Observer design for nonlinear systems represented by Takagi-Sugeno models

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Abstract: In this paper, the problem of synthesis of a multiple observer for a class of uncertain nonlinear system represented by a Takagi-Sugeno multiple model is studied. The measure's uncertainties are considered as unknown outputs. To conceive the observer a mathematical transformation is considered to conceive an augmented system in which the unknown output appear as an unknown input. Convergence conditions are established in order to guarantee the convergence of the state estimation error. These conditions are expressed in Linear Matrix Inequality (LMI) formulation. An example of simulation is given to illustrate the proposed method.

Key–Words: Takagi-Sugeno models, multiple observer, state estimation, unknown inputs, unknown outputs, measures uncertainties.

1 Nomenclature

$x(t) \in R^n$: the state vector
$\hat{x}(t) \in R^n$: the estimate state vector
$u(t) \in R^m$: the input vector
$y(t) \in R^p$: the measured output
$A_i \in R^{n \times n}$: are the state matrices
$B_i \in R^{n \times m}$: are the matrices of input
$C \in R^{p \times n}$: is the output matrix
$\xi(t)$: the vector of decision
M: is the number of local models.

2 Introduction

The state reconstruction of an uncertain system is a traditional problem of the automatic. The observer of Luenberger is not always sufficient for the fault detection, because the state estimation error given by this observer for an uncertain system or with unknown inputs does not converge inevitably towards zero.

In the case of linear systems, observers can be designed for uncertain system with time-delay perturbations [8] and unknown input systems [6]. Many researchers have paid attention to the problem of state estimation of dynamic linear systems subjected to both known and unknown inputs [6, 23]. These works can be gathered into two categories [2]. The first one supposes an a priori knowledge of information on these nonmeasurable inputs; in particular, Johansen [16] proposes a polynomial approach and Meditch [19] suggests approximating the unknown inputs by the response of a known dynamic system. The second category proceeds either by estimation of the unknown inputs, or by their complete elimination from the equations of the system [11].

However, in the majority of real cases the nonlinear nature of the process cannot be neglected. The assumption of linearity is checked only in a limited vicinity of a particular operating point. Indeed, the physical systems present complex behaviours utilizing nonlinear laws. As, it is delicate to synthesize an observer for a nonlinear system, the multiple model approach constitutes a tool which is largely used in the modeling of nonlinear systems [20]. The idea of the multiple model approach is to apprehend the total behavior of a system by a set of local models. Each local model can be for example a linear time invariant system valid around an operation point. The relative contribution of each submodel is quantified with the help of a weighting function. Finally, the approximation of the system behaviour is performed by associating the submodels and by taking into consideration their respective contributions.

The choice of the structure used to associate the submodels constitutes a key point in the multiple modelling framework. Indeed, the submodels can be aggregated using various structures [9]. Classically, the association of submodels is performed in the dynamic equation of the multiple model using a common state vector. This model, known as Takagi-Sugeno multiple model, has been initially proposed, in a fuzzy modelling framework, by Takagi and Sugeno [24] and in a multiple model modelling framework by Johansen and Foss [15].

In the case of nonlinear systems, various studies dealing with the presence of uncertainties were published [1, 21]. The problem of state estimation of nonlinear systems submitted to unknown inputs has received considerable attention [3, 12, 17, 18]. The recourse to the use of an unknown input observer is necessary in order to be able to estimate the state of the nonlinear system [1]. For state estimation, the suggested technique consists in associating to each local model a local unknown input observer. The considered observer is then a convex interpolation of these local observers. This interpolation is obtained throughout the same activation functions as the Takagi-Sugeno model.

Our contribution lies in the design of observers for nonlinear systems using Takagi-Sugeno models. This paper proposes a method to extend the approach of synthesis of observers with unknown outputs to taking into account, on the one hand, the unknown outputs, and on the other hand, the measures uncertainties. A mathematical transformation, proposed in the linear case in [7] and used in [13, 17, 28], is used allowing us to consider unknown outputs in the form of unknown inputs. So, one gets to the conception of multiple observers based on the elimination of these unknown inputs.

