+y'^2)^2 erfc(\frac{y'}{\sqrt{2\pi T'}}) \right]$.

where, σ^2 and γ_0^2 are variances of the elevations and slopes, respectively. And the definitions of τ and T are the same with Fuks and Godin [12]. It is clearly that the probability of the travel time of the first arrival is only the function of the dimensionless parameter T, which depends on the height of the source and variances of the surface statistics.
Secondly, when the surface satisfies Exponential distribution **(ED)**, the PDFs of the surface elevation ξ and slope $\dot{\xi}$ are written:

$$\omega(\xi) = \begin{cases} \frac{1}{\sigma_2}\exp(-\frac{\xi}{\sigma_2}), \xi > 0 \\ 0, \xi \le 0 \end{cases}$$
(11)

$$\omega(\dot{\xi}) = \begin{cases} \frac{1}{\gamma_2}\exp(-\frac{\dot{\xi}}{\gamma_2}), \dot{\xi} > 0 \\ 0, \dot{\xi} \le 0 \end{cases}$$
(12)

where, σ_2^2 and γ_2^2 are the variances of the elevations and slopes, respectively. The mean value of ξ and $\dot{\xi}$ are σ_2 and γ_2.

At this case, the cumulative **PDF** also satisfied Equation (8). Substituting (8)-(12) into (1), (2) and (4), yields the probability function $F_1(\tau'')$:

$$F_1(\tau'') = \exp\left\{ \begin{array}{l} -2\sqrt{\pi T''}\exp(-\tau'') \\ \int_{-L_2''}^{L_2''} \exp[-(y''^2 + \frac{y''}{\sqrt{\pi T''}})dy''] \end{array} \right\}$$
(13a)

$$F_1(\tau'') = \exp\left\{ \begin{array}{l} -2\sqrt{\pi T''}\exp(-\tau'') \\ \int_{-L_2''}^{L_2''} \frac{\exp[-(y''^2 + \frac{y''}{\sqrt{\pi T''}})}{1-\exp[-(\tau''+y''^2)]}dy''] \end{array} \right\}$$ (13b
)

Where

$\tau'' = \tau + 1$,
$T'' = T$,
$y'' = \frac{x}{\sqrt{2H\sigma_2}}$,
$L_2{}'' = \frac{L}{\sqrt{2H\sigma_2}}$.

The PDFs of the first and the second travel times are given by:

$$\omega_n(\tau) = \frac{F_n(\tau)}{d\tau}, n = 1,2 \quad (14)$$

We conclude from (13) that the probability distribution of the travel time of the first arrival is only determined by the dimensionless parameter T. Furthermore, the discrepancy between (10a) and (10b) or between (13a) and (13b) can be neglected for any $\tau > 0$ when $T >> 1$ in terms of our numerical results. Therefore, it is reasonable that we only apply the Equation (a) in the following analysis.

In Figure 2, the probability distribution functions of $F_1(\tau)$ *, and the corresponding PDFs are shown for a set of T parameters for the surfaces with Gaussian (solid), Rayleigh (dotted) and Exponential (dashed) distributions, respectively.

According to [12], the probability distribution function $F_2(\tau)$ of the travel time of the second arrival satisfies the following equation:

* From equations (10) and (13), we may obtain $F_1(\tau) = F_1(\sqrt{\frac{2}{4-\pi}}\tau' - \sqrt{\frac{\pi}{4-\pi}})$ for Rayleigh distribution and $F_1(\tau) = F_1(\tau''-1)$ for Exponential distribution

$$F_2(\tau) = [1 - \ln F_1(\tau)] \times F_1(\tau)$$
(15)

Then the probability functions F_2 , the PDFs $\omega_2(\tau)$ of the second travel time are easily obtained by using Equations (10)-(15), as shown in Figure 3. The parameters are taken same as that in Figure 2. From Figure 2 and 3, it is seen that for a given T $F_1(\tau)$ and $F_2(\tau)$ are different for three rough surfaces. For the peak values of PDFs of travel times of the first and second arrivals, the Gaussian surface correspond to the maximum, and the Exponential surface is minimum. The maximums of PDFs of travel times increase with T increasing for Rayleigh and Gaussian surfaces and are not related with T approximately for Exponential distributions.

