Design of an adaptive faults tolerant control: case of sensor faults

ATEF KHEDHER
LARA/ENIT
BP 37, le Belvédère
1002 Tunis,TUNISIA
khedher_atef@yahoo.fr

KAMEL BEN OTHMAN
LARA/ENIT
BP 37, le Belvédère
1002 Tunis,TUNISIA
kamalbenothman@yahoo.fr

DIDIER MAQUIN
CRAN, UMR 7039, Nancy-Université
CNRS, 2, avenue de la Forêt de Haye
54516 Vandœuvre-lès-Nancy,
didier.maquin@ensem.inpl-nancy.fr

MOHAMED BENREJEB
LARA/ENIT
BP 37, le Belvédère
1002 Tunis,TUNISIA
mohamed.benrejeb@enit.rnu.tn

Abstract: This paper presents a method of design of a sensor faults tolerant control. The method is presented for the case of linear systems and then for the case of non linear systems described by Takagi-Sugeno models. The faults are initially estimated using a proportional integral observer. A mathematical transformation is used to conceive an augmented system in which the sensor fault appear as an unknown inputs. The synthesized control depends on the estimated faults and the error between the state of a reference reference and the faulty system state. The fault tolearnt control is conceived using the augmented state. The conditions of the observer convergence and of the control existence are formulated in terms of Linear Matrix Inequalities (LMI). The formulation in LMI shows that the synthesis of the control and the observer can be independently made. For both cases (linear and non linear) The theoretical results are validated by their application to a noisy system affected by sensor faults.

Key–Words: multiple model, estimation, proportional integral observer, sensor faults, faults tolerant control.

1 Introduction

Physical processes are generally subjected to disturbances affecting their inputs or their outputs. The evolution in time of these disturbances is unknown and can damage the smooth running of the system. The consideration of the disturbances during the modelling and the state estimation becomes necessary to establish diagnosis procedures allowing faults detection and localization.

Faults estimation can be made using a proportional integral state observer in the case of non linear systems represented by multiple models [12, 16]. PMI Observers can also be used for faults estimation [6]. That kind of observers gives some robustness property of the state estimation with respect to the system uncertainties and perturbations [2, 6, 13, 17, 21]. Once the fault is estimated, its effect can be limited or eliminated using a fault tolerant control strategy.

The objective of a fault tolerant control is to find a control strategy which can limit or cancel, the fault effects on the system performances [18]. There are two approaches of faults tolerant control synthesis: the passive approach and the active approach.

In the passive approach, the faults are taken into account during the design of the control. The method considers faults as disturbances which the control has to consider from its initial conception [4, 15].

The active approach reacts "actively" on the faults in on-line reconfiguration of the control so as to keep the stability and the nominal performances of the system [3, 20]. This approach allows then to treat unforeseen faults but requires an effective method of faults detection and isolation allowing giving exactly information about the faults.

Our contribution in this paper lies in the synthesis of an active sensor fault tolerant control. For faults estimation, a mathematical transformation is used. It allows conceiving an augmented system in which the initial sensor fault appears as an actuator fault. By considering the augmented system, a proportional integral observer is conceived to estimate the faults [11]. It is possible to estimate simultaneously actuator and sensor faults using this approach [8, 10]. The fault tolerant control is then synthesized. By being inspired of the works of Witczak and al. [22] made in the context of the discrete systems affected by actuator faults, and works presented in [5, 23] treating linear systems, Khedher and al. [9] have proposed an approach to conceive a fault tolerant control for actuator faults. this work is interested to the case of nonlinear systems described by Takagi-Sugeno fuzzy models affected by sensor faults.

The paper is structured in the following way. The section II presents the proposed method for the case of linear systems, the obtained results is applied to a numerical example. The section III recalls the Takagi-Sugeno models and details the proposed method for the multiple models case. An example of application of the proposed method to the Takagi-Sugeno models showing the efficiency of the method is presented in the section IV.

2 Proposed method for the linear system case

2.1 Problem formulation

The main objective of this part is to synthesize an active sensor fault tolerant control for the case of linear systems. The application of this control to a linear system presenting sensors faults allows to restore its original behaviour. A linear system can be described by the following state equation:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ represents the system state, $y(t) \in \mathbb{R}^m$ is the measured output, $u(t) \in \mathbb{R}^r$ is the system input. A, B and C are known constant matrices with appropriate dimensions.

Let consider a linear system with sensor fault described by the following state equation:

$$\begin{cases} \dot{x}_f(t) = Ax_f(t) + Bu_f(t) \\ y_f(t) = Cx_f(t) + Ef(t) + Dw(t) \end{cases} \tag{2}$$

where $x_f(t) \in R^n$ represents the system state, $y_f(t) \in R^m$ is the measured output, $u_f(t) \in R^r$ is the system input, $f(t)$ represents the fault which is assumed to be bounded and $w(t)$ is the measurement noise. E and D are respectively the fault and the noise distribution matrices which are assumed to be known.

Let us define the following states [5] :

$$\begin{array}{l} \dot{z}(t) = \bar{A}Cx(t) - \bar{A}z(t) \\ \dot{z}_f(t) = \bar{A}Cx_f(t) - \bar{A}z_f(t) + \bar{A}Ef(t) + \bar{A}Dw(t) \end{array} \tag{3}$$

where $-\bar{A}$ is a stable matrix.