The paper is organized as follows. Section 3 presents an overview of the multiple model approach. Section 4 describes the principle of the synthesis of a multiple observer with unknown inputs. In section 5, the main results to design observers under LMI formulation are given. Finally, in section 6, a simulation example is given to show the validity of the proposed method.

3 Multiple model approach

The principle of the multiple model approach is based on the reduction of the system complexity by the decomposition of its operation space in a finished number of operation zone. Each zone is characterized by a local model or sub-model. Each sub-model is a simple and linear system around an operation point. The total behavior of the nonlinear system is obtained thus by the sum of the local models balanced by weighting functions associated to each of them.

Two main structures of multiple models, uncoupled structure and coupled structure, can be distinguished according to nature from the coupling between local models. The coupled structure or the Takagi-Sugeno structure provides a useful tool to represent with a good precision a large class of nonlinear systems [25]. The main advantage of T-S structure is its simplicity because it originates from the interpolation between linear systems. Thus, analysis and design methods developed for linear systems can be generalized to nonlinear systems.

The Takagi-Sugeno multiple model representation is given by:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + D_i) \\ y(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(C_i x(t) + E_i u(t) + N_i) \end{cases} \tag{1}$$

where $\mu_i(\xi(t))$ are the activation functions. $\xi(t)$ may depend on the known input and/or the measured state variables.

If $E_i = N_i = 0$ and the output $y(t)$ is linear, i.e. $(C_1 = C_2 = ... = C_M = C)$, the structure of the Takagi-Sugeno multiple model becomes:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + D_i) \\ y(t) = Cx(t) \end{cases} \tag{2}$$

where D_i is introduced to take into account the operating point of the system. M is the number of local models. It depends on the precision of desired modeling, the complexity of the nonlinear system and the choice of the structure of the weighting functions.

Matrices A_i, B_i, D_i and C can be obtained by using the direct linearisation of an a priori nonlinear model around operating points, or alternatively by using an identification procedure [4, 10, 14].

The weighting functions $\mu_i(\xi(t))$ are associated to each operating zone. They are nonlinear in $\xi(t)$. They satisfy the following convex sum properties:

$$\sum_{i=1}^{M} \mu_i(\xi(t)) = 1 \quad , \quad 0 \leq \mu_i(\xi(t)) \leq 1 \tag{3}$$

4 Multiple observer with unknown inputs

This part is dedicated to the state estimation of a nonlinear system perturbed by unknown inputs. The design of this multiple observer is based on the elimination of these unknown inputs.

The following Takagi-Sugeno multiple model representing a nonlinear system with unknown inputs is considered:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + R\bar{u}(t) \\ \qquad\qquad\qquad\qquad\qquad\qquad + D_i) \\ y(t) = Cx(t) \end{cases} \tag{4}$$

where $\bar{u}(t) \in R^q, q < n$ represents the vector of unknown inputs and R is the distribution matrix of unknown inputs.

Consider the global multiple observer described as follows [3]:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i z(t) + G_{i1} u(t) + G_{i2} \\ \qquad\qquad\qquad\qquad\qquad\qquad + L_i y(t)) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases} \tag{5}$$

$N_i \in R^{n*n}$, $G_{i1} \in R^{n*m}$, $L_i \in R^{n*p}$ is the gain of the ith local observer, $G_{i2} \in R^n$ is a constant vector and E is a matrix transformation. All these matrices or vectors have to be determined in order to guarantee the asymptotic convergence of $\hat{x}(t)$ towards $x(t)$.

The state estimation error is given by:

$$e(t) = x(t) - \hat{x}(t) = (I + EC)x(t) - z(t) \tag{6}$$

and its dynamic is:

$$\dot{e}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i e(t) + (PA_i - N_i - K_i C)x(t) \\ + (PB_i - G_{i1})u(t) + (PD_i - G_{i2}) + PR\overline{u}(t)) \tag{7}$$

with $K_i = N_i E + L_i$.

The state estimation error between the multiple model (4) and the unknown input multiple observer (5) converges towards zero, if all the pairs (A_i, C) are observables and if the following conditions hold $\forall i \in \{1, ..., M\}$ [3]:

$$N_i^T X + X N_i < 0 \tag{8a}$$
$$N_i = PA_i - K_i C \tag{8b}$$
$$P = I + EC \tag{8c}$$
$$PR = 0 \tag{8d}$$
$$L_i = K_i - N_i E \tag{8e}$$
$$G_{i1} = PB_i \tag{8f}$$
$$G_{i2} = PD_i \tag{8g}$$

where $X \in R^{n*n}$ is a positive definite symmetric matrix.

