Dynamical Model of a Power Plant Superheater

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Abstract: - The paper deals with simulation of both dynamics and control of power plant superheaters. Superheaters are heat exchangers that transfer energy from flue gas to superheated steam. A composition of superheater, its input and output pipelines, and fittings is called a superheater assembly. Inertias of superheater assembly are often decisive for design of a steam temperature control system. Mathematical model of a superheater assemble is described by sets of nonlinear partial differential equations. The accuracy of the mathematical model is the center of the problem of simulation. Dominant role plays the accuracy of the mathematical model of the superheater. To discuss the accuracy of the mathematical model, the model was applied to the output superheater assemble of the 200 MW generating block of the actual operating power plant. To analyze the accuracy of the mathematical model, the system was agitated by test signals. Experiments carried out at the power plant were simulated mathematically. Then, data obtained by the measurement were compared with simulation results. Comparison leads to the verification of both the accuracy and the serviceability of the mathematical model discussed.

Key-Words: - Simulation, Measurement, Superheaters, Partial differential equations, Model verification

1 Introduction

The interchange of energy from chemical to electrical one made in fossil thermal power plant is a complex process. Mathematical model of this process enables operator to optimize the control of the actual plant and the designer to optimize the design of the future plants.

There are many units that are situated in the main technological chain of the thermal power plant. All of them can be described mathematically and included in the mathematical model of the plant. This paper deals with power plant heat exchangers, particularly with superheaters. Superheaters are parts of the power plant boiler. They transfer heat energy from flue gas to superheated steam. Superheaters are connected to the other parts of the boiler by pipelines and headers. Inertias of heat exchangers and their pipelines are often decisive for the design of the power plant steam temperature control system.

Mathematical model of the steam exchanger was developed in [6]. It is given by equations (1) - (5) below. Mathematical model of a pipeline or a header can be developed from the mathematical model of the heat exchanger. The models comprise many coefficients. Coefficients of pipelines and headers are usually known with the relatively good accuracy. Let us consider the mathematical model of the superheater assembly comprising superheater, its associated pipelines and pipe fittings. The accuracy of the model would depend on both the accuracy and correctness of coefficients of the model of the superheater.

In this paper, the deterministic verification of the mathematical model of the superheater and its associated parts is presented.

The verification process was as follows. The superheater assembly of operating 200 MW power plant was agitated by the set of long term forced input signals. The dynamic responses were both measured and simulated. The measured and calculated results were compared. The paper presents results of selected experiments.

2 Mathematical model of a superheater

Superheater is a heat exchanger that transfers heat energy from a heating media to a heated media. The heating media is usually flue gas. The heated media is usually steam, sometimes it is a mixture of air and steam or some other media. There are many types and configurations of superheaters. One energy block of a power station usually contains several different units which increase temperature of superheated steam in successive cascade. The last superheater in the cascade is called the output superheater.

The interconnections of superheaters differ from case to case. They are parts of the control loops that generate steam of desired state values.

Technical designs of superheaters result in constructions that are complicated and complex. The

following paragraphs deal with actual output fuel gas to steam superheater that operates in a normal operating mode.
Typical superheater is a steam tube bundle sank into a flue gas channel. To simulate the dynamics of a superheater as a single tube, the values of some parameters of the actual superheater have to be converted.

The mathematical model of a superheater is defined by seven state variables, [6]. They are as follows:

$T_1(x,t)$	temperature of steam
$T_2(x,t)$	temperature of flue gas
$T_S(x,t)$	temperature of the wall of the heat exchanging surface of the superheater
$p_1(x,t)$	pressure of steam
$u_1(x,t)$	velocity of steam

$p_2(0,t) = p_2(x,t) = p_2(L,t)$ pressure of flue gas
$u_2(0,t) = u_2(x,t) = u_2(L,t)$ velocity of flue gas

where x is the space variable along the active length of the wall of the heat exchanging surface of the superheater
t is time.

Fig. 1 shows the principal scheme of the physical state variables of a parallel flow steam superheater.

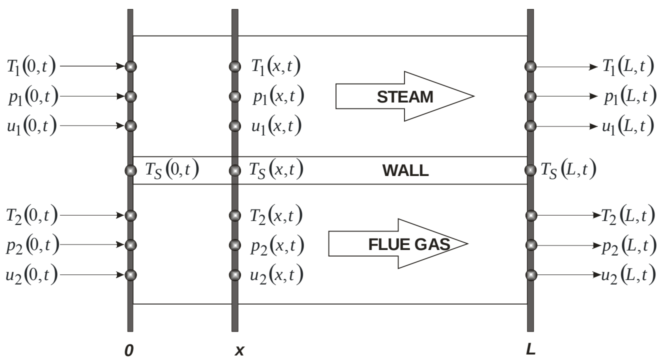

Fig. 1 Principal scheme of the physical state variables at a parallel flow steam superheater

Applying the energy equations, Newton's equation, and heat transfer equation, and principle of continuity the behavior of five state variables of superheater can be well described by five nonlinear partial differential equations, PDE, as follows:

Reduced energy equation for flue gas:

$$\frac{c_2\rho_2F_2}{\alpha_{S2}O_2}\left[u_2\frac{\partial T_2}{\partial x}+\frac{\partial T_2}{\partial t}\right]+(T_2-T_S)=0 \qquad (1)$$

Heat transfer equation describes the transfer of heat from burned gases to steam via the wall:

$$\frac{\partial T_S}{\partial t}-\frac{T_1-T_S}{\dfrac{c_SG}{\alpha_{S1}O_1}}-\frac{T_2-T_S}{\dfrac{c_SG}{\alpha_{S2}O_2}}=0 \qquad (2)$$

