# Algorithms for Estimation in Distributed Parameter Systems Based on Sensor Networks and ANFIS

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*Abstract:* - This paper presents some algorithms for estimation of the state variables in distributed parameter systems of parabolic and hyperbolic types. These algorithms are expressed on regression using anterior values of adjacent state variables and on auto-regression using the anterior values of the same variable. The momentary values may be obtained using sensors from a network placed in the field of the distributed parameter systems. The computation of the estimates is done using the adaptive-network-based fuzzy inference scheme. The structure of the ANFIS is derived based on training using measured values obtained form the sensor network. The algorithms and the method of estimation, emerged from three powerful concepts as theory of distributed parameter systems, artificial intelligence, with its tool adaptive-network-based fuzzy inference and the intelligent wireless ad-hoc sensor networks allow treatment of large and complex systems with many variables by learning and extrapolation. They have applications in monitoring, fault estimation, detection and diagnosis of large and complex physical processes. The paper presents some case studies as applications of all four algorithms.

*Keywords:* - System identification, fault detection and diagnosis, wireless sensor networks, non-linear system identification, distributed parameter systems, adaptive-network-based fuzzy inference, multivariable estimation techniques, auto-regression, non-linear auto-regression exogenous model, heat distribution, wave equation, partial differential equations.

# **1** Introduction

Many practical applications may be seen as distributed parameter systems, described with partial differential equations. The supervision, fault detection and fault diagnosis are important to improve reliability, safety and efficiency in maintenance of industrial processes. For these purpose some analytical methods in a classical approach in the field of fault detection and diagnosis are based on linear models as: parameter estimation, state space observers and parity equations [1]. In the last decades these methods were applied with success in electrical drives, power plants, aircrafts or chemical plants. For non-linear systems identification usage of the artificial intelligence concepts as fuzzy logic, neural networks and the adaptive-network-based fuzzy inference [2] represents powerful tools in system identification. In the theory of non-linear system identification there is the non-linear auto-regression exogenous model, as a time series [3, 4]. The model uses the present and the past values of the time series. Like any other estimation model it has an error as a residual term, a difference between the estimate and the measured value. The distributed parameter systems are in practice more complex processes, described using differential equations, partial such as the propagation of sound or heat, electrostatic phenomena, fluid flows or elasticity. Processes considered with variables distributed in space may be watched using modern wireless intelligent sensor networks. The wireless sensor networks are made by tiny spatial distributed autonomous sensors [5, 6]. They cooperate to monitor physical variables such as temperature, sound, acceleration, vibration or pressure. Some classical methods are developed for identification of the general distributed parameter system identification [7, 8].

Some recent references cited in the international conferences are presented as it follows. The paper [9] presents an identification method, a structure and algorithms based on ANFIS, as a better solution

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then large set of continuous functions defined on a compact set to an arbitrary accuracy. A non parametric modeling problem for a distributed parameter plant is analyzed to verify its quality behavior. The paper [11] proposed a strategy for the identification of the parameters in a mathematical model described by partial differential equations based on differential neural networks, on a tubular reactor system as application. In [12] ANFIS is used to predict chaotic and traffic flow time series, to achieve both high accuracy and less computational complexity for time series prediction. In [13] ANFIS is combined to tune the premise parameters and consequent parameters by means of a hybrid gradient descent and least-squares estimation. A simulation to a dynamic nonlinear system demonstrates the effective of this method. Paper [14] introduces a new approach for predicting and modeling of total solar radiation data from only the mean sunshine duration and air temperature using an ANFIS. This technique is suitable for time series prediction. In this study a database of daily sunshine duration, ambient temperature and total solar radiation data, which had been recorded for 10 years. An ANFIS model has been trained based on 9 years known data from a database. Paper [15] presents some aspects related to the estimation of average air temperature in the built environment by using integer neural networks, ANFIS and inferential sensor models. The paper compares the results of these models, presenting their advantages and disadvantages. In the emerging area of research of wireless sensor network applications [16] the design of a wireless network sensor information and identification system database which archives the data reported by distributed sensors is outlined here, as well as the implemented support for queries and data presentation. The innovations in this research include real-time support for data presentation and visual presentation of sensor nodes reporting in a geographical and temporal context. It provides support for pattern identification and data mining in sensor systems. The identification of non-linear systems continues to be a contemporary problem, trying to be solved using different methods, for example in [17], for unknown nonlinear system, given the distribution knowledge of the system inputs.