5 Main results

5.1 Multiple observer of a system with unknown outputs

This section addresses the design of a multiple observer with unknown outputs.

In the case of linear systems affected by unknown outputs [7], a mathematical transformation is used to consider these unknown outputs in the form of unknown inputs of an augmented system. This result is then extended to nonlinear systems represented by multiple model [13,17]. In so doing, a multiple observer based on the elimination of these unknown inputs is conceived.

5.1.1 Linear system case

Consider the linear model affected by a sensor fault described by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + D\bar{u}(t) \end{cases} \tag{9}$$

where $\bar{u}(t) \in R^r$ represents the sensor fault. D is of full column rank.

Consider a new state $z(t)$ [7, 28] that is a filtered version of $y(t)$:

$$\dot{z}(t) \quad = \quad -\bar{A}z(t) + \bar{A}Cx(t) + \bar{A}D\bar{u}(t) \tag{10}$$

where $-\bar{A}$ is a stable matrix, $\bar{A} \in R^{p\times p}$.
Denote

$$X(t) = \begin{bmatrix} x(t)^T & z(t)^T \end{bmatrix}^T \tag{11}$$

The augmented system $X(t)$ is given by the following expression:

$$\begin{cases} \dot{X}(t) = A_a X(t) + B_a u(t) + D_a \bar{u}(t) \\ Y(t) = C_a X(t) \end{cases} \tag{12}$$

where $A_a \in R^{(n+p)\times(n+p)}$, $B_a \in R^{(n+p)\times m}$, $D_a \in R^{(n+p)\times r}$ and $C_a \in R^{p\times(n+p)}$.

These matrices are described as follows:

$$A_a = \begin{bmatrix} A & 0 \\ \bar{A}C & -\bar{A} \end{bmatrix}, \quad B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$$
$$C_a = \begin{bmatrix} C & 0 \end{bmatrix} \text{ and } D_a = \begin{bmatrix} 0 \\ \bar{A}D \end{bmatrix}$$

Sensor fault of (9) appears as an actuator fault of the augmented system (12).

The structure of the chosen observer is described as follows:

$$\begin{cases} \dot{Z}(t) = NZ(t) + Gu(t) + LY(t) \\ \hat{X}(t) = Z(t) - EY(t) \end{cases} \tag{13}$$

where $\hat{X}(t)$ is the augmented estimated state, $\hat{Y}(t)$ is the estimated output. N, G, L is the gain of the local observer and E is a matrix of transformation.

- Exemple

Consider a linear system described by the following matrices:

$$A = \begin{bmatrix} -1 & .2 & -.3 \\ -0.1 & -0.2 & -0.3 \\ 0.25 & 0.35 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}$$
$$D = \begin{bmatrix} 0.2 \\ 0.35 \\ 0.55 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The new state $z(t)$ is defined in (10) with $\bar{A} = 25 * I$.
The sensor fault is defined by $\bar{u}(t) = 0.1sin(.25\pi t)$.
The computation of the matrices of the multiple observer (13) gives:

$$N = \begin{bmatrix} 31.57 & 49.48 & 75.38 & \ldots \\ 35.33 & 15.34 & 73.36 & \ldots \\ 9.29 & 25.74 & -2.76 & \ldots \\ -35.21 & -20.90 & -18.76 & \ldots \\ -149.34 & -72.06 & -44.16 & \ldots \\ -204.52 & -261.52 & -32.78 & \ldots \end{bmatrix}$$
$$\begin{bmatrix} \ldots & -43.11 & 92.39 & 75.68 \\ \ldots & -38.23 & 53.77 & 73.66 \\ \ldots & 11.90 & 13.48 & -2.86 \\ \ldots & -4.75 & -3.48 & -3.94 \\ \ldots & 2.79 & -15.18 & -18.23 \\ \ldots & -2.06 & -87.11 & -42.04 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.10 \\ 0.20 \\ 0.30 \\ -0.50 \\ -0.875 \\ -1.375 \end{bmatrix}, \quad E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1.25 & 0 & -0.31 \\ -2.18 & 0 & -0.54 \\ -3.43 & 0 & -0.85 \end{bmatrix}$$

$$L = \begin{bmatrix} 451.51 & -92.39 & 26.41 \\ 361.29 & -53.77 & 7.10 \\ 22.63 & -13.48 & 11.50 \\ -16.13 & 3.48 & 4.97 \\ -40.51 & -9.81 & 8.80 \\ -249.67 & 87.11 & -45.88 \end{bmatrix}$$