Defining X and X_f as: $X = \begin{bmatrix} x^T & z^T \end{bmatrix}^T$ and $X_f = \begin{bmatrix} x_f^T & z_f^T \end{bmatrix}^T$, these two state vectors can be written:

$$\begin{cases} \dot{X}(t) = A_a X(t) + B_a u(t) \\ Y(t) = C_a X(t) \end{cases} \tag{4}$$

and:

$$\begin{cases} \dot{X}_f(t) = A_a X_f(t) + B_a u_f(t) + E_a f(t) + F_a w(t) \\ Y_f(t) = C_a X_f(t) \end{cases} \tag{5}$$

with:

$$A_a = \begin{bmatrix} A & 0 \\ \bar{A}C & -\bar{A} \end{bmatrix}, \; B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, \; C_a = \begin{bmatrix} 0 & I \end{bmatrix}$$

$$F_a = \begin{bmatrix} 0 \\ \bar{A}D \end{bmatrix} \text{ and } E_a = \begin{bmatrix} 0 \\ \bar{A}E \end{bmatrix} \tag{6}$$

where I is the identity matrix with appropriate dimensions.

A proportional integral observer which permits the estimation of X_f and f is considered:

$$\begin{cases} \dot{\hat{X}}_f(t) = A_a \hat{X}_f(t) + B_a u_f(t) + E_a \hat{f}(t) + K\tilde{Y}(t) \\ \dot{\hat{f}}(t) = L\tilde{Y}(t) \\ \hat{Y}_f(t) = C_a \hat{X}_f(t) \end{cases} \tag{7}$$

where $\hat{X}_f(t)$ is the estimated state, $\hat{f}(t)$ represents the estimated fault, $\hat{Y}_f(t)$ is the estimated output, K is the proportional observer gain and L is its integral gain which must be computed and $\tilde{Y}(t) = Y_f(t) - \hat{Y}_f(t)$. The control $u_f(t)$ is given by [22]:

$$u_f(t) = -S\hat{f}(t) + N(X(t) - \hat{X}_f(t)) + u(t) \tag{8}$$

where S and N are two constant matrices with appropriate dimensions.

The objective is to find the matrices S and N which permit to the state X_f to converge to X.
Let define $\tilde{X}(t)$ the error between $X(t)$ and $X_f(t)$, $\tilde{X}_f(t)$ the estimation error of the state $X_f(t)$ and $\tilde{f}(t)$ the fault estimation error.

$$\tilde{X}(t) = X(t) - X_f(t) \tag{9}$$

$$\tilde{X}_f(t) = X_f(t) - \hat{X}_f(t) \tag{10}$$

$$\tilde{f}(t) = f(t) - \hat{f}(t) \tag{11}$$

The dynamics of $\tilde{X}(t)$ can be written:

$$\begin{array}{l} \dot{\tilde{X}}(t) = \dot{X}(t) - \dot{X}_f(t) = (A_a - B_a N)\tilde{X}(t) \\ +B_a S\hat{f}(t) - B_a N\tilde{X}_f(t) - E_a f(t) - F_a w(t) \end{array} \tag{12}$$

S is chosen so that $E_a = B_aS$, The dynamics of $\tilde{X}(t)$ becomes:

$$\begin{aligned}\dot{\tilde{X}}(t) &= (A_a - B_aN)\tilde{X}(t) - B_aN\tilde{X}_f(t) \\ &- E_a\tilde{f}(t) - F_aw(t) \end{aligned} \quad (13)$$

The dynamics of $\tilde{X}_f(t)$ can be written:

$$\begin{aligned}\dot{\tilde{X}}_f(t) &= \dot{X}_f(t) - \dot{\hat{X}}_f(t) \\ &= (A_a - KC_a)\tilde{X}_f(t) + E_a\tilde{f}(t) + F_aw(t) \end{aligned} \quad (14)$$

The dynamics of $\tilde{f}$ is written:

$$\begin{aligned}\dot{\tilde{f}}(t) &= \dot{f}(t) - \dot{\hat{f}}(t) \\ &= \dot{f}(t) - LC_a\tilde{X}_f(t) \end{aligned} \quad (15)$$

The following vectors are introduced:

$$\varphi(t) = \begin{bmatrix} \tilde{X}(t) \\ \tilde{X}_f(t) \\ \tilde{f}(t) \end{bmatrix} \text{ and } \psi(t) = \begin{bmatrix} w(t) \\ \dot{f}(t) \end{bmatrix} \quad (16)$$

(13), (14) and (15) can be written as:

$$\dot{\varphi}(t) = A_0\varphi(t) + B_0\psi(t) \quad (17)$$

with:

$$A_0 = \begin{bmatrix} A_a - B_aN & -B_aN & -E_a \\ 0 & A_a - KC_a & E_a \\ 0 & -LC_a & 0 \end{bmatrix}$$

$$\text{and } B_0 = \begin{bmatrix} -F_a & 0 \\ F_a & 0 \\ 0 & I \end{bmatrix} \quad (18)$$

In order to analyse the convergence of the generalized error $\varphi(t)$, let us consider the following quadratic Lyapunov candidate function $V(t)$:

$$V(t) = \varphi(t)^TP\varphi(t) \quad (19)$$

where P denotes a symmetric positive matrix.

The problem of robust state and fault estimation is to find the gains K and L of the observer to ensure an asymptotic convergence of φ toward zero when $\psi(t) = 0$ and to ensure a bounded error when $\psi(t) \neq 0$. This problem is reduced to find P verifying $\dot{V} < 0$, ie. $A_0^TP + PA_0 < 0$.