From the international journals we may refer some papers as it follows. In the paper [18] a spectral approximation based on intelligent modeling is proposed for the distributed thermal processing in semiconductor industry. Other approaches as: the parabolic model for the thermal distributed parameter system and state estimation with neural networks are used. Real time experiments are done. In the paper [19] ANFIS is applied to sensor data processing, for a calibration technique. The paper [20] describes the use of an ANFIS model to reduce the delay effects in gaze control, by prediction of the target movement. A study of fault diagnosis to a rolling bearing used in a reciprocating machine by adaptive filtering technique and fuzzy neural network is presented in [21]. An application of intelligent sensor fault detection and identification for temperature control is presented in [22]. An application of nonlinear system identification with a feedforward neural network and an optimal bounded ellipsoid algorithm is presented in [23].

The author has developed and published several papers related of using multivariable estimation techniques based on artificial intelligence for the identification of distributed parameter systems [24, 25], in the new context of intelligent sensor networks. As a distributed tool they may be used to measure time variables in the complex distributed parameter systems. In this application, with a large field of interest in science and engineering, all the above topics contribute, converging to the same objective – identification, detection and diagnosis of faults in distributed parameter systems.

The paper presents a general theory for developing four estimation algorithms in distributed parameter systems, using ANFIS as a non-linear estimator obtained by training. Also a general method is provided for monitoring of distributed parameter systems based on measurements made with sensor networks and ANFIS as a practical platform these estimation algorithms. In the chapter 2 the development starts from general equations in continuous time, using discretization to obtain general equations in discrete time, to obtain the basic models for estimation. From these time discrete equations the algorithms are derived as an obvious consequence in chapter 3. The estimator mechanism based on a non-linear ANFIS function, which may be trained from measured data obtained from the sensor networks is presented. In the fourth chapter some information on sensor networks is presented: technical characteristics, measuring capabilities with the practical application structure. The fifth chapter presents the considerations on estimation and detection, with the estimator equation, the detection structure and the monitoring method. In the sixth chapter four study cases are discussed, for each estimation algorithm presented in chapter 3. In the end the conclusion reviews the main things of the paper, advantages and possible applications.

# 2 Primary General Models

### 2.1 Continuous Time Models

The distributed parameter systems are of different types. In the following theory we will make references to the parabolic and hyperbolic types, with have many applications in practice. For these systems the general models in continuous time, as partial differential equations, are:

$$\frac{\partial \theta}{\partial t} = c_1 \nabla (c_2 \nabla \theta) + c_3 \theta + Q \tag{1}$$

$$\frac{\partial^2 \theta}{\partial t^2} = c_1 \nabla (c_2 \nabla \theta) + c_3 \theta + Q \tag{2}$$

where the variables  $\theta(\zeta, t)$  are depending on time  $t \ge 0$  and on space  $\zeta \in V$ , where  $\zeta$  is x for one axis, (x, y) for two axis or (x, y, z) for three axis,  $c_1$ ,  $c_2$  and  $c_3$  are coefficients, which could be also time variant and  $Q(\zeta, t)$ is an exterior excitation, variable on time and space. In the general case, an implicit equation may be used:

$$f(\frac{\partial \theta}{\partial t}, \frac{\partial^2 \theta}{\partial t^2}, \frac{\partial \theta}{\partial \zeta}, \frac{\partial^2 \theta}{\partial \zeta^2} \dots) = 0$$
(3)

For the partial differential equations (1, 2) some boundary conditions may be imposed to establish a solution. So, when the variable value of the boundary is specified there are Dirichlet conditions:

$$c_4 \theta = q \tag{4}$$

And, when the variable flux and transfer coefficient are specified there are Neumann conditions:

$$c_5 \nabla \theta + c_6 \theta = 0 \tag{5}$$

Limit and initial conditions of the equations (1, 2) are imposed in the practical application case studies:

$$\begin{aligned} \theta(0,t) &= \theta_{\zeta_0}, \ t \in [0,T], \\ \theta(l,t) &= \theta_{\zeta_l}, \ t \in [0,T] \end{aligned}$$
(6)