Principle of continuity for steam:

$$\frac{1}{F\rho_1}\left\{Fu_1\left(\frac{\partial\rho_1}{\partial p_1}\frac{\partial p_1}{\partial x}+\frac{\partial\rho_1}{\partial T_1}\frac{\partial T_1}{\partial x}\right)+u_1\rho_1\left(\frac{\partial F}{\partial p_1}\frac{\partial p_1}{\partial x}+\frac{\partial F}{\partial T_1}\frac{\partial T_1}{\partial x}\right)\right.$$
$$\left.+\rho_1\left(\frac{\partial F}{\partial p_1}\frac{\partial p_1}{\partial t}+\frac{\partial F}{\partial T_1}\frac{\partial T_1}{\partial t}\right)+F\left(\frac{\partial\rho_1}{\partial p_1}\frac{\partial p_1}{\partial t}+\frac{\partial\rho_1}{\partial T_1}\frac{\partial T_1}{\partial t}\right)\right\}+\frac{\partial u_1}{\partial x}=0 \qquad (3)$$

Newton's equation for steam:

$$\frac{\partial p_1}{\partial x}+\rho_1u_1\frac{\partial u_1}{\partial x}+\rho_1\frac{\partial u_1}{\partial t}+\rho_1g\sin(\theta)+\frac{\rho_1\lambda_1u_1|u_1|}{2d_n}=0 \qquad (4)$$

Energy equation for steam:

$$\frac{\partial}{\partial t}\left\{\rho_1\left(c_1T_1+\frac{u_1^2}{2}\right)\right\}+\frac{\partial}{\partial x}\left\{\rho_1u_1\left(c_1T_1+\frac{u_1^2}{2}\right)\right\}+\frac{\partial}{\partial x}\{p_1.u_1\}+\frac{\partial}{\partial x}\{\rho_1.u_1.g.z\}-\alpha_{1S}O_1(T_S-T_1)\frac{1}{F}=0 \qquad (5)$$

Where

$c_1=c_1(p,T)$	heat capacity of steam at constant pressure, $J.kg^{-1}K^{-1}$
$c_2=c_2(p,T)$	heat capacity of flue gas at constant pressure, $J.kg^{-1}K^{-1}$
c_S	heat capacity of superheater's wall material, $J.kg^{-1}K^{-1}$
d_n	diameter of pipeline, m
$F_1=F_1(x)$	steam pass crossection, m^2
$F_2=F_2(x)$	flue gas channel crossection, m^2
g	acceleration of gravity, $m.s^{-2}$

$G = G(x)$	weight of wall per unit of length in x direction, kg m^{-1}
L	active length of the wall, m
$O_1 = O_1(x)$	surface of wall per unit of length in x direction for steam, m
$O_2 = O_2(x)$	surface of wall per unit of length in x direction for flue gas, m
$p_1 = p_1(x,t)$	pressure of steam, Pa
$p_2 = p_2(x,t)$	pressure of flue gas, Pa
t	time, s
$T_1 = T_1(x,t)$	temperature of steam, ºC
$T_2 = T_2(x,t)$	temperature of flue gas, ºC
$T_S = T_S(x,t)$	temperature of the wall, ºC
$u_1 = u_1(x,t)$	velocity of steam in x direction, m.s^{-1}
$u_2 = u_2(x,t)$	velocity of flue gas in x direction, m.s^{-1}
x	space variable along the active length of the wall, m
$z = z(x)$	ground elevation of the superheater, m
α_{S1}	heat transfer coefficient between the wall and steam, J.m^{-2}s^{-1}K^{-1}
α_{S2}	heat transfer coefficient between the wall and flue gas, J.m^{-2}s^{-1}K^{-1}
$\lambda_1(x)$	steam friction coefficient, 1
θ	superheater's constructional gradient, 1
$\rho_1 = \rho_1(p,T)$	density of steam, kg.m^{-3}
$\rho_2 = \rho_2(p,T)$	density of flue gas, kg.m^{-3}

Equations (1) - (5) define the basic mathematical model of a superheater

As shown in [6], near stabilized operating state of superheater, the derivatives of parameters of in PDE (3) can be neglected. Then, also the flow velocity and the pressure of steam can be assumed to be the known functions of time. Under these presumptions, the mathematical model of superheater describes only the relatively slow heat transfer between media.

For constant steam pass crossection $F_1(x) = F_1$ steam pressure and steam velocity act as known inputs independent of length x ,

$$p_1(0,t) = p_1(x,t) = p_1(L,t)$$
$$u_1(0,t) = u_1(x,t) = u_1(L,t)$$

For horizontal wall the equations (1) - (5) can be reduced to the system of three equations. These three equations define the simplified mathematical model of a superheater, see [6] for details.

3 Mathematical model of a pipeline

Superheater is operated as a unit that is connected to the preceding and consecutive units via pipelines and headers. The header can be considered to be a sort of pipeline.

To simulate the processes measured on the superheater at the actual power plant, mathematical model of the pipeline is necessary.

There is not any principal physical difference between the heat exchange that is in progress in a boiler heat exchanger and the heat exchange that runs in a pipeline. In superheater the flue gas heats steam, in pipeline steam warms air. In consequence, mathematical model of both a pipeline and a header is given by the same system of equations. Here, the flue gas has to be substituted by air. For an insulated pipeline the thermal losses to the external environment can be often neglected. Then, $\alpha_{S2} \to 0$, and the mathematical model of pipeline can be reduced.

A composition of superheater, its input and output pipelines, and fittings is called a superheater assembly. At an actual power plant, there is necessary to respect the technical feasibility of both the insertion of input signals and the measurement of output signals. To compare the measured and calculated signals the mathematical model simulating the actual superheater assembly is indispensable.

4 Superheater assembly

The mathematical model of the heat exchanger was specified for the parallel flow output superheater of the 200 MW block of Detmarovice thermal power station, EDE. The EDE is the 800 MW coal power plant of CEZ joint-stock company. The factory is in 2010 in operation for 40 years. It is equipped with very modern digital controllers and computer control system.

The specification of the model was made with the assistance of the thermal and hydraulic boiler calculation. The thermal and hydraulic boiler calculation defines operating parameters of the superheater. It also defines various operating steady-state values of state variables at both the input and the output of the superheater. It does not cover all parameters of the model and functional dependences of parameters.