The matrix A_0 can be expressed as:

$$A_0 = \begin{bmatrix} A_a - B_aN & E_1 \\ 0 & \tilde{A} - \tilde{K}\tilde{C} \end{bmatrix} \quad (20)$$

where:

$$\tilde{A} = \begin{bmatrix} A_a & E_a \\ 0 & 0 \end{bmatrix}, \; \tilde{K} = \begin{bmatrix} K \\ L \end{bmatrix}$$

$$E_1 = \begin{bmatrix} -B_aN & -E_a \end{bmatrix}, \; \tilde{C} = \begin{bmatrix} C_a & 0 \end{bmatrix} \quad (21)$$

Assuming that P has the block diagonal form $P = diag(P_1, P_2)$, it can be observed from the structure of A_0 that the eigenvalues of the matrix A_0 are the union of those of $A_a - B_aN$ and $\tilde{A} - \tilde{K}\tilde{C}$. This indicates that the design of the control $v(t)$ and the observer can be carried out independently (separation principle). Thus, it is clear from the expression of P that φ converges to zero if there exist matrices $P_1 > 0$ and $P_2 > 0$ such that these inequalities are satisfied:

$$(A_a - B_aN)^TP_1 + P_1(A_a - B_aN) < 0 \quad (22)$$

$$(\tilde{A} - \tilde{K}\tilde{C})^TP_2 + P_2(\tilde{A} - \tilde{K}\tilde{C}) < 0 \quad (23)$$

By multiplying (22) from left and right by $W = P_1^{-1}$ one obtains:

$$W(A_a - B_aN)^T + (A_a - B_aN)W < 0 \quad (24)$$

The inequalities (23) and (24) are not linear. Substituting $U = NW$, and $G = P_2\tilde{K}$, they become:

$$WA_a^T + A_aW - U^TB_a^T - B_aU < 0 \quad (25)$$

$$\tilde{A}^TP_2 + P_2\tilde{A} - G\tilde{C} - \tilde{C}^TG^T < 0 \quad (26)$$

After the resolution of the linear matrices inequalities (LMI) (25) and (26), N and $\tilde{K}$ are computed using the equations:

$$N = UW^{-1} \quad (27)$$

$$\tilde{K} = P_2^{-1}G \quad (28)$$

2.2 Example

Consider the linear systems described by the equations (1) and (2) with $C = I$ and:

$$A = \begin{bmatrix} -0.2 & -3 & -0.6 & 0.3 \\ -0.6 & -4 & 1 & -0.6 \\ 3 & -0.9 & -7 & -0.2 \\ -0.5 & -1 & -2 & -0.8 \end{bmatrix}, \; B \begin{bmatrix} 1 & 2 \\ 5 & 1 \\ 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \\ 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \; E = \begin{bmatrix} 2.5 & 4 \\ 2.5 & 0.5 \\ 0 & -0.5 \\ 4 & 4 \end{bmatrix}$$

The system input $u(t) = \begin{bmatrix} u_1(t)^T & u_2(t)^T \end{bmatrix}^T$ with:

$u_1(t)$ is a telegraph type signal varying between zero and one, $u_2(t) = 0.3 + 0.1\sin(\pi t)$
The fault $f(t) = \begin{bmatrix} f_1(t)^T & f_2(t)^T \end{bmatrix}^T$ with :

$$f_1 = \begin{cases} 0, & t \leq 4sec \\ 0.1 * sin(\pi t), & t > 4sec \end{cases}$$

and $\quad f_2 = \begin{cases} 0, & t \leq 1.5sec \\ 0.4, & t > 1.5sec \end{cases}$

The computation of the matrices K, L and N gives :

$$L = \begin{bmatrix} 65.0202 & 66.3220 & 3.9470 & 100.6591 \\ 98.0943 & 10.9744 & -19.4836 & 100.0499 \end{bmatrix}$$

$$K = \begin{bmatrix} 27.8211 & -3.0638 & 19.3552 & -3.9180 \\ -5.2349 & 17.8149 & -5.8514 & -3.0006 \\ -0.2878 & -0.4806 & 24.4244 & -0.1831 \\ -2.3542 & -8.6612 & 12.0768 & 20.7143 \\ -26.6008 & -0.0654 & 1.2754 & 0.4694 \\ -0.2478 & -17.8359 & 0.9772 & 0.8980 \\ 0.8879 & 0.9559 & -20.9804 & -1.6446 \\ 0.0411 & 0.5892 & -1.4298 & -18.0905 \end{bmatrix}$$

$$N = \begin{bmatrix} -0.9721 & -0.6599 & -5.2898 & 1.3076 & \ldots \\ 1.1387 & -10.6258 & 3.0007 & 0.1747 & \ldots \\ \ldots & 0.7685 & 6.0930 & 6.8601 & 0.8939 \\ \ldots & 7.0937 & 9.0879 & -6.2236 & 6.5954 \end{bmatrix}$$

The simulation results are shown in the figures (1) to (3) :

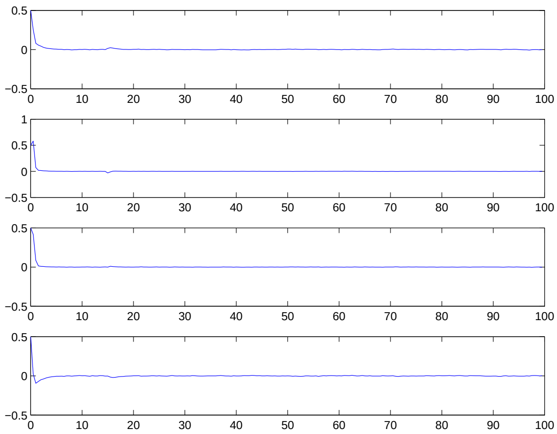

Figure 1: Error between x and x_f

The input $u_f(t)$ is computed using the equation (8), this input permits to the system (2) to have the same behaviour with the system (1). This input is shown in figure (4).