#### 2.2 Discrete Time Models

Models with finite differences may be associated to the equations (1) and (2). For this purpose the space S is divided into small pieces of dimension  $l_p$ :

$$l_p = l/n \tag{7}$$

In each small piece  $S_{pi}$ , i=1,...,n of the space S the variable  $\theta$  could be measured at each moment  $t_k$ , using a sensor from the sensor network, in a

characteristic point  $P_i(\zeta_i)$ , of coordinate  $\zeta_i$ . Let it be  $\theta_i^k$  the variable value in the point  $P_i(\zeta_i)$  at the moment  $t_k$ . It is a general known method to approximate the derivatives of a variable with small variations. In the equation with partial derivatives there are derivatives of first order, in time, and derivatives of first and second order in space.

So, theoretically, we may approximate the variable derivative in time with a small variation in time, with the following relation:

$$\frac{\partial \theta}{\partial t} = \frac{\theta_i^{k+1} - \theta_i^k}{t_{k+1} - t_k}$$

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{\theta_i^{k+1} - 2\theta_i^k + \theta_i^{k-1}}{(t_{k+1} - t_k)^2}$$
(8)

The first and the second derivatives in space may be approximated with small variations in space to obtain the following relations:

$$\frac{\partial \theta}{\partial \zeta} = \frac{\theta_i^k - \theta_{i-1}^k}{l_p}$$

$$\frac{\partial^2 \theta}{\partial \zeta^2} = \frac{\theta_{i+1}^k - 2\theta_i^k + \theta_{i-1}^k}{l_p^2}$$
(9)

We may consider the variable is measured as samples at equal time intervals with the value:

$$h = t_{k+1} - t_k \tag{10}$$

called sample period, in a sampling procedure, with a digital equipment, from a sensor network.

A linear approximate system of derivative equations of first order may be used:

$$\frac{d\theta}{dt} = A\theta + BQ \tag{11}$$

where, this time,  $\theta$  is a vector containing the values of the variable  $\theta(\zeta, t)$  in different points of the space, at different time moments.

Combining the equations (8, 9) in the equation (1) a system of equations with differences results for the parabolic equation:

$$f_p(\theta_i^k, \theta_{i-1}^k, \theta_i^{k+1}, \theta_{i-1}^{k+1}) = 0$$
(12)

and combining the equations (8, 9) in the equation (2) an equivalent system with differences results as a model for the hyperbolic equation:

Combining the equations (8, 9) in the equation

(2) an equivalent system with differences results as a model for the hyperbolic equation:

$$f_h(\theta_i^k, \theta_{i-1}^k, \theta_i^{k+1}, \theta_{i-1}^{k+1}, \theta_i^{k-1}, \theta_i^{k-1}, \theta_{i-1}^{k-1}) = 0$$
(13)

Taking account of equations (12, 13) it is obvious that several estimation algorithms may be developed as it follows, based on the discrete models of the partial differential equations. We may use several estimation algorithms based on discrete models of the partial derivative equation.

# **3** Estimation Algorithms

## 3.1 Algorithms for Parabolic Systems

*Estimation algorithm 1*. It estimates the value of the variable  $\theta_i^{k+1}$  at the moment  $t_{k+1}$ , measuring the values of the variables  $\theta_{i-1}^k, \theta_{i+1}^k, \theta_i^k$  at the anterior moment  $t_k$ :

$$\theta_i^{k+1} = f_1 \left( \theta_{i-1}^k, \theta_{i+1}^k, \theta_i^k \right) \tag{14}$$

This is a multivariable estimation algorithm, based on the adjacent nodes.

*Estimation algorithm 2.* It estimates the value of the variable  $\theta_i^{k+1}$  at the moment  $t_{k+1}$ , measuring the values of the same variable  $\theta_i^k, \theta_i^{k-1}, \theta_i^{k-2}, \theta_i^{k-3}$ , but at four anterior moments  $t_k, t_{k-1}, t_{k-2}$  and  $t_{k-3}$ .

$$\theta_i^{k+1} = f_2\left(\theta_i^k, \theta_i^{k-1}, \theta_i^{k-2}, \theta_i^{k-3}\right)$$
(15)

This is an autoregressive algorithm.