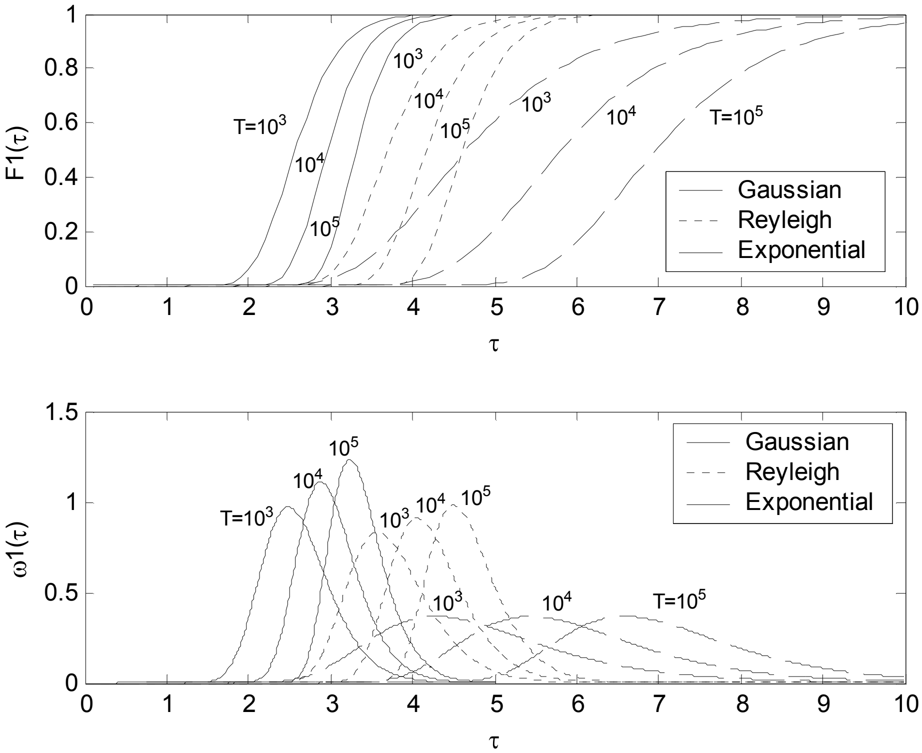


Fig. 2. Comparison of probability distributions of travel time of the first arrival of a short pulse backscattered by Gaussian (solid lines), Rayleigh(dotted lines) and Exponential(dashed lines) surfaces.

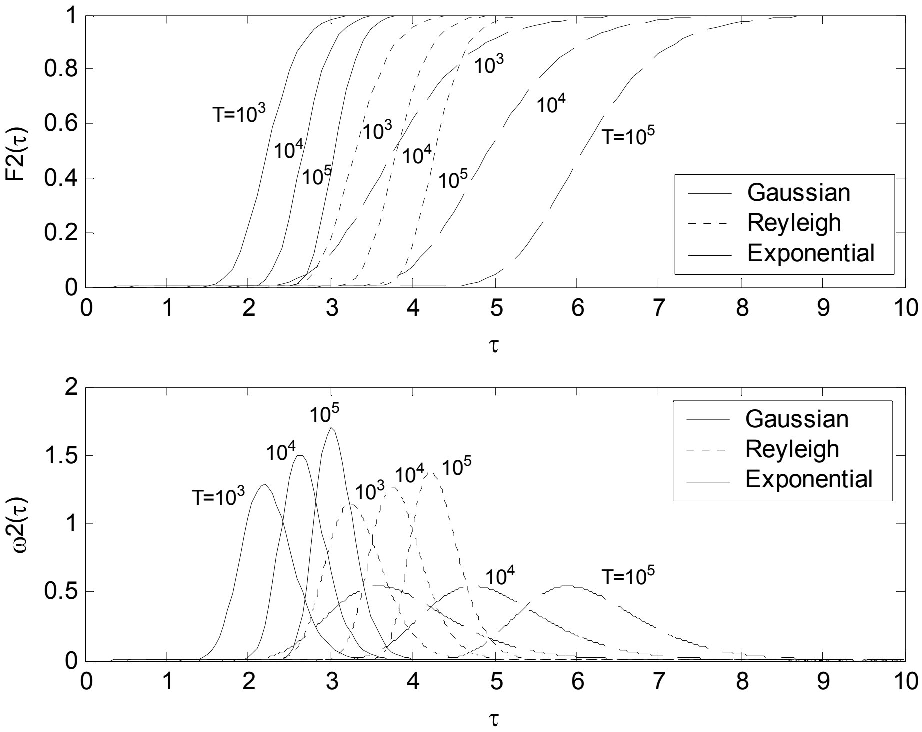


Fig.3. Comparison of probability distributions of travel time of the second arrival of a short pulse backscattered by Gaussian (solid lines), Rayleigh(dotted lines) and Exponential(dashed lines) surfaces.