The known input is shown in Figure (1). Figure (2) presents the states and their estimations.

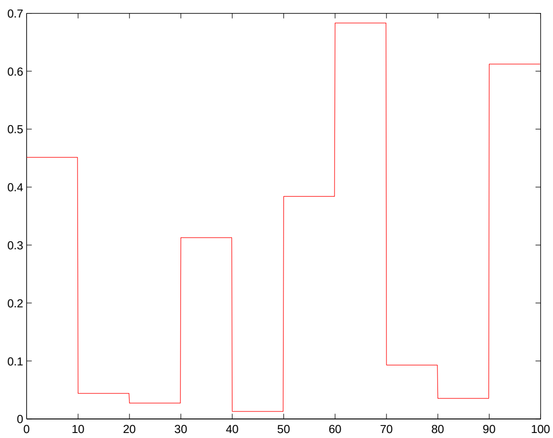

Figure 1: The known input

The proposed method provides good estimates of the system state.

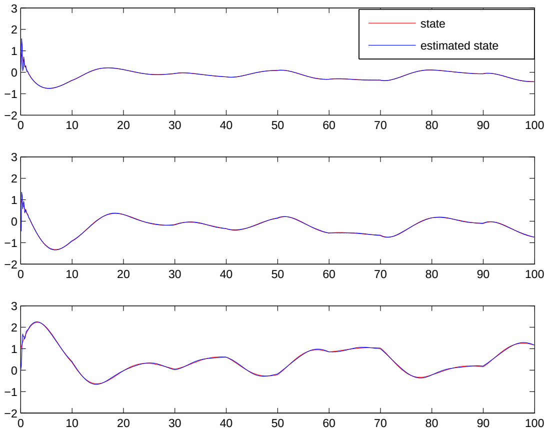

Figure 2: States and their estimates

5.1.2 Nonlinear system case

The aim in this part is to extend the method described above to be employed to nonlinear systems represented by Takagi-Sugeno models. Let us consider the following Takagi-Sugeno model affected by a sensor fault:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) + D\bar{u}(t) \end{cases} \quad (14)$$

where $\bar{u}(t) \in R^r$ represents the sensor fault. The matrix D is of full column rank.

Using the property given by (3), the new state $z(t)$ given in (10) can be rewritten:

$$\dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(-\bar{A}z(t) + \bar{A}Cx(t) + \bar{A}D\bar{u}(t)) \quad (15)$$

where $-\bar{A}$ is a stable matrix, $\bar{A} \in R^{p\times p}$.
Denote

$$X(t) = \left[\begin{array}{cc} x(t)^T & z(t)^T \end{array} \right]^T \quad (16)$$

This augmented state $X(t)$ can be expressed as:

$$\begin{cases} \dot{X}(t) = \sum\limits_{i=1}^{M} \mu_i(\xi(t))(A_{ai}X(t) + B_{ai}u(t) + D_a\bar{u}(t)) \\ Y(t) = C_a X(t) \end{cases} \quad (17)$$

where

$$A_{ai} = \left[\begin{array}{cc} A_i & 0 \\ \bar{A}C & -\bar{A} \end{array} \right],\ B_{ai} = \left[\begin{array}{c} B_i \\ 0 \end{array} \right],$$

$$D_a = \left[\begin{array}{c} 0 \\ \bar{A}D \end{array} \right] \text{ and } C_a = \left[\begin{array}{cc} C & I \end{array} \right].$$

From the obtained results, sensor fault of the system (14) appears as an actuator fault of the augmented system (17). This fault is considered as an unknown input. In so doing, fault estimation method is similar to the method of conception of multiple observer with unknown inputs.