The basic useful method to check the model accuracy is to compare selected steady-state values of physical variables obtained by simulation with values specified by the thermal and hydraulic boiler calculation. As presented above, such quantification of accuracy is partial und incomplete.

The better method to check the model accuracy is to compare selected characteristics and time responses obtained by superheater simulation with characteristics and time responses obtained by measurement on the actual power plant superheater. Such quantification of accuracy needs the suitable selection of characteristics and responses.

In this paper, there are compared the responses of both the actual superheater plus associated piping and its mathematical model to forced input signals perturbations. The closed loop control system is not suitable for this purpose. The effect of accuracy of coefficients of mathematical model of superheater on the resulting transients is due the feedback very small. To assess the accuracy of the mathematical model, experiments have to be done on the open loop system, see paragraph 5.

Fig. 2 shows the scheme comprising the superheater, piping, and the basic controllers that stabilize the temperature of steam at the output of superheater assembly. The inlet superheated steam enters the mixer. The outlet superheated steam leaves the last pipeline.

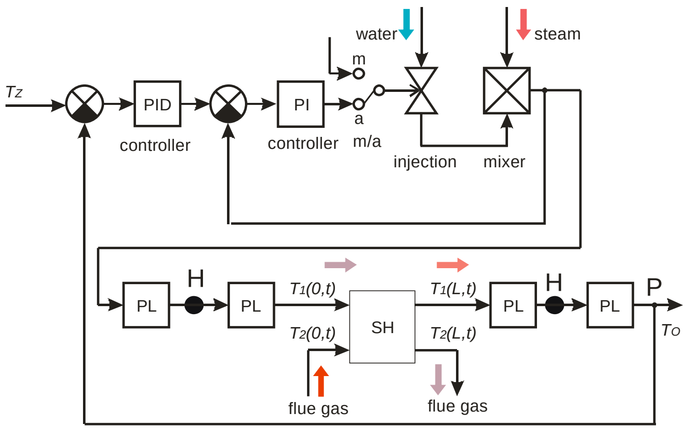

Fig. 2 Scheme of the superheater assembly

The control circuit includes two control loops. The fast loop with **PI** controller regulates the water flow rate by the valve **injection** to balance the temperature behind the **mixer**. The main loop with **PID** controller stabilizes superheater assembly outlet steam temperature T_o.

Superheater assembly being controlled consists of the input section, parallel flow superheater **SH**, and the output section. Both input and output section consists from two pipelines **PL** separated with a header **H**. The manual to automatic control switch **m/a** is set to the automatic control mode, and the assembly outlet steam temperature T_o measured at point **P** is stabilized to the set point value $T_z = 540^\circ\text{C}$.

The closed loop control loop process was simulated in MATLAB&Simulink. Data for simulation were accumulated by measurement on EDE. The basic scheme is shown in Fig. 3.

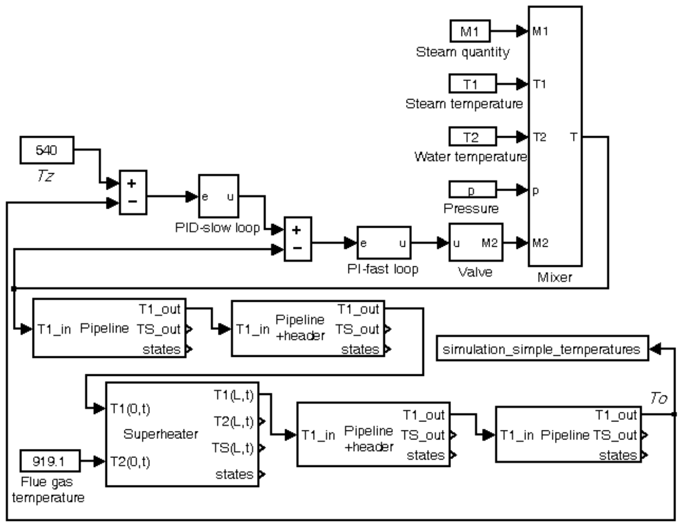

Fig. 3 Closed loop temperature control MATLAB&Simulink scheme

Fig. 4 shows one typical simulation task. This experiment cannot be carried out on the operating power plant. It is not possible to enter such a set point difference to the power plant equipped with the actual closed loop control system. Fig. 4 relates to the superheater that is operating under standard operating conditions. Superheater and its feedback control system are shown in Fig. 2. At time $t = 0$, the superheater is in its steady state, and the set point value T_z is changed from 520°C to 540°C. The simulated time response of the outlet temperature T_o of the superheater assembly initiated by both the outlet temperature set point step change of 20°C and actual deviations of input signals is displayed in Fig. 4. Positions of signals are shown in Fig. 2.

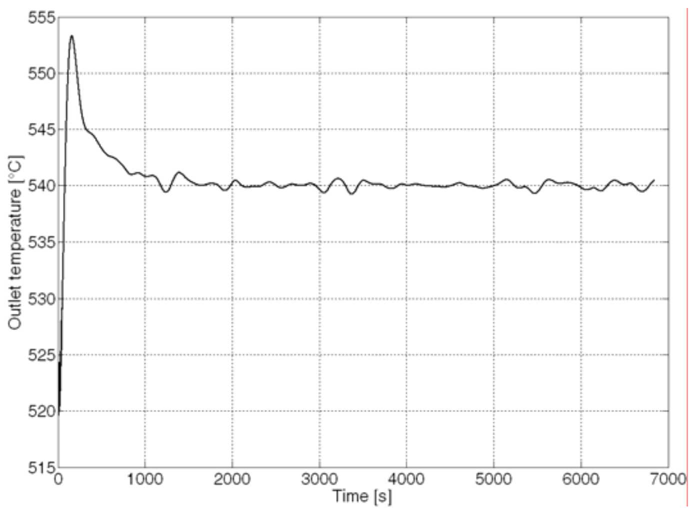

Fig. 4 Simulated outlet temperature at the feedback control system

5 Verification of the mathematical model

There are six input variables in the mathematical model (1) - (5): $T_1(0,t)$, $p_1(0,t)$, $u_1(0,t)$, $T_2(0,t)$ $p_2(0,t)$, $u_2(0,t)$. The output variables of interest are temperatures $T_1(L,t)$ and $T_2(L,t)$ of steam and flue gas and pressure $p_1(L,t)$ of steam. The change of each input variable results in time responses all of output variables. It would be advantageous to set all but one input signals constant and study the responses of the system item-by-item. At the operating power plant, it is not a simple problem.