#### **3.2** Algorithms for Hyperbolic Systems

*Estimation algorithm 3*. It estimates the value of the variable  $\theta_i^{k+1}$  at the moment  $t_{k+1}$ , measuring the values of the variables  $\theta_{i-1}^k, \theta_{i+1}^k, \theta_i^k$  at the anterior moments  $t_k$  and  $t_{k-1}$ :

$$\theta_i^{k+1} = f_1 \Big( \theta_{i-1}^k, \theta_{i+1}^k, \theta_i^k, \theta_{i-1}^{k-1}, \theta_{i+1}^{k-1}, \theta_i^{k-1} \Big)$$
(16)

This is a multivariable estimation algorithm, based on the adjacent nodes and 2 time anterior moments.

*Estimation algorithm 4.* It estimates the value of the variable  $\theta_i^{k+1}$  at the moment  $t_{k+1}$ , measuring the values of the same variable  $\theta_i^k, \theta_i^{k-1}, \theta_i^{k-2}, \theta_i^{k-3}, \theta_i^{k-4}, \theta_i^{k-5}$ , but at six anterior moments  $t_k, t_{k-1}, t_{k-2}, t_{k-3}, t_{k-4}$  and  $t_{k-5}$ :

$$\theta_i^{k+1} = f_2\left(\theta_i^k, \theta_i^{k-1}, \theta_i^{k-2}, \theta_i^{k-3}, \theta_i^{k-4}, \theta_i^{k-5}\right)$$
(17)

### **3.3 Estimator Mechanism**

The estimator is a non-linear one, described by the function  $y=f(u_1, u_2, u_3, u_4)$ , using the adaptivenetwork-based fuzzy inference [2, 10]. Its general structure is presented in Fig. 1.



Fig. 1. The estimator input-output general structure

The estimator has *n*r inputs  $u_1, u_2, u_3, ..., u_4$  and one output *y*. The number of inputs depends on the algorithm type. For the 1<sup>st</sup> and 2<sup>nd</sup> algorithms there are 4 inputs, because of the first order derivation in time of the parabolic model. For the 3<sup>rd</sup> and the 4<sup>th</sup> algorithms there are 6 inputs, because of the second order derivation in time of the hyperbolic model.

The ANFIS procedure may use a hybrid learning algorithm to identify the membership function parameters of single-output, Sugeno type fuzzy inference system. A combination of least-squares and backpropagation gradient descent methods may be used for training membership function parameters, modeling a given set of input/output data.

In the inference method *and* may be implemented with product or minimum, *or* with maximum or summation, implication with product or minimum and aggregation with maximum or arithmetic media.

The first layer is the input layer. The second layer represents the input membership or fuzzification layer. The neurons represent fuzzy sets used in the antecedents of fuzzy rules determine the membership degree of the input. The activation function represents the membership functions. The 3<sup>rd</sup> layer represents the fuzzy rule base layer. Each neuron corresponds to a single fuzzy rule from the rule base. The inference is in this case the sum-prod inference method, the conjunction of the rule antecedents being made with product. The weights of the 3<sup>rd</sup> and 4<sup>th</sup> layers are the normalized degree of confidence of the corresponding fuzzy rules. These weights are obtained by training in the learning

process. The  $4^{th}$  layer represents the output membership function. The activation function is the output membership function. The  $5^{th}$  layer represents the defuzzification layer, with single output, and the defuzzification method may be the centre of gravity.

## 4 Sensor Network Capabilities 4.1 General Technical Characteristics

A Crossbow sensor network was used in practice. It has the following components: a starter kit, a MICA2 2,4 GHz wireless module, and an MTS320 sensor board. Their nodes are 2 MICAz 2,4 GHz modules, with 2 sensors MTS400, which are measuring temperature, humidity, pressure, ambient light intensity; 1 MICAz 2,4 GHz with 2 sensors MTS310 and 1 module MICAz 2,4 GHz working as a central node when it is connected through the UB port. A gateway MIB520 for node programming and a data acquisition board MDA320 with 8 analogue channels are provided. The network has the following software: MoteView for history sensor network monitoring and real time graphics and MoteWorks for nod programming in MesC language. The user interface allows some facilities, as: administration, searching, connections options and so on.

#### 4.2 Measuring Capabilities

This modern wireless sensor network has multiple measuring capabilities. So, it can measure temperature, humidity, light intensity or acceleration on 2 axes. For these kind of physical variables the mathematical models are as follows.