The conceived observer allows to estimate the state x_f and the control $u_f(t)$ is a fault tolerant

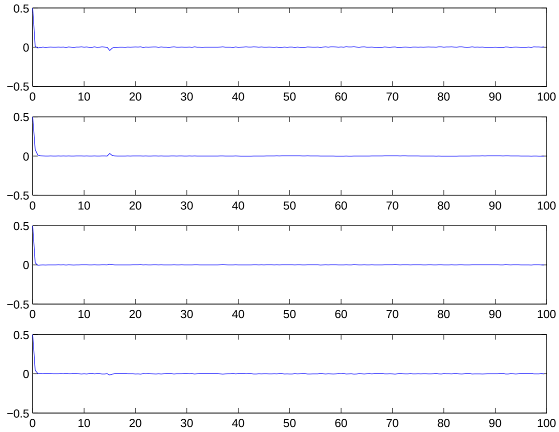

Figure 2: Estimation error of x_f

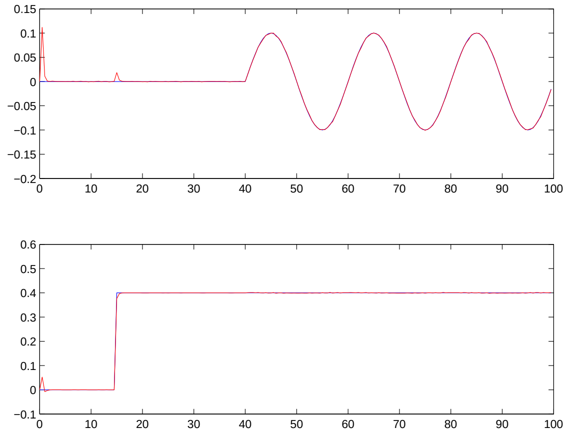

Figure 3: Faults and their estimations

control applied to the system (2). The effect of the conceived fault tolerant control is clear because the faulty state converge to the state of the reference model even the fault exists.

2.3 Conclusion

A method which permits simultaneously the fault estimation and the conception of the fault tolerant control is proposed in this section. This control is computed using the fault estimate and the error between the faulty system state and a reference model state. In the next section the proposed method will be extended to nonlinear systems described with multiple models.

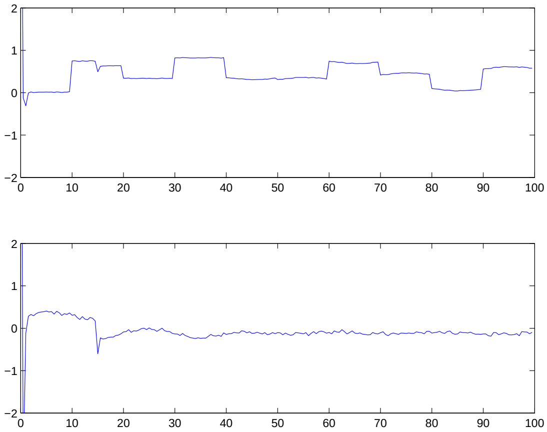

Figure 4: fault tolerant control u_f

3 Extention to multiple models case

3.1 On the multiple model representation

Multiple model approach is considered by many researches as an approach giving a simple structure for control or diagnosis of nonlinear systems [1,14]. This approach has been initially proposed, in a fuzzy modelling framework, by Takagi and Sugeno [19] and in a multiple model modelling framework by Johansen and Foss [7]. This model has been largely considered for analysis, modelling, control and state estimation of nonlinear systems.

The multiple model approach constitutes a tool which is largely used in the modelling of nonlinear systems [14]. The basis of the multiple model approach is the decomposition of the operating space of the system into a finite number of operating zones. Hence, the dynamic behaviour of the system inside each operating zone can be modelled using a simple submodel, for example a linear model. The relative contribution of each submodel is quantified with the help of a weighting function. Finally, the approximation of the system behaviour is performed by associating the submodels and by taking into consideration their respective contributions. The choice of the structure used to associate the submodels constitutes a key point in the multiple modelling frameworks. This model has been largely considered for analysis, modelling, control and state estimation of nonlinear systems. The structure of a Takagi-Sugeno model is:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{M} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum\limits_{i=1}^{M} \mu_i(\xi(t)) C_i x(t) \end{cases} \quad (29)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ control vector, $y(t) \in R^m$ vector of measures and A_i, B_i and C_i are known constant matrices with appropriate dimensions.

The membership functions $\mu_i(\xi(t))$ assure a progressive passage between the local models. These have the following proprieties:

$$\sum_{i=1}^{M} \mu_i(\xi(t)) = 1, \forall t \quad (30)$$

$$\text{and} \quad 0 \leq \mu_i(\xi(t)) \leq 1, \ \ \forall i = 1...M, \forall t \quad (31)$$

The variable of decision $\xi(t)$ is accessible in real time and it depends of measurable variables like system inputs or outputs.

Let's remark that state matrix of this kind of multiple model are built by the made of a level-headed sum, with variable weight of different matrix of local models. One can also make a similarity between multiple model and system with variables parameters in time.