3.2 The median value of travel time

The median values of the first and second travel times are defined as the following forms:

$$F_1(\tau_{1m}) = \frac{1}{2} \quad , \quad F_2(\tau_{2m}) = \frac{1}{2}$$
(16)

In the limiting case $\tau >> 1$, the equations (10) and (13) have the following simple expressions

$$F_1(\tau') = \exp[-\pi\sqrt{2\pi\tau' T'}\exp(-\frac{\tau'^2}{2})]$$
(17)

$$F_1(\tau'') = \exp[-2\pi\sqrt{T''}\exp(-\tau'')]$$
(18)

Accepting the equations (17) and (18), in term of the definitions of median value of travel times of the first and the second arrival, the median values τ'_{1m} and τ''_{1m} should satisfy the following equations

$$\exp[-\pi\sqrt{2\pi\tau'_{1m}T'}\exp(-\frac{\tau'^{\,2}_{1m}}{2})] = \frac{1}{2} \qquad \textbf{RD}$$
(19)

$$\exp[-2\pi\sqrt{T''}\exp(-\tau''_{1m})] = \frac{1}{2} \qquad \textbf{ED}$$
(20)

When $\ln T >> 1$, applying the similar method in [12], the asymptotic solution to (19) is given

$$\tau_{1m} \approx \sqrt{\frac{2}{4-\pi}\times(\ln T + 5.98)} - \sqrt{\frac{\pi}{4-\pi}} \qquad \textbf{RD}$$
(21)

We may easily obtain the solution to (20)

$$\tau_{1m} \approx \frac{1}{2}\ln T + 2.2 \qquad \textbf{ED}$$
(22)

Then the median value τ'_{2m} and τ''_{2m} of the second arrivals takes the form similar to (21) and (22):

$$\tau_{2m} \approx \sqrt{\frac{2}{4-\pi}(\ln T + 4.21)} - \sqrt{\frac{\pi}{4-\pi}} \quad \textbf{RD}$$
(23)

$$\tau_{2m} \approx \frac{1}{2}\ln T + 1.32 \quad \textbf{ED}$$
(24)

At the case of $\ln T >> 1$, the difference between the median values is given

$$\delta\tau_{12} = \tau_{1m} - \tau_{2m} \approx \frac{1.34}{\sqrt{\ln T}} \quad \textbf{RD}$$
(25)

$$\delta\tau_{12} = \tau_{1m} - \tau_{2m} \approx 0.88 \quad \textbf{ED}$$
(26)

Table 1 shows the median values and time delay of travel times of the first and the second arrivals for the Gaussian, Rayleigh and Exponential cases. It can be seen from Table 1 that for the Rayleigh distribution, the median values of the travel times of the first and second arrivals are proportional to $\sqrt{\ln T}$, and the difference between τ_{1m} and τ_{2m} are inversely proportional to $\sqrt{\ln T}$. There exits similar relation in the case of Gaussian distribution[12], but these proportional constants are all different to the that of Gaussian surface. For example, the proportional constant between τ_{1m} (or τ_{2m}) and $\sqrt{\ln T}$ is $\sqrt{\frac{2}{4-\pi}}$ at the case of Rayleigh surface, but 1 for the Gaussian surface. And for the Exponential distribution, it is found that the mean values τ_{1m}, τ_{2m} are proportional to $\ln T$, and $\delta\tau_{12}$ is a constant and not associated with the parameter T.

3.3 Relations between the median values of travel times and the altitude of source

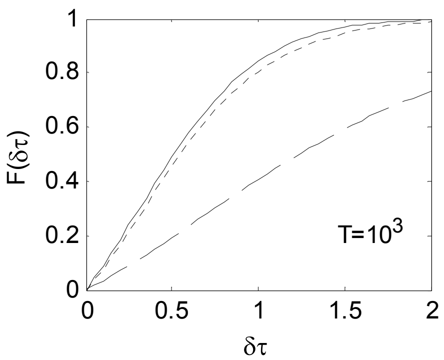
The dimensionless parameter T can be written as:

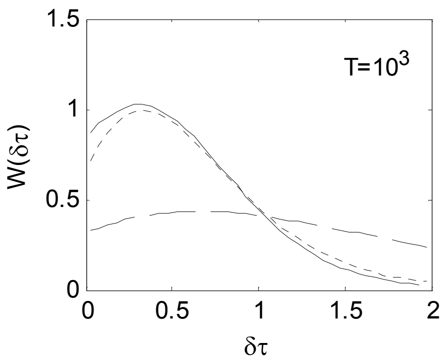
$$T = H \times S \tag{27}$$

where H is the altitude of source, $S = \frac{\gamma_0^2}{2\pi\sigma}$. As is known that $\ln(H \times S) = \ln H + \ln S$, the relations between the median values of travel times and the altitude of source are given

$$\mathrm{Z} = \mathrm{K} \times \ln H + \mathrm{B} \tag{28}$$

where Z is the function of τ_{1m} (or τ_{2m}), which is different for the Gaussian, Rayleigh and Exponential surfaces cases (see Table 2), K is the slope, B is a constant. The relations between the median values of travel times and the altitude of source for the Gaussian, Rayleigh and Exponential cases are shown in Table 2. Z and K are calculated in terms of the analytical expresses shown in Table 1. K_1, K_2 and K_3 are the numerical results when H is varying in the range $[1,1.5]\times 10^4$, 10^7 and 10^{10}, respectively. It can be





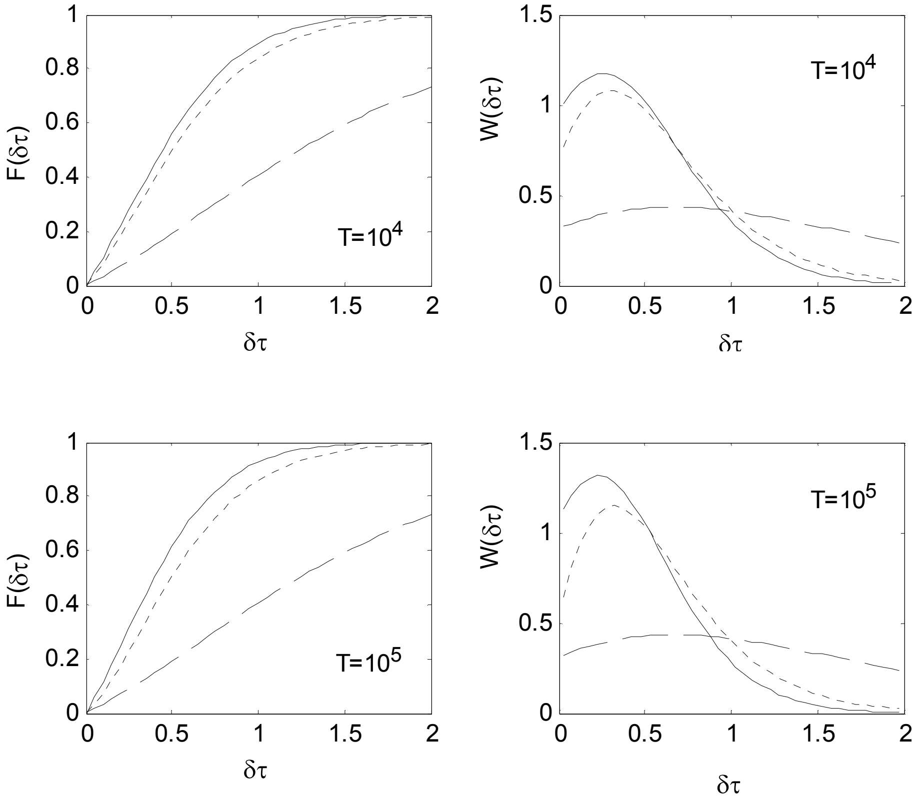


Fig. 4. The probability distributions and corresponding **PDF**s of the travel time delay between the first and the second arrivals of a short pulse backscattered by Gaussian (solid lines), Rayleigh (dotted lines) and Exponential (dashed lines) surfaces.

seen from Table 2 that there is the linear relation between Z and K. And the values of K are almost a constant. It is clear that the relations between the median values of travel times and the altitude of source are different for the Gaussian, Rayleigh and Exponential surfaces cases. Therefore, the results shown in Table 2 suggest that the relation between τ_{1m} (τ_{2m}) and the altitude of source can be used to judge whether the scattering surface is Gaussian distribution or not. After the statistical characteristic of the rough surface is specified in advance, we may apply the similar method in Refs.[] to obtain the parameters of the rough surface.