For temperature:

$$\frac{\partial \theta}{\partial t} = a \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + Q(x, y, t)$$
(18)

where Q is the time variable source of heating, positioned in space and  $\theta$  is the temperature.

For light intensity:

$$E(x) = \frac{I}{h^2 + x^2}, \ E = \frac{\Delta \Phi}{\Delta S}, \ \Delta \Phi = I \frac{\Delta S}{r^2} = I \Delta \alpha$$
(19)

where *I* is the luminous intensity of the light source, at the distance *x* and high *h*, as a measure of the source intensity as seen by the eye, *E* is the luminance at the specific point, defined as a ratio, with  $\Delta \Phi$  representing the flux that strikes a tiny area  $\Delta S$ , calculated considering a spherical surface of radius r, with  $\Delta \alpha$  representing the solid angle.

For acceleration:

$$a_x = \frac{dv_x}{dt}, \ a_y = \frac{dv_y}{dt}, \ v_x = \frac{dx}{dt}, \ v_y = \frac{dy}{dt}$$
(20)

where the above notations represents the acceleration  $a_x$ ,  $a_y$ , the speed  $v_x$ ,  $v_y$  and the space x, y on two axis for an object of the mass m, under a force F. Some characteristics measured for the sensor network are presented in Fig. 2.



Fig. 2. Temperature am humidity transient characteristics measured with the sensor network

### 4.3 Application Structure

A sensor network is made by ad-hoc tiny sensor nodes spread across the space S. Sensor nodes collaborate among themselves, and the sensor network provides information anytime, by collecting, processing, analyzing and disseminating temperature measured data. Sensor network is working as a distributed sensor. The constructive and functional representation of a sensor network is presented in Fig. 3.



Fig. 3. A sensor network with mobile access

# **5** Estimation and Detection

### 5.1 Estimation Equation

The present paper considers four multivariable estimation models, two as regressive and other two as autoregressive, both based on nonlinear ANFIS estimator, which can efficiently approximate the time evolution in space of the measured values provided by each and every sensor within the coverage area. An estimation model describes the evolution of a variable measured over the same sample period as a non-linear function of past evolutions. This kind of systems evolves due to its "non-linear memory", generating internal dynamics. The estimation model definition is:

$$y(t) = f(u_1(t), ..., u_n(t))$$
(21)

where u(t) is a vector of the time series under investigation (in our case is the series of values measured by the sensors from the network):

$$\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix}^T \tag{22}$$

and f is the non-linear estimation function of nonlinear regression, n is the order of the regression. By convention all the components  $u_1(t),...,u_n(t)$  of the multivariable time series u(t) are assumed to be zero mean. The function f may be estimated in case that the time series u(t), u(t-1),..., u(t-n) is known (recursive parameter estimation), either predict future value in case that the function f and past values u(t-1),..., u(t-n) are known (AR prediction).

### 5.2 Detection Structure

The method uses the time series of measured data provided by each sensor and relies on an (auto)regressive multivariable predictor placed in base stations as it is presented in Fig. 4.



Fig. 4. Detection structure

The principle is the following: the sensor nodes will be identified by comparing their output values

 $\theta(t)$  with the values y(t) predicted using past/present values provided by the same sensors or adjacent sensors (adj). After this initialization, at every instant time t the estimated values are computed relying only on past values  $\theta_A(t-1)$ , ...,  $\theta_A(0)$  and both parameter estimation and prediction are used as in the following steps. First the parameters of the function f are estimated using training from measured values with a training algorithm as backpropagation for example. After that, the present values  $\theta_A(t)$  measured by the sensor nodes may be compared with their estimated values y(t) by computing the errors:

$$e_A(t) = \left| \Theta_A(t) - y(t) \right| \tag{23}$$

If these errors are higher than the thresholds  $\varepsilon_A$ at the sensor measuring point a fault occurs. Here, based on a database containing the known models, on a knowledge-based system we may see the case as a multi-agent system, which can do critics, learning and changes, taking decision based on node analysis from network topology. Two parameters can influence the decision: the type of data measured by sensors and the computing limitations. Because both of them are a priori known an off-line methodology is proposed. Realistic values are between 3 and 6.We are choosing 4 for the algorithms 1 and 2 and 6 for the algorithms 3 and 4. So, the method for fault detection and diagnosis provided by this paper may be synthesized as follows.

## 5.3 Monitoring Method

The method recommended for fault detection and diagnosis based on identification, sensor network and ANFIS is the following. It is according to the objectives of monitoring of defined distributed parameter system from the practical application in the real world, as heat distribution, wave propagation and so on. These systems have known mathematical model as partial differential equations as primary models from physics, with well-defined boundary and initial conditions for the system in practice. These represent the basic knowledge for a reference model from real data observation. The primary physical model must be discretized, to obtain a mathematical model as a multi input - multi output state space model in discrete time. The unstructured meshes may be generated. The sensors must be placed in the field according to the meshes structured under the form of nodes and triangles. A scenario for practical applications could be chosen and simulated. The simulation and the practical measurements are producing transient regime characteristics. On these transient characteristics, seen as times series, the estimation algorithms may be applied. ANFIS is used to implement the nonlinear estimation algorithms. With these algorithms future states of the process may be estimated. Possible fault in the system are chosen and strategies for detection may be developed, to identify and to diagnose them, base on the state estimation. In practice applying the method presumes the following steps: -placing a sensor network in the field of the distributed parameter system; -acquiring data, in time, from the sensor nodes, for the system variables; -using measured data to determine an estimation model based on ANFIS; -using measured data to estimate the future values of the system variables; -imposing an error threshold for the system variables; -comparing the measured data with the estimated values; -if the determined error is greater then the threshold a default occurs; -diagnosing the default, based on estimated data, determining its place in the sensor network and in the distribute parameter system field.

### 6 Case Studies

### 6.1 Parabolic Case

In this paper a parabolic case study consisting in a heat distribution flux through a plane square surface of dimensions l=1, with Dirichlet boundary conditions as constant temperature on three margins:

$$h_{\theta}\theta = r \tag{24}$$

with r=0, and a Neuman boundary condition as a flux temperature from a source

$$nk\nabla\theta + q\theta = g \tag{25}$$

where q is the heat transfer coefficient q=0, g=0,  $h_0=1$ .

The heat equation, of a parabolic type, is:

$$\rho C \frac{\partial \theta}{\partial t} = \nabla (k \nabla \theta) + Q + h_{\theta} (\theta_{ext} - \theta)$$
(26)

where  $\rho$  is the density of the medium, C is the thermal (heat) capacity, k is the thermal conductivity, coefficient of heat conduction, Q is the heat source,  $h_{\theta}$  is the convective heat transfer coefficient,  $\theta_{\text{ext}}$  is the external temperature. Relative values are chosen for the equation parameters:  $\rho C=1$ , Q=10, k=1.

With the above conditions the equation may be

solved using the finite element method. The optimized mean meshes and nodes are presenting in Fig. 5.



Fig. 5. The optimized meshes and nodes

The temperature represented height 3D over the surface analyzed is presented in Fig. 6.



Fig. 6. The temperature over the plane

In practice we are using a reduced number of sensors, which could be equivalent to a number reduced of nodes and meshes, for example a sensor network with only 13 nodes, placed like in Fig. 7.



Fig. 7. Sensor network position in the field For this case of approximation with a reduced

number of meshes the solution with the finite element method is represented in Fig. 8.



Fig. 8. Solution for 13 meshes

This could be the worst case of approximation, equivalent to the worst case of estimation.

The repartition of temperature on isotherms in plane is presented in Fig. 9.



Fig. 9. Temperature in plane

In the application we are choosing the nodes 8, 13, 12 5 and 11 from the Fig. 7 to apply the estimation method. The transient characteristics of the temperature are presented in Fig. 10 for 101 samples.



Fig.10. Temperature transient characteristics

The time period was 1 and the sampling period was 0,01. In Fig. 10 the temperature for nodes 13 and 12 are the same, because they are on the same isotherm.

We are chosen as an example the node 5 to be the node with the estimated temperature, based on the first recursive algorithm:

$$\theta_5^{k+1} = f(\theta_8^k, \theta_{13}^k, \theta_{12}^k, \theta_{11}^k)$$
(27)

And also for the node 5 we will apply the second algorithm, auto-recursive:

$$\theta_5^{k+1} = f(\theta_5^k, \theta_5^{k-1}, \theta_5^{k-2}, \theta_5^{k-3})$$
(28)

The fuzzy inference system structure is presented in Fig. 11.



Fig. 11. FIS structure

The comparison transient characteristics for training and testing output data are presented in Fig. 12.



Fig. 12. Comparison between training and testing output

The average testing error is  $2,017.10^{-5}$ . Number of training epochs is 3.

For the second algorithm the training error was of 0,007, number of epochs 3 and the testing error 0,007.

The FIS general structure is the same, but with different parameter values.

The estimated output for the second algorithm is

presented in Fig. 13.



Fig. 13. The estimated output for the second algorithm

Comparing the two algorithms the first one had a better testing error.

If a fault appears at the sensor 5 an error occurs in estimation, like in Fig. 14.



Fig. 14. Error at the fifth node for a fault in the network

Detection of this error is equivalent to a default at sensor 5, from other point of view in the place of the sensor 5 in the space of the distributed parameter systems and in the heat flow around the sensor 5.

#### 6.2 Hyperbolic Case

In this paper a hyperbolic case study is presented, useful for example at wave propagation in plane.

The equation used in analysis is:

$$\frac{\partial \theta^2}{\partial^2 t} = c_1 \nabla (c_2 \nabla \theta) + c_3 \theta + c_4$$
<sup>(29)</sup>

where the parameter have the following values:  $c_1=1, c_2=1, c_3=0, c_4=10$ .

The space on which is made the analysis is a square with unitary dimension l=1. Boundary conditions were imposed as follows: on the left, right and front Dirichlet conditions: h=1, r=0. On the square's base Neumann conditions: q=0, g=0.

The discrete optimized number and position of

### meshes are presented in Fig. 15.



Fig.15 The optimized meshes

For these meshes the approximated solution is presented in Fig. 16 in 3D.





The contour solution is presented in Fig. 17.



Fig. 17 Contour plotted solution

We are considering the case of using of a reduced number of sensors ( $n_s = 15$ ), placed in the field as in Fig. 18.



Fig. 18 Positions of sensors in the field

The solution approximated in this case, as samples in space for these sensors, is presented in Fig. 19.



Fig. 19 3D approximated values for 15 points

The contour values for 15 approximation points are presented in Fig. 20.



Fig. 20 Contour of measurements for 15 points

We are using the node 15, according to Fig. 18, as a node for monitoring to obtain the estimate  $\hat{\theta}_{15}$ , at the moment *t*+1, based on measurements  $\theta_{13}$ ,  $\theta_{14}$ ,  $\theta_6$ ,  $\theta_5$ ,  $\theta_8$  and  $\theta_6$  from the adjacent nodes 13, 14, 6, 5, 8 and 6 at the moment *t*.

$$\hat{\theta}_{15}^{\wedge t+1} = f(\theta_{13}^t, \theta_{14}^t, \theta_5^t, \theta_7^t, \theta_8^t, \theta_6^t)$$
(30)

for the algorithm 3.

To apply the algorithm 4 we are using the past values of the sensor 15:  $\theta_{15}^{t}$ ,  $\theta_{15}^{t-1}$ ,  $\theta_{15}^{t-2}$ ,  $\theta_{15}^{t-3}$ ,  $\theta_{15}^{t-4}$ ,  $\theta_{15}^{t-5}$  at six anterior moments, to obtain the estimate at the moment *t*+1:

$${}^{\wedge t+1}_{\theta_{15}} = f(\theta_{15}^t, \theta_{15}^{t-1}, \theta_{15}^{t-2}, \theta_{15}^{t-3}, \theta_{15}^{t-4}, \theta_{15}^{t-5})$$
(31)

The first operation is to train the ANFIS scheme. This training is done using the transient characteristics of the seven nodes presented in Fig. 21, obtained from the approximated values for 15 sensors.



Fig. 21 The transient characteristics used for ANFIS training

From these characteristics the variation in the 15<sup>th</sup> point is presented in Fig. 22.



Fig. 22 Transient characteristic for the 15<sup>th</sup> node

The FIS structure from Fig. 23 has 6 inputs with 3 membership functions for each input.



Fig. 23 The FIS structure for the hyperbolic case

Number of training epochs was 3. The compared outputs