As listed above, there are eighteen principal combinations of choice of the input to output pair of a superheater. There is also possible to insert some input signals and measure some output signals in different points of superheater assembly. It is beyond the scope of this paper to present here all possible combinations of responses. To discuss the quality and accuracy of the mathematical model, the example of presentation has been selected as follows.

The input was the disturbance of the water flow rate at the controlling water **injection**. Note that the change of the water flow rate results in a change of both steam velocity and steam temperature at the output of the **mixer**. The output was the superheater assembly outlet steam temperature T_o. Layout of the experiment is shown at Fig. 5.

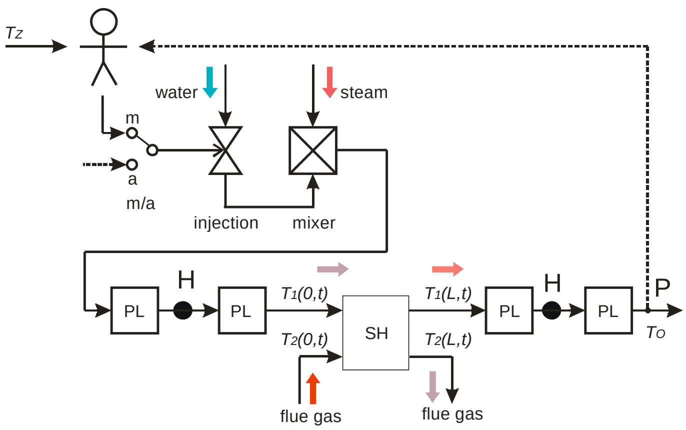

Fig.5 Layout of the open loop experiment

To obtain sufficiently large values of deviations of state values and output signals, the superheater's automatic feedback control loops were disconnected during experiments. At a 200 MW superheater, it is a rather challenging task. To deal with this problem, the presented experiments were realized at the derated power of 180 MW. Note that at the output superheater the outlet steam is technologically stabilized and lead to the high-pressure part of turbine. The discussion of technological stabilization is beyond the scope of this paper.

To disconnect the feedback loops, the control of the controlling water injection was set to the manual mode. The superheater assembly outlet temperature T_o was controlled, roughly, by the operator.

The open loop temperature control process was simulated in MATLAB&Simulink. The basic scheme is shown in Fig. 6.

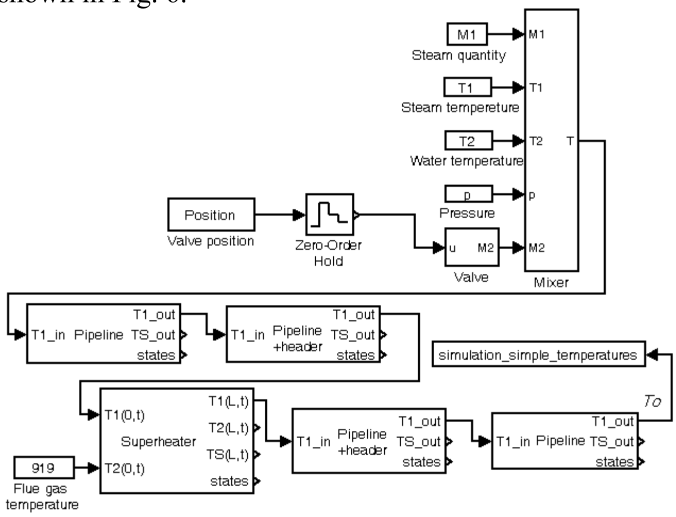

Fig. 6 Open loop temperature control MATLAB&Simulink scheme

Every measurement was approximately for two hours in length. All necessary input and output variables were measured automatically and processed and evaluated by the model. Data were measured in three second sampling interval.

6 Measured and simulated results

Fig. 7 compares assembly steam outlet temperatures obtained by both measurement and simplified mathematical model. Fig. 8 presents the same measurement and compares the simulated results for the basic mathematical model (1) - (5).

The position of output signal is shown in Fig. 5. The intensity of the forced disturbance of the water flow rate at the controlling water injection applied was the part of the experiment. The disturbance in the standard operating regime of the superheater is much smaller. So are the deviations of the outlet temperature.

As the basic model is more complex than the simplified model, it gives more precise results at both steady states and dynamics of the time responses. Comparison of Fig. 7 with Fig. 8 illustrates, that at the standard operating state the simplified model approximates the basic mathematical model (1) - (5) very well. Outside the vicinity of the set point, the accuracy of the simplified model decreases.