If, in the equation which defines the output, we impose that $C_1 = C_2 = ... = C_M = C$, the output of the multiple model (29) is reduced to : $y(t) = Cx(t)$ and the multiple model becomes:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \quad (32)$$

3.2 Problem formulation

In this section the method proposed for linear system will be extended to nonlinear system described by multiple models. Suppose that the matrices B_i are equals.

Consider the nonlinear system described by the following multiple model structure:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{M} \mu_i(\xi(t)) A_i x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (33)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ control vector, $y(t) \in R^m$ vector of measures and A_i, B and C are known constant matrices with appropriate dimensions. The scalar M represents

the number of local models. $\xi(t)$ is the variable of decision which can depend on the input and/or the output and/or the system state.
Consider the following Takagi-Sugeno model affected by sensor faults and measurement noise:

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{M} \mu_i(\xi(t))A_i x_f(t) + Bu_f(t) \\ y_f(t) = Cx_f(t) + Ef(t) + Dw(t) \end{cases} \quad (34)$$

where $x_f(t) \in R^n$ is the state vector, $u_f(t) \in R^r$ is the input vector, $y_f(t) \in R^m$ the output vector. $f(t)$ represents the fault which is assumed to be bounded and $w(t)$ is the measurement noise. E and D are respectively the fault and the noise distribution matrices which are assumed to be known.

The weighting functions must verify $\sum_{i=1}^{M} \mu_i(\xi(t)) = 1$, so, the states z and z_f defined in (3) can be written :

$$\begin{aligned} \dot{z}(t) &= \sum_{i=1}^{M} \mu_i(\xi(t))(-\bar{A}z(t) + \bar{A}Cx(t)) \\ \dot{z_f}(t) &= \sum_{i=1}^{M} \mu_i(\xi(t))(-\bar{A}z_f(t) + \bar{A}Cx_f(t) \\ &\qquad + \bar{A}Ef(t) + \bar{A}Dw(t)) \end{aligned} \quad (35)$$

The two augmented state vectors X and X_f are:

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))A_{ai}X(t) + B_a u(t) \\ Y(t) = C_a X(t) \end{cases} \quad (36)$$

and :

$$\begin{cases} \dot{X}_f(t) = \sum_{i=1}^{M} \mu_i(\xi(t))A_{ai}X_f(t) + B_a u_f(t) \\ \qquad\qquad + E_a f(t) + F_a w(t) \\ Y_f(t) = C_a X_f(t) \end{cases} \quad (37)$$

with :

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}C & -\bar{A} \end{bmatrix} \quad (38)$$

The Other matrices are given in (2.1).
The structure of the proportional integral observer is chosen as follows:

$$\begin{cases} \dot{\hat{X}}_f(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}\hat{X}_f(t) + K_i(\tilde{Y}(t))) \\ \qquad\qquad + B_a u_f(t) + E_a \hat{f}(t) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))L_i(\tilde{Y}(t)) \\ \hat{Y}_f(t) = C_a \hat{X}_f(t) \end{cases} \quad (39)$$

where $\hat{X}_f(t)$ is the estimated state, $\hat{f}(t)$ represents the estimated fault, $\hat{Y}_f(t)$ is the estimated output, K is the proportional observer gain, L is its integral gain which must be computed and $\tilde{Y}(t) = Y_f(t) - \hat{Y}_f(t)$. K_i are the local model proportional observer gains and L_i are the local model integral gains to be computed. The control strategy is conceived using the following form of $u_f(t)$:

$$u_f(t) = -S\hat{f}(t) + u(t) \quad (40)$$

Using the same notations given in (9), (10) and (11), the following is obtained:

$$\begin{aligned} \dot{\tilde{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))A_{ai}\tilde{X}(t) + B_a S\hat{f}(t) \\ - E_a f(t) - F_a w(t) \end{aligned} \quad (41)$$

If S verify $E_a = B_a S$, $\dot{\tilde{X}}$ becomes :

$$\dot{\tilde{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))A_{ai}\tilde{X}(t) - E_a\tilde{f}(t) - F_a w(t) \quad (42)$$

The dynamics of the error $\tilde{X}_f(t)$ described by the equation (10) is written in multiple model case:

$$\begin{aligned} \dot{\tilde{X}}_f(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_{ai} - K_i C_a)\tilde{X}_f(t)) \\ + E_a\tilde{f}(t) + F_a w(t) \end{aligned} \quad (43)$$

The dynamics of the fault estimation error is:

$$\dot{\tilde{f}} = \dot{f}(t) - \sum_{i=1}^{M} \mu_i(\xi(t))L_i C_a \tilde{X}_f \quad (44)$$

The equations (42), (43) and (44) can be written:

$$\dot{\varphi}(t) = A_m\varphi(t) + B_m\psi(t) \quad (45)$$

where φ and ψ are given in (16) and:

$$A_m = \sum_{i=1}^{M} \mu_i(\xi(t))A_{mi} \quad (46)$$

where:

$$A_{mi} = \begin{bmatrix} A_{ai} & 0 & -E_a \\ 0 & A_{ai} - K_i C_a & E_a \\ 0 & -L_i C_a & 0 \end{bmatrix}$$

$$\text{and} \quad B_m = \begin{bmatrix} -F_a & 0 \\ F_a & 0 \\ 0 & I \end{bmatrix} \quad (47)$$