3.4 Time delay between the first and the second arrival

In [12], Fuks and Godin think the scale of time delay τ being sensitive only to σ. Thus, we should discuss the statistical properties of the time delay between the first and second arrival in this subsection. It is helpful to determine the parameters of surface with non-Gaussian distributions under our considering circumstance.

Table. 1. The median values and time delay of the first and the second arrivals of a short pulse backscattered by Gaussian (GD), Rayleigh (RD) and Exponential surfaces (ED)

	GD	RD	ED
$\tau_{1m} \approx$	$\sqrt{\ln T + 0.73}$	$\sqrt{\frac{2}{4-\pi} \times (\ln T + 5.98)} - \sqrt{\frac{\pi}{4-\pi}}$	$\frac{1}{2}\ln T + 2.2$
$\tau_{2m} \approx$	$\sqrt{\ln T - 1.04}$	$\sqrt{\frac{2}{4-\pi} (\ln T + 4.21)} - \sqrt{\frac{\pi}{4-\pi}}$	$\frac{1}{2}\ln T + 1.32$

$\delta\tau_{12} \approx$	$\frac{0.88}{\sqrt{\ln T}}$	$\frac{1.34}{\sqrt{\ln T}}$	0.88

Table. 2. The relations between the median values of travel times and the altitude of source for Gaussian (GD), Rayleigh (RD) and Exponential cases (ED)

	GD	RD	SD
Z	τ_{1m}^2 、 τ_{2m}^2	$\left(\tau_{1m} + \sqrt{\frac{\pi}{4-\pi}}\right)^2$ 、 $\left(\tau_{2m} + \sqrt{\frac{\pi}{4-\pi}}\right)^2$	τ_{1m} 、 τ_{2m}
K	1	2.33	0.5
K_1	0.95	2.40	0.5
K_2	0.97	2.38	0.5
K_3	0.98	2.36	0.5

The probability distribution function of the time delay between the first and second arrivals is also derived:

$$F(\delta\tau) = 1 - C\int_0^\infty \omega_1(\tau+\delta\tau)F_2(\tau)d\tau$$

(29)

$$W(\delta\tau) = C\theta(\delta\tau)\int_0^\infty \omega_1(\tau+\delta\tau)\omega_2(\tau)d\tau$$

(30)

Where

$$C = \left[\int_0^\infty \omega_1(\tau)F_2(\tau)d\tau\right]^{-1} = \left[1-\int_0^\infty \omega_2(\tau)F_1(\tau)d\tau\right]^{-1},$$

$\theta(x)$ is the unit step function:

$$\theta(x) = \begin{cases} 1, x \ge 0 \\ 0, x < 0 \end{cases}$$

(31)

Figure 4 is the probability distributions and corresponding **PDF**s of the travel time delay between the first and second arrivals for Gaussian (solid lines), Rayleigh (dotted lines) and Exponential (dashed lines) surfaces. The values of T in Figure 4 are chosen to 10^3 , 10^4 and 10^5 , respectively.

From Figure 4, the **PDF** of time delay between the first and second arrivals is determined by the parameter T for Rayleigh surfaces. At the case of Exponential distributions, the **PDF**s of time delay are holding constant for different values of T . When $\delta\tau$ is small, the **PDF** of the case of Gaussian is maximum, Rayleigh is following and Exponential is minimum, while for the large $\delta\tau$, on the contrary, that is, Exponential is the maximum and Gaussian is the minimum.

4 Conclusion

We have considered the problem of travel time of a short pulse backscattered from rough surface with special non-Gaussian (Rayleigh and Exponential) distributions, which are different from [12]. The main results obtained are summarized below:

(1) For all consider cases, the probability distributions of travel times of the first and second arrival are only the function of the dimensionless parameter T .

(2) For the Rayleigh distribution, the median values of the travel times of the first and second

arrivals are proportional to $\sqrt{\ln T}$, and the difference $\delta\tau_{12}$ between τ_{1m} and τ_{2m} are inversely proportional to $\sqrt{\ln T}$. The proportional constants of these relations above are all different to that of Gaussian surface. For the Exponential distribution, τ_{1m} and τ_{2m} are proportional to $\ln T$, and $\delta\tau_{12}$ is independent approximately with the parameter T .

(3) It is confident that we are able to distinguish surfaces among Gaussian, Rayleigh and Exponential by use of the results in Table 2 in section 3.3. After the statistical characteristic of the rough surface is specified in advance, the results of this paper can be applied to retrieve the parameters of the non-Gaussian (Rayleigh and Exponential) random surface.

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