The structure of the multiple observer is chosen as follows:

$$\begin{cases} \dot{Z}(t) = \sum\limits_{i=1}^{M} \mu_i(\xi(t))(N_i Z(t) + G_{i1}u(t) + G_{i2} \\ \qquad\qquad + L_i Y(t)) \\ \hat{X}(t) = Z(t) - EY(t) \end{cases} \quad (18)$$

where $\hat{X}(t) \in R^n$ is the state vector, $Y(t) \in R^p$ is the measured output. N_i, G_{i1}, L_i is the gain of the local observer, $G_{i2} \in R^n$ is a constant vector and E a matrix of transformation.

Let us consider the augmented state estimation error:

$$\begin{aligned} \tilde{X}(t) &= X(t) - \hat{X}(t) \\ &= PX(t) - Z(t) \end{aligned} \quad (19)$$

with:
$P = I + EC_a$

By direct time derivative, the dynamic evolution of $\tilde{X}(t)$ is given as follows:

$$\dot{\tilde{X}}(t) = \dot{X}(t) - \dot{\hat{X}}(t) \quad (20)$$

that can be expressed as:

$$\dot{\tilde{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(P(A_{ai}X(t) + B_{ai}u(t) + D_a\bar{u}(t)) - N_i Z(t) - G_{i1}u(t) - G_{i2} - L_i Y(t)) \quad (21)$$

Replacing $Z(t)$ and $Y(t)$ by their expressions, (21) becomes:

$$\dot{\tilde{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i\tilde{X}(t) + (PA_{ai} - N_i)X(t) - K_i C_a X(t) + (PB_{ai} - G_{i1})u(t) - G_{i2} + PD_a\bar{u}(t)) \quad (22)$$

with: $K_i = N_iE + L_i$.
If the following conditions are fullfilled:

$$\begin{cases} PD_a = 0 \\ P = I + EC_a \\ N_i = PA_{ai} - K_iC_a \\ L_i = K_i - N_iE \\ G_{i1} = PB_{ai} \\ G_{i2} = 0 \\ \sum_{i=1}^{M} \mu_i(\xi(t))N_i \text{ stable} \end{cases} \tag{23}$$

The reconstruction error of the augmented state tends asymptotically towards zero and (21) is reduced to:

$$\dot{\tilde{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))N_i\tilde{X}(t) \tag{24}$$

- Global convergence of the multiple observer

The augmented state estimation error converges towards zero, if all the pairs (A_{ai}, C_a) are observables, and if the following conditions are checked $\forall i \in \{1, ..., M\}$:

$$N_i^TX + XN_i < 0 \tag{25a}$$
$$N_i = PA_{ai} - K_iCa \tag{25b}$$
$$P = I + EC_a \tag{25c}$$
$$PD_a = 0 \tag{25d}$$
$$L_i = K_i - N_iE \tag{25e}$$
$$G_{i1} = PB_{ai} \tag{25f}$$
$$G_{i2} = 0 \tag{25g}$$

where $X \in R^{n*n}$ is a positive definite symmetric matrix.

Using the expression (25b), the inequality (25a) can be written as:

$$(PA_{ai} - K_iC_a)^TX + X(PA_{ai} - K_iC_a) < 0, \quad \forall i \in \{1, ..., M\} \tag{26}$$

The inequalities (26) are nonlinear in X and K_i. LMI formulation can thus be used only after linearization of these inequalities. A technique of change of variable is used.

- Method of resolution

Three steps are needed to resolve the system (25):

1. $\text{rank}(C_aD_a) = \text{rank}(D_a)$, the matrix E is given by using the expression (25d), where $(C_aD_a)^{(-)}$ is the pseudo-inverse of (C_aD_a):

$$E = -D_a(C_aD_a)^{(-)} \tag{27}$$

The matrix P may be deduced from (25c):

$$P = I - D_a(C_aD_a)^{(-)}C_a \tag{28}$$

2. To linearize the inequalities (26), the following change of variables is used:

$$W_i = XK_i \tag{29}$$

Equation (26) is rewritten:

$$(PA_{ai})^TX + X(PA_{ai}) - C_a^TW_i^T - W_iC_a < 0, \quad \forall i \in \{1, ..., M\} \tag{30}$$