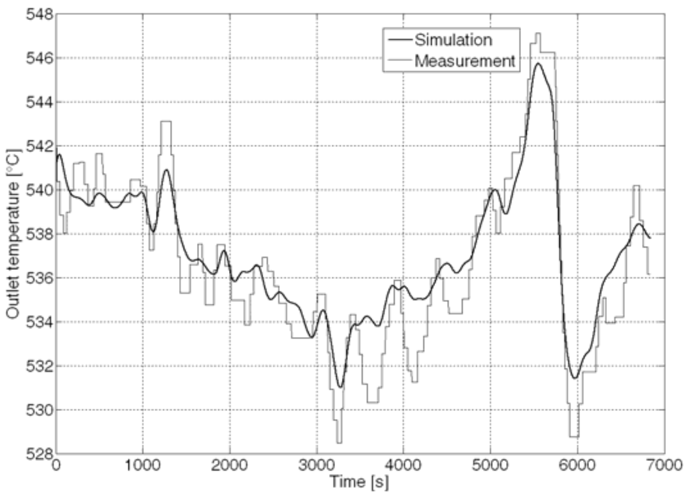

Fig. 7 Comparison of measured and simulated outlet temperatures at the open loop control system experiment. Simplified model of superheater assembly

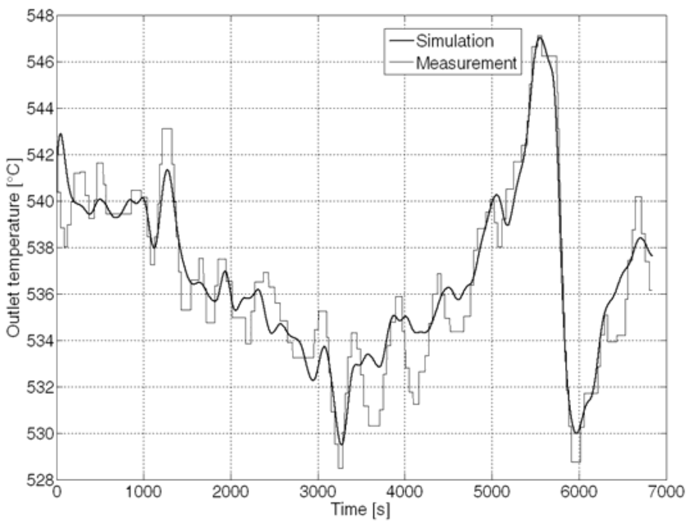

Fig. 8 Comparison of measured and simulated outlet temperatures at the open loop control system experiment. Basic model of superheater assembly (1) - (5)

7 Verification of control circuit by methods of statistic dynamics

The whole model of control circuit described above is more complex in fact. Control injection in control circuit is affected by two valves. One of them is active in the range of 37,5% to 100% and the second one from 0% to 100% of manipulated value of PI controller in the fast loop. Active areas of both valves are affected by function generators that adjust static characteristic of the valves. These valves are arranged in cascade and their cooperative regulation serves for linearization of water flow into the mixer. To control these valves in simple way, the percentages of opening positions are recalculated into the range of 0 to 100% of the range of a single fictive valve. This percentage affects the proportional element of fast loop controller through another function generator and time delay. Due to the fact that controllers of both fast and slow loops are expressed in multiplicative form of control algorithms, it is possible to say that percentage of valve opening affects both P elements of controllers. Model of control circuit of output superheater focused on detail composition of PI controller is shown in Fig. 9. The arrangement of unheated and heated areas was merged into one block to become transparent. The whole setup can be seen from Fig. 2.

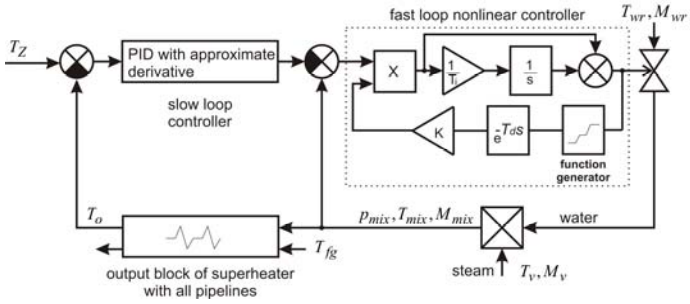

Fig. 9 Detail control circuit scheme for output superheater

Fig. 9 shows the following signals measured under real operation and consequently used for running and verification of the simulation by use of the methods of statistic dynamic as follows:

T_v	steam temperature at mixer inlet
M_v	steam quantity at the mixer inlet
T_{wr}	water temperature at mixer inlet
M_{wr}	water quantity at the mixer inlet
T_{mix}	steam quantity at the mixer outlet
M_{mix}	water quantity at the mixer outlet
p_{mix}	steam pressure at the mixer outlet
T_z	desire temperature in slow loop, constant
T_{fg}	flue gas temperature
T_o	superheater's output temperature

Firstly it is necessary to determine the course of flue gas temperature, which cannot be directly measured due to the technological reasons. It is measured by other technological blocks that already affect the temperature course. Special algorithm was made up for calculation of flue gas temperature. Based on knowledge of temperatures T_{mix} and T_o it computes the flue gas temperature backward. Particularly it uses the splitting intervals method when the steady state of temperature T_o from simulation (hereafter denoted as T_{osim}) is compared with a temperature T_o measured under real operation. The temperature T_{osim} is a function of known (measured) steam temperature at the inlet of the superheater T_{mix} and working temperature T_{fg}, which is determined from a predefined interval. Based on given acceptable value of relative error between temperatures T_o a T_{osim} and its difference, the temperature T_{fg} is being refined until the relative error between T_o a T_{osim} is less than a given threshold. Resulting temperature T_{fg} and comparison of T_o a T_{osim} is given in Fig. 10.

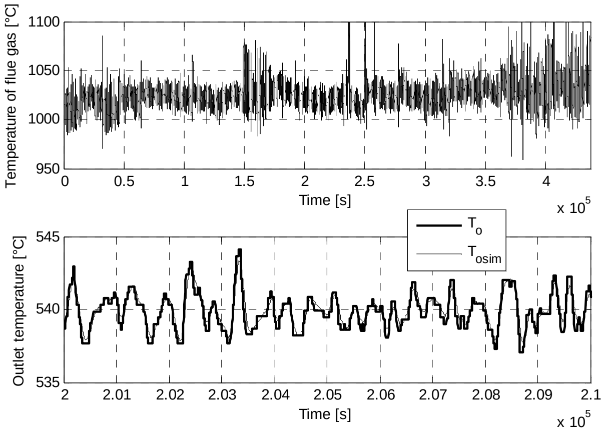

Fig. 10 Calculated flue gas temperature T_{fg} together with comparison of output superheater temperatures T_o a T_{osim}

These two temperatures are almost identical because the comparison is carried out for a setup with superheaters and unheated areas which is not involved in control circuit.