Considering the Lyapunov fonction given in (19), the errors converge to zero if $\dot{V} < 0$. $\dot{V} < 0$ if $A_{mi}^T P + P A_{mi} < 0, \ \forall i \in \{1, ..., M\}$. The matrix A_{mi} can be written:

$$A_{mi} = \begin{bmatrix} A_{ai} & E_1 \\ 0 & \tilde{A}_i - \tilde{K}_i \tilde{C} \end{bmatrix} \quad (48)$$

with:

$$\tilde{A}_i = \begin{bmatrix} A_{ai} & E_a \\ 0 & 0 \end{bmatrix}, \quad \tilde{K}_i = \begin{bmatrix} K_i \\ L_i \end{bmatrix}$$
$$E_1 = \begin{bmatrix} 0 & -E_a \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} C_a & 0 \end{bmatrix} \quad (49)$$

Assuming that P has the block diagonal form $P = diag(P_1, P_2)$, φ converges to zero if there exist matrices $P_1 > 0$ and $P_2 > 0$ such that following inequality is satisfied:

$$\begin{bmatrix} A_{ai}^T P_1 + P_1 A_{ai} & E_1 P_2 + P_1 E_1 \\ P_2 E_1^T + E_1^T P_1 & \Xi \end{bmatrix} < 0 \quad (50)$$

with :

$$\Xi = (\tilde{A}_i - \tilde{K}_i \tilde{C})^T P_2 + P_2 (\tilde{A}_i - \tilde{K}_i \tilde{C}) \quad (51)$$

Substituing $G_i = P_2 \tilde{K}_i$, (51) becomes:

$$\Xi = \tilde{A}^T P_2 + P_2 \tilde{A} - G_i \tilde{C} - \tilde{C}^T G_i^T \quad (52)$$

The resolution of the linear matrix inequality (LMI) (50), which is now linear, permits to find the matrices P_1, P_2 and G_i.
The matrices $\tilde{K}_i$ are computed using $\tilde{K}_i = P_2^{-1} G_i$.

Summarizing the following theorem can be proposed:

Theorem 1 *The system (45) describing the evolution of the errors $\tilde{X}(t)$, $\tilde{X}_f(t)$ and $\tilde{f}(t)$ is stable if there exist symmetric definite positive matrices P_1 et P_2 and matrices G_i, $i \in \{1...M\}$, so that the following LMI are verified :*

$$\begin{bmatrix} A_{ai}^T P_1 + P_1 A_{ai} & E_1 P_2 + P_1 E_1 \\ P_2 E_1^T + E_1^T P_1 & \Xi \end{bmatrix} < 0 \quad (53)$$

where

$$\Xi = \tilde{A}^T P_2 + P_2 \tilde{A} - G_i \tilde{C} - \tilde{C}^T G_i^T \quad (54)$$

The observer gains (proportional and integral) are obtained by : $\tilde{K}_i = P_2^{-1} G_i$.

4 Illustrative example

Consider the non linear systems described by (33) and (34) where: $C = I$, $\xi(t) = u(t)$ and :

$$A_1 = \begin{bmatrix} -0.4 & -2 & 0.8 & 0.3 \\ 0.6 & -5 & 1 & -0.2 \\ -0.5 & 0.6 & -9 & 0.3 \\ 0.4 & 3 & 2 & -0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 2 \\ -1 & -2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.7 & -7 & -1.5 & -7 \\ -0.2 & -2 & 0.6 & 1.3 \\ 5 & -1.5 & -9 & -3.9 \\ -0.4 & -1 & 0.3 & -1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 2 \\ 5 & 1 \\ 4 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\text{and } D = \begin{bmatrix} 0.5 & 0.2 & 0.1 & 0 \\ 0.5 & 0.2 & 0.1 & 0.1 \end{bmatrix}^T$$

The system input is: $u = \begin{bmatrix} {u_1}^T & {u_2}^T \end{bmatrix}^T$ with : $u_1(t)$ is a telegraph type signal varying in $[0, 0.5]$ and $u_2(t) = 0.4 + 0.25 \sin(\pi t)$. It is shown in the figure (5). The fault: $f = \begin{bmatrix} {f_1}^T & {f_2}^T \end{bmatrix}^T$ with:
$f_1 = \begin{cases} sin(0.5\pi t), 15 < t < 75 \\ 0, \quad \text{otherwise} \end{cases}$ and $f_2 = \begin{cases} 0, \quad t < 20 \\ 0.3, 20 < t < 70 \\ 0.5, 70 < t < 100 \end{cases}$

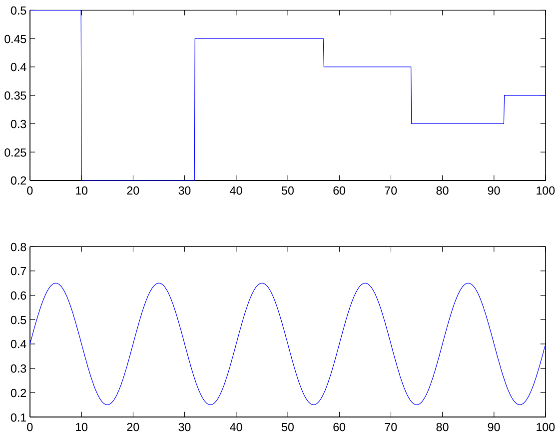

Figure 5: System input

The chosen weighting functions depends on the two inputs of the system. They have been created on the basis of Gaussian membership functions. Figure 6 shows their time-evolution showing that the system is clearly nonlinear since μ_1 and μ_2 are not constant functions.