The inequalities (30) are of LMI type, LMI Matlab Toolbox may be used for that resolution. Then, one deduces:

$$K_i = X^{-1}W_i \tag{31}$$

3. The other matrices defining the observer are deduced knowing E, P and K_i:

$$N_i = PA_{ai} - K_iC_a \tag{32a}$$
$$L_i = K_i - N_iE \tag{32b}$$
$$G_{i1} = PB_{ai} \tag{32c}$$

5.2 Multiple observers designing for systems with measures uncertainties

5.2.1 Multiple model representation of an uncertain system

Generally, process can present uncertainties of inputs, measures and models. In the context of Takagi-Sugeno fuzzy systems, the general representation of an uncertain system is given by the following equation :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_i \pm \Delta A_i)x(t) + \\ \qquad\qquad (B_i \pm \Delta B_i)u(t)) \\ y(t) = (C \pm \Delta C)x(t) \end{cases} \tag{33}$$

where ΔA_i are the matrices of modeling uncertainties, ΔB_i represent the input uncertainties of the system and ΔC represents the measures uncertainties. The weighting functions $\mu_i(\xi(t))$ are selected in order to check the conditions (3).

In this paper, only the measures uncertainties are considered. The modeling and inputs uncertainties are supposed certains, ie. $\Delta A_i = \Delta B_i = 0$. In so doing, the system given by (33) becomes :

$$\dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \quad (34a)$$

$$y(t) = (C \pm \Delta C)x(t) \quad (34b)$$

By developing the expression given by the equation (34b), one obtains:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \pm \Delta C x(t) \end{cases} \quad (35)$$

Noting $\bar{u}(t) = \pm \Delta C x(t)$, the system (35) becomes:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) + \bar{u}(t) \end{cases} \quad (36)$$

By comparing the equation (36) with the equation (14), one notices that the two equations are almost identical. The only difference is that the matrix D is replaced by the matrix identity. It is possible under these conditions to adapte the results obtained in the case of unknown outputs for the design of a multiple observer in the presence of measures uncertainties.

One uses for this fact the mathematical transformation given by the equation (15) allowing to obtain a new model (37) for which the uncertainty of measurement affecting the first system (35) appears as an unknown input.

$$\begin{cases} \dot{X}(t) = \sum\limits_{i=1}^{M} \mu_i(\xi(t))(A_{ai} X(t) + B_{ai} u(t) + D_a \bar{u}(t)) \\ Y(t) = C_a X(t) \end{cases} \quad (37)$$

where

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}C & -\bar{A} \end{bmatrix},\ B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix},\ D_a = \begin{bmatrix} 0 \\ \bar{A} \end{bmatrix}$$
$$\text{and } \ C_a = \begin{bmatrix} C & I \end{bmatrix}$$

5.2.2 Design of the multiple observer

The structure of the multiple observer is given by (18). The methodology used in the case of multiple observer with unknown outputs is adapted for the design of an observer with uncertainties of measurement. The method of resolution allowing us to determine the gains of local observers is that given above. The deduction of the matrices N_i and gains L_i and G_{i1} are given respectively by the equations (32a), (32b) and (32c).

6 Simulation example

In this section, the proposed method is illustrated through an academic example. Consider the Takagi-Sugeno model described in (35) with $M = 2$ defined by:

$$A_1 = \begin{bmatrix} -2 & -1 & -1 \\ -1 & -4 & 2 \\ -3 & 1 & -4 \end{bmatrix},\ A_2 = \begin{bmatrix} -4 & 2 & -1 \\ 3 & -1 & -1 \\ 2 & 3 & -2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.5 \\ 1 \\ 2 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

one takes $\Delta C = \pm 0.1 * C$.

During simulation, one distinguishes the 2 borderline cases: $+\Delta C$ and $-\Delta C$. The study of these two cases makes it possible to give an idea on the effectiveness of the method.

The decision vector $\xi(t)$ is depending on the system input. The weighting functions $\mu_i(\xi(t))$ are represented on Figure (3). They can be obtained from normalised Gaussian functions:

$$\omega_i(\xi(t)) = \exp\left(\frac{-(\xi(t) - (\xi(t))^{(i)})^2}{\sigma^2}\right)$$

$$\mu_i(\xi(t)) = \frac{\omega_i(\xi(t))}{\sum\limits_{j=1}^{M} \omega_j(\xi(t))}$$

with $\sigma = 0.15$, $\xi^1 = 0.25$ and $\xi^2 = 0.81$.