7.1 Measuring the plant by stochastic signals

Measured signals from real operation make up ten-day record from July/August 2009. The records are separated from daily periods when the power plant's wattage was 180MW, with sampling period of T_s = 3 seconds.

The control circuit (see Fig. 9) was fed with stochastic signals T_v, T_{wr}, T_{fg}, M_{mix} a p_{mix}, measured in real operation. The following pictures show comparison of chosen signals from simulation and real operation. Fig. 11 compares output temperatures T_o and simulated T_{osimCL}.

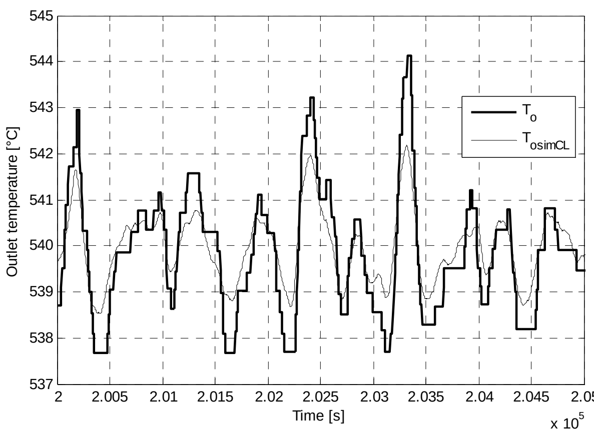

Fig. 11 Comparison of output temperatures T_o and T_{osimCL} , simulation of the whole control circuit

The difference compared to Fig. 10 is obvious. Temperature T_{mix}, coming into the superheater is no longer course from real operation, but simulated course control circuit consisting of simplified model of superheater and unheated areas. It causes differences between real and simulated data, as shown for valve opening positions in Fig. 12.

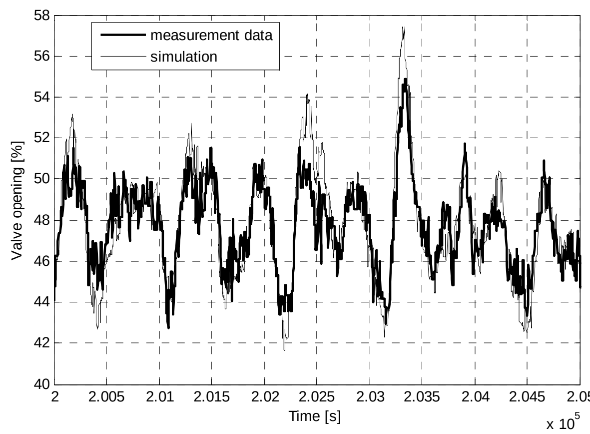

Fig. 12 Comparison of percentages of valve opening

Fig. 13 shows the course of proportional element of fast PI loop, which is not measures in real operation but it is necessary to know its mean value for consequent analysis, particular for further linearization in operating point.

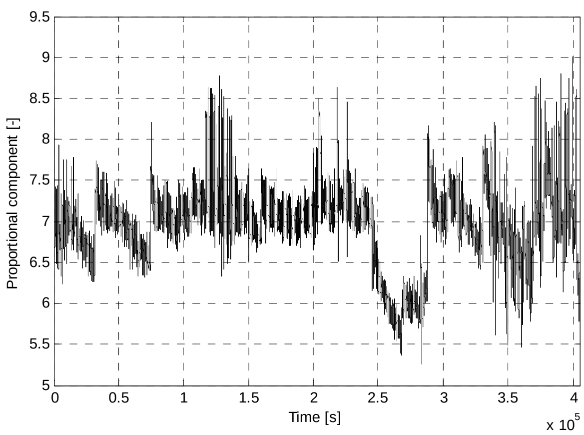

Fig. 13 Proportional element time course, fast loop

7.2 Measurements of correlation functions

Measurements of correlation functions of stationary stochastic signals is based on definition

$$R_{uy}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y(t) \cdot y(t+\tau) dt \qquad (6)$$

With respect to finite length of the record T_N and getting equidistant sample with sampling T_s, this formula can be transformed into summation:

$$R_{uy}[\tau] = \frac{1}{N - \frac{\tau}{T_{sampling}}} \cdot \sum_{k=0}^{N-\frac{\tau}{T_s}} y[t_k] \cdot u[t_k + \tau] \qquad (7)$$

Values $y[t_k]$ a $u[t_k]$ stand for discrete samples of signals $y(t)$ a $u(t)$ in equidistant time intervals with T_s. Parameter N in equation (7) must be high enough since it is whole number of elements in record.

Reaching the solution requires choosing several parameters:

a) Whole length of measurement T_N must be quite long, so as all of the frequencies of the signals can be captured, especially lower ones. Calculation of autocorrelation functions for highest time shift τ_{max} requires

$$T_N = (10 \div 20) \cdot \tau_{max} \qquad (8)$$

Calculating correlation functions according (7) for $\tau \neq 0$ causes distortion of resulting correlation function. This distortion grows with increasing distance from zero shift $\tau = 0$. That's why T_N is chosen the same size or bigger than the period of lowest importance elements of the signal according (8) to assure that distortion would for time much higher than τ_{max}.

b) Sampling time T_s must be so low to ensure that measured signals doesn't significantly change during T_s second. Once T_s is set, it's not possible to measure elements of signals with frequencies higher than

$$f_{max} = \frac{1}{2T_{sampling}} \qquad (9)$$

By means of the term (7) three correlation functions were calculated. First one is autocorrelation function of the signal that indicates detrended temperature of a steam at the inlet of mixer T_v (see Fig. 14). Other two correlation functions define time dependencies between detrended temperatures T_v , To and T_v , T_{osimCL} (see Fig. 15).