The computation of the matrices K_1, L_1, K_2

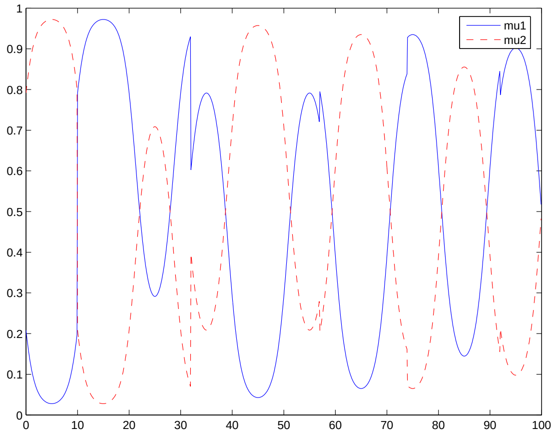

Figure 6: Weighting functions

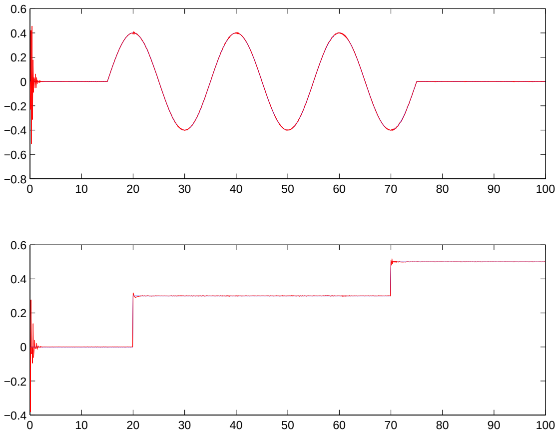

Figure 7: Faults and their estimations

and L_2 gives:

$$L_1 = \begin{bmatrix} 1.0794 & 8.1861 & 6.6330 & 0.7608 \\ 4.3498 & 1.5344 & -3.7962 & 8.9428 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -2.1645 & 8.1289 & 7.0558 & 2.2015 \\ -0.1195 & 4.2296 & -5.7418 & 10.1167 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 3.8612 & 0.2280 & -1.1161 & 1.1617 \\ -0.6495 & 0.7253 & 2.4040 & 0.4932 \\ 0.8769 & 2.6781 & -3.4879 & -0.6686 \\ 2.5354 & 3.3790 & 1.0535 & 5.9432 \\ 0.3170 & 2.0343 & -9.7468 & 0.2042 \\ 1.2085 & 0.3383 & -0.8709 & -9.0551 \\ -0.0015 & 0.0186 & 17.0517 & 0.0178 \\ 0.0808 & -0.0122 & 0.0454 & 16.8713 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 3.5902 & 1.1476 & -0.6030 & 1.5530 \\ -7.3246 & 3.7181 & 2.5477 & 0.9748 \\ 3.5981 & 0.0388 & -3.4950 & -0.5782 \\ -5.9167 & 0.3522 & -4.9351 & 5.5285 \\ 1.7989 & 0.0015 & 0.0047 & -0.0007 \\ -7.1948 & 0.4824 & 0.0018 & 0.0001 \\ 3.4984 & -0.9159 & -6.5076 & 0.0011 \\ -7.3971 & 0.2961 & -3.6144 & 1.5070 \end{bmatrix}$$

Simulation results are shown in figures (7) to (9).

The figure 7 shows the sensor faults and their estimated. It is clear that the proposed method allows the faults estimation even in the case of the faults varying in the time. The figure 9 shows the evolution of the fault tolerant control, if a fault appears the control changes in a way that the system guards its original behaviour. This result is verified as shown it the figure 8 (error between the reference state x and the state z affected by the fault). The error is practically equal to zero and the action of the fault tolerant control is quick.

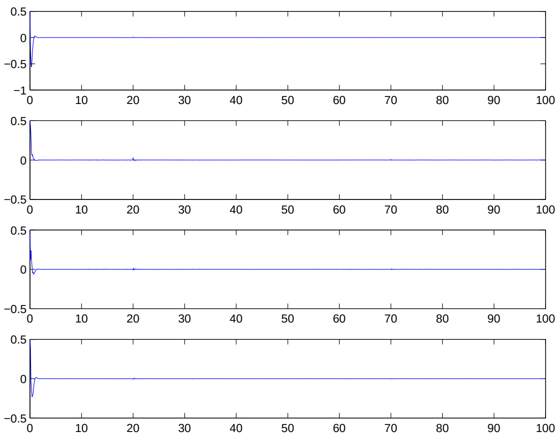

Figure 8: Error between x and z

5 Conclusion

This work has presented a method of synthesis of active sensor fault tolerant control. The proposed method uses the fault estimation and the error between the reference state and the faulty system state to synthesize the fault tolerant control strategy. To estimate the sensor fault, an augmented system is conceived, this system has the advantage to let the sensor fault affecting the initial system appears as an unknown input which make its estimation simple. The advantage of this method

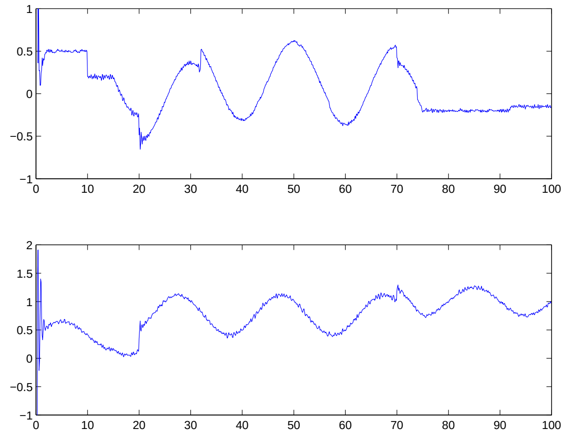

Figure 9: fault tolerant control

is to estimate non constant faults and to conceive the observer and the fault tolerant control independently. An example of simulation allowing to validate the proposed method is proposed in the end of the paper.