The known system input $u(t)$ is shown in Figure (4).

The structure of the multiple observer is:

$$\begin{cases} \dot{Z}(t) = \sum\limits_{i=1}^{2} \mu_i(u(t))(N_i Z(t) + G_{i1} u(t) + L_i Y(t)) \\ \hat{X}(t) = Z(t) - EY(t) \end{cases} \quad (38)$$

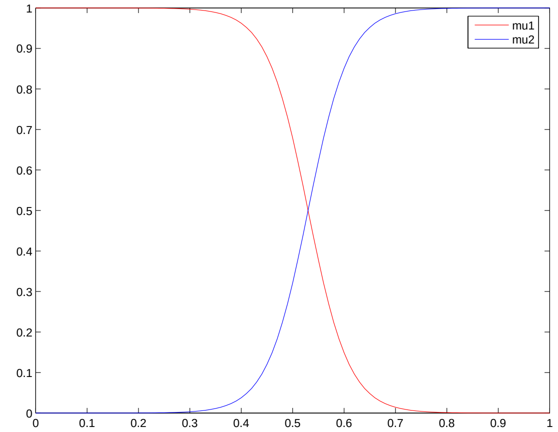

Figure 3: the weighting functions

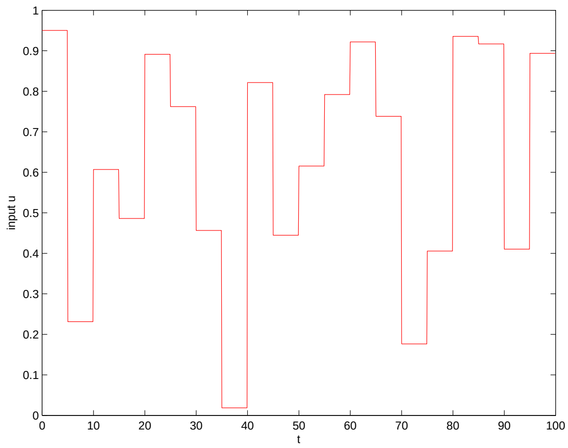

Figure 4: The known input $u(t)$

The matrices computation of this multiple observer gives:

$$N_1 = \begin{bmatrix} -1 & 0.0153 & 0.0763 & \ldots \\ -1.2653 & -2.75 & 1.7732 & \ldots \\ -0.5763 & 1.7268 & -1.75 & \ldots \\ 1.8109 & 1.0385 & -1.4005 & \ldots \\ 1.6737 & 0.9615 & -0.8263 & \ldots \\ 1.4390 & -0.7732 & 1.4768 & \ldots \end{bmatrix}$$
$$\begin{bmatrix} \ldots & 0.9391 & 0.0763 & 0.0609 \\ \ldots & 1.2115 & 0.0385 & -1.4768 \\ \ldots & 0.9005 & -0.1737 & 1.5232 \\ \ldots & -2.7500 & -0.2115 & -1.4391 \\ \ldots & -1.5385 & -0.50 & -0.7877 \\ \ldots & -0.8109 & 0.0377 & -2.75 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} -2 & 1.2946 & -0.0473 & \ldots \\ 1.7054 & -0.50 & -1.3302 & \ldots \\ 1.2973 & 2.0802 & -2.50 & \ldots \\ -1.5918 & -3.2857 & 3.1225 & \ldots \\ -3.2027 & -1.7144 & 3.0473 & \ldots \\ 0.0918 & -2.9198 & 2.3302 & \ldots \end{bmatrix}$$
$$\begin{bmatrix} \ldots & 0.3418 & -1.0473 & 1.6582 \\ \ldots & -0.4644 & 0.9644 & -0.8302 \\ \ldots & -1.1225 & 0.2027 & 0.4198 \\ \ldots & 1 & -0.2856 & -1.5918 \\ \ldots & 2.0357 & -1.75 & -0.2384 \\ \ldots & 0.8418 & 1.2384 & -2.75 \end{bmatrix}$$

$$\mathrm{G}_{11} = \begin{bmatrix} 0.50 \\ 1 \\ 2 \\ -3.50 \\ -3 \\ -2.50 \end{bmatrix}, \quad \mathrm{G}_{21} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -4 \\ -3 \\ -2 \end{bmatrix} \quad \text{and}$$

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

L_1 and L_2 are null matrices.

The simulation results are shown in Figures (5) and (6).

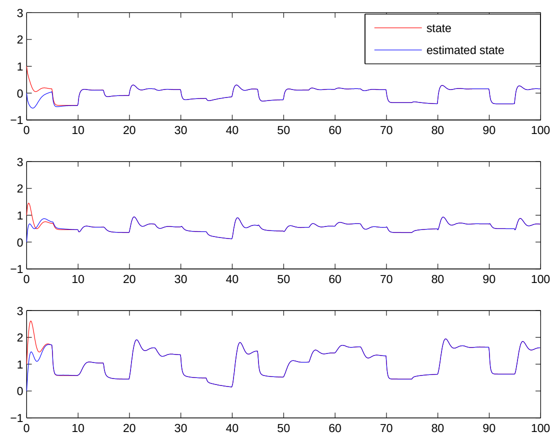

Figure 5: States and their estimates associated for $+\Delta C$

Thus, one succeeds in estimating the system state for nonlinear systems described by Takagi-Sugeno models subject to measures uncertainties.

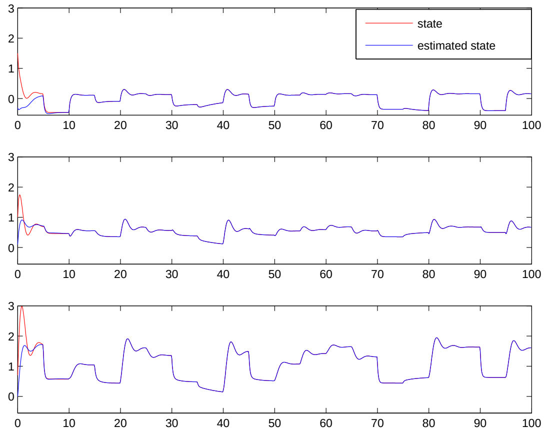

Figure 6: States and their estimates associated for $-\Delta C$

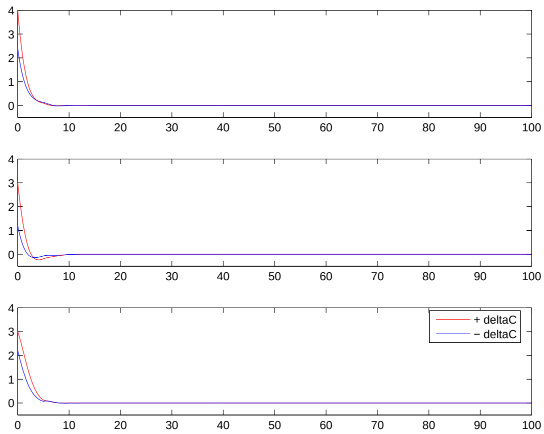

Figure 7: State estimation error

The proposed method shows the good estimation between the states of the system and their estimated. At $t = 0$, the disparity between actual and estimated state is due to the choice of initial conditions.

Figure (7) shows the evolution of the state estimation error. The two studied cases of simulation are considered and it is shown that the two errors converge towards zero. It can be conclude that the proposed method allows to estimate well the system state even in the presence of measure imprecisions.

7 Conclusion

Using a Takagi-Sugeno model representation, this paper showed a new method to design multiple observers for nonlinear systems influenced by unknown outputs, in the one hand, and measures uncertainties, on the other hand. A mathematical transformation is used in order to considerate these disturbances as unknown inputs. The proposed method is based on the principle of unknown input multiple observer which used the principle of the interpolation of local observers. The synthesis conditions of that observer are expressed in LMI terms. Then, the calculation of the gains of the multiple observer is returned to a calculation of gains of the local observers. The simulation results show that the estimation of the state is very satisfactory. The future works will concern the design of multiple observers of systems subject to modelling and measures uncertainties.

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