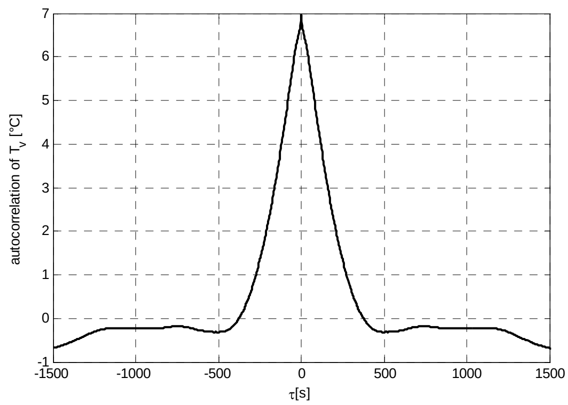

Fig. 14 Autocorrelation function of detrended temperature course T_v after applying ergodic hypothesis

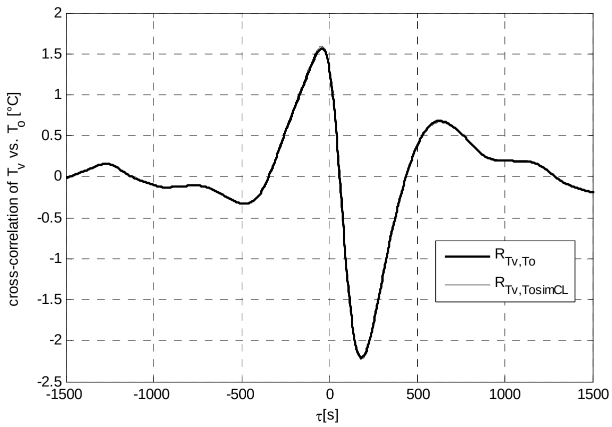

Fig. 15 Comparison of cross-correlation functions of detrended temperatures T_v vs. T_o and T_v vs. T_{osimCL} after applying ergodic hypothesis

7.3 Ergodic hypothesis

Stochastic signal, as a name of continuous variable depending on time, can be stored in two different ways. It is either possible to make one record of infinite length or infinite number of finite length records. Despite the finite length of record of stochastic signal, infinite time interval is necessary to describe time dependence and sequence of the values. Ergodic hypothesis allows transition between these ways.

Due to the fact that length of the data to be processed would exceed the size of inverse matrix several times when computing numerical deconvolution, the whole record was divided into approximately 200 same time intervals. Then 200 correlation function of the same type were calculated and summed up, and the final result was divided by the number of intervals. Using this way, so called ergodic hypothesis has been implemented. As a result of this, the estimation of correlation functions were refined. Fig. 14 and 15 show courses in the surrounding $\tau = 0$, where the error caused by shifting τ (7) is not relevant yet. In some cases, it is even effective to omit this correction and to divide whole correlation function by its length (in Matlab syntax using `xcorr` function, parameter 'biased' refers to this situation). This substitution can be applied for correlation functions that are long enough and for surrounding $\tau = 0$.

7.4 Identification the dynamics of control circuit with steam superheater

Method of identification the system by statistic dynamics is designed for linear systems. This paper describes its use for comparison of modeled control circuit in Simulink and real control circuit. The result of this identification is response of steam temperatures at the superheater outlet to Heaviside step of superheater inlet temperature. In simple words, it is response of the

control circuit to step change of disturbance, representing steam temperature at the mixer inlet T_v.
When computing numerical deconvolution, Wiener – Hoppf equation

$$R_{uy}(\tau) = \int_0^{\infty} h(t) R_{uu}(\tau - t) dt \tag{10}$$

represents stochastic formulation of dynamic system.
Under a special condition, in case of bringing white noise into input of the system having the following autocorrelation function

$$R_{uu}(\tau) = \delta(\tau), \tag{11}$$

we get

$$R_{uy}(\tau) = \int_0^{\infty} h(t) \delta(\tau - t) dt = h(\tau) \tag{12}$$

Numerical calculation of weighting function is based on replacing integration process by summation and numeric deconvolution. Discretizing equation (12) leads to

$$R_{uy}(\tau) \approx \sum_{k=0}^{N} R_{uu}(\tau - kT_s) h[kT_s] T_s \tag{14}$$

If time shift τ is expressed as multiple of time step T_s, that is $\tau = 0, T_s, 2\,T_s, \ldots, N$, it is possible, using the last equation, a set of $(N + 1)$ linear algebraic equations, from which it is possible to compute unknown values of weighting function $h(0), h(T_s), \ldots, h(NT_s)$:

$$\begin{aligned} R_{uy}[0] &= (R_{uu}[0]h[0] + R_{uu}[-T_s]h[T_s] + \ldots + R_{uu}[-NT_s]h[NT_s])T_s \\ R_{uy}[T_s] &= (R_{uu}[T_s]h[0] + R_{uu}[0]h[T_s] + \ldots + R_{uu}[T_S - NT_s]h[NT_s])T_s \\ &\vdots \\ R_{uy}[NT_s] &= (R_{uu}[NT_s]h[0] + R_{uu}[NT_s - T_s]h[T_s] + \ldots + R_{uu}[0]h[NT_s])T_s \end{aligned} \tag{15}$$

Using following feature of autocorrelation function,

$$R_{uu}(\tau) = R_{uu}(-\tau) \tag{16}$$

and after introduction of shortened notation of weighting function $h_k = h[kT_s]$

The set of equation can be rewritten into matrix form:

$$\begin{bmatrix} \frac{R_{uy}[0]}{T_s} \\ \frac{R_{uy}[T_s]}{T_s} \\ \vdots \\ \frac{R_{uy}[N \cdot T_s]}{T_s} \end{bmatrix} = \begin{bmatrix} R_{uu}[0] & R_{uu}[T_s] & \cdots & R_{uu}[NT_s] \\ R_{uu}[T_s] & R_{uu}[0] & \cdots & R_{uu}[(N-1)T_s] \\ \vdots & \vdots & & \vdots \\ R_{uu}[NT_s] & R_{uu}[(N-1)T_s] & \cdots & R_{uu}[0] \end{bmatrix} \cdot \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_N \end{bmatrix} \tag{17}$$

Or in the matrix form

$$\mathbf{r} = \mathbf{R} \cdot \mathbf{h} \tag{18}$$

Solution of weighting function can be reached by use of inverse matrix $\mathbf{R}^{-1}$ as follows:

$$\mathbf{h} = \mathbf{R}^{-1} \cdot \mathbf{r} \tag{19}$$

This numerical solution of deconvolution in Matlab is limited by matrix until approximately 3000×3000 elements.
Concerning that the length of measured data exceeds the size of the matrix that would be created during numerical solution of deconvolution, it is suitable to split the record into several same sections and compute particular impulse characteristics. The second reason for splitting is the fact that time constant of superheater is smaller than time of calculated impulse response that would be computed in case of maximal possible solution of numeric deconvolution (3000 x T_s = 9000 seconds). Due to this reason, the ergodic hypothesis was used for estimation of impulse characteristic.

Applying numerical deconvolution of Wiener – Hopf equation (10) leads to estimation of impulse characteristic of disturbance transfer function (see Fig. 16). In equation (10) signal u denotes temperature T_v and signal y stands for temperature T_o, resp. T_{osimCL}. To get worked this method in proper way it is necessary to detrend the temperatures.

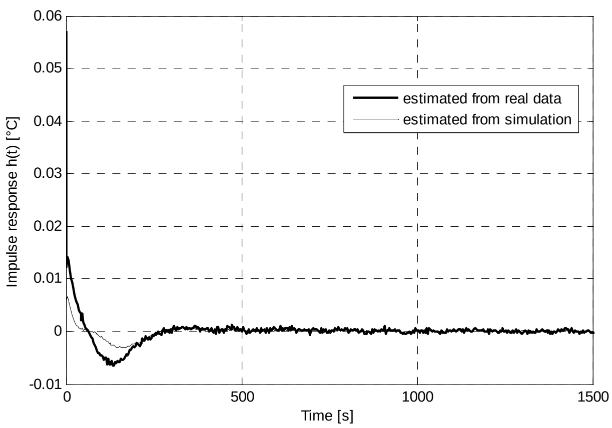

Fig. 16 Comparison of estimations of impulse characteristics of disturbance transfer function

Integrating impulse characteristic we get estimation of the step response of disturbance transfer function (see Fig. 17).

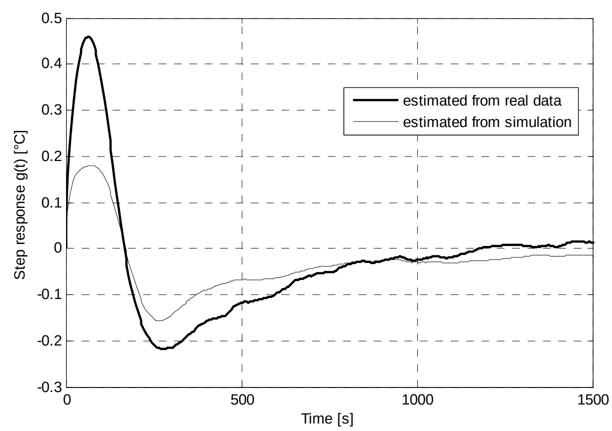

Fig. 17 Comparison of estimations of step characteristics of disturbance transfer function

Disturbance transfer function figuratively means "black box" systems representing modeled control circuit, see

Fig. 9. This control circuit compensates error corresponding to steam temperature at the inlet of mixer T_v. Fig. 17 shows comparison of estimated step responses. The first one is computed from measured data while the second one is calculated from simulation of modeled control circuit driven by measured data. Evaluating dynamics of these responses, it is possible to conclude that real control circuit has very similar dynamic as its model. The difference overshoot can be resolved by the fact that temperatures T_v and T_o are correlated to a certain degree. The steam of temperature T_v which is brought into mixer, is output product of previous second degree of the superheater. This superheater is heated with the same flue gas as the output superheater described in this paper. It results in affecting estimation of impulse, resp. step response of the circuit. In case of the comparison the result of this identification with the response to the Heaviside step, it would be necessary to change the flue gas temperature proportionally to the value of the step at the mixer inlet, with adequate advanced time interval corresponding to the soaking all of the superheaters so that inlet mixer temperature rises by 1 °C.

8 Conclusion

The results of comparison of measured and simulated time responses show that mathematical model (1) - (5) guaranties very good description of static regime of the superheater.

The differences between measured and calculated technological steady state values are minimal. For the steady state steam inlet temperature of 320 ºC and measured steady state steam outlet temperature of 540 ºC, the maximal difference between measured and calculated values was 3.8 ºC. Similar good results were obtained for other technological state variables.

The dynamical congruence of the basic model and the simulated system is also satisfactory. Simulated waveforms of technological state variables correspond to values measured at actual superheater, they are not shifted in time. In the presented example, the dominant time constant of the mathematical model of the superheater assembly is about 320 s. The simulated steam outlet temperature tracks the measured values with the average absolute time deviation of about 20 s.

Verification of statistic qualities of the regulation was impossible due to the large number of nonlinearities affecting the control circuit. Identification by the method of statistic dynamics in this case clarified approximated compliance between modeled and real control circuit. To a great extend it is caused by statistic dependence of inlet mixer temperature T_v and output superheater temperature T_o. Both of these temperatures are to a certain degree correlated by flue gas temperature, whose time course is unknown because it's not measure under real operation. For comparison purposes the flue gas temperature time course was computed backward based on analytical model of output superheater.

In this paper, the verification of the mathematical model of the power plant superheater was described. Presented results demonstrate that the accuracy of the model is sufficient for both power plant operators and boiler designers.

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