References:

[1] A. Akhenak, M. Chadli, J. Ragot and D. Maquin, *Design of observers for Takagi-Sugeno fuzzy models for Fault Detection and Isolation.* 7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes SAFEPROCESS'09, Barcelona, Spain, June 30th - July 3rd, 2009.

[2] Beale S. and Shafai B. *Robust control system design with a proportional integral observer*, International Journal of Control, 50(1):97-111, 1989.

[3] Blanke M. , Kinnaert M., Lunze J., et Staroswiecki M. *Diagnosis and fault-tolerant control.* Springer-Verlag, 2003.

[4] Chen J. et Patton R.J. *Fault-tolerant control systems design using the linear matrix inequality approach.* 6th Europeen Control Conference, Porto, Portugal, 4-7 September, 2001

[5] Edwards C. *A comparison of sliding mode and unknown input observers for fault reconstruction*, IEEE Conference on Decision and Control, vol. 5, pp. 5279-5284, 2004.

[6] D. Ichalal, B. Marx, J. Ragot and D. Maquin, *Simultaneous state and unknown inputs estimation with PI and PMI observers for Takagi-Sugeno model with unmeasurable premise variables.* 17th Mediterranean Conference on Control and Automation, MED'09, Thessaloniki, Greece, June 24-26, 2009.

[7] Johansen T. A. et Foss A. B., *Non linear local model representation for adaptive systems.* IEEE International Conference on Intelligent control and instrumentation, Vol. 2, pp. 677-682, 1992.

[8] Khedher A., Benothman K., Maquin D., Benrejeb M. *An approach of faults estimation in Takagi-Sugeno fuzzy systems* Accepted in the 8^{th} ACS/IEEE International Conference on Computer Systems and Applications AICCSA'10, Hammamet, Tunisia, May 16-19th, 2010

[9] Khedher A., Benothman K., Maquin D., Benrejeb M. *Fault tolerant control for nonliner system described by Takagi-Sugeno models* 8^{th} International Conference of Modeling and Simulation - MOSIM'10 - Hammamet - Tunisia - May 10-12, 2010.

[10] Khedher A., Benothman K., Maquin D., Benrejeb M. *Sensor fault estimation for nonlinear systems described by Takagi-Sugeno models* Accepted for publication in the International Journal Transaction on system, signal & devices, Issues on Systems, Analysis & Automatic Control.

[11] Khedher A., Benothman K., Maquin D., Benrejeb M. *State and sensor faults estimation via a proportional integral observer.* 6th international multi-conference on Systems signals & devices SSD'09 March 23-26, Djerba, Tunisia, 2009.

[12] Khedher A., Benothman K., Maquin D., Benrejeb M. 2008. *State and unknown input estimation via a proportional integral observer with unkbown inputs.* 9th international conference on Sciences and Techniques of Automatic control and computer engineering, STA'2008, Sousse, Tunisia, December 20-23, 2008.

[13] Linder S. and Shafai B. *Rejecting disturbances to flexible structures using PI Kalman filters*, IEEE International Conference on Control Applications, Hartford, CT, USA, October 5-7,1997.

[14] Murray-Smith R., Johansen T. *Multiple model approaches to modeling and control.* Taylor and Francis, London, 1997.

[15] Niemann H. et Stoustrup J. *Passive fault tolerant control of double inverted pendulum-a case study example. 5th IFAC Symposium*

on Fault Detection, Supervision and Safety of Technical Processes SAFEPROCESS'03, Washington, D.C., USA, 9-11 june 2003.

[16] R. Orjuela, B. Marx, J. Ragot and D. Maquin, *On the simultaneous state and unknown inputs estimation of complex systems via a multiple model strategy.* IET Control Theory & Applications, 3(7):877-890, 2009.

[17] Orjuela R., Marx B., Ragot J. et Maquin D., *Proportional-Integral observer design for nonlinear uncertain systems modelled by a multiple model approach*, 47th IEEE Conference on Decision and Control, Cancun, Mexico, December 9-11, 2008

[18] oudghiri M. *Commande multi-modèles tolérante aux défauts : Application au contrôle de la dynamique d'un véhicule automobile.* thèse de doctorat, Université Picardie Jules Verne, France, 20 Octobre 2008.

[19] Takagi T. et Sugeno M., *Fuzzy identification of systems and its applications to model and control.* IEEE Transactions on Systems, Man, and Cybernetics vol.15, pp. 116-132, 1985

[20] Weng Z., Patton R., et Cui P. *Active fault tolerant control of a double inverted pendulum.* 6th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes SAFEPROCESS'06, 2006.

[21] Weinmann A. *Uncertain models and robust control.* Springer-Verlag, Vienna, 1991.

[22] Witczak M., Dziekan L., Puig V. and Korbicz J. *A fault-tolerant control strategy for Takagi-Sugeno fuzzy systems.* 17th World Congress The International Federation of Automatic Control, Seoul, Korea, July 6-11, 2008.

[23] Zhang K., Jiang B., Coquempot V. *Adaptive Observer-based Fast Fault Estimation.* Internation Journal of Control Automation, and systems Vol. 6 no 3, pp 320-326 june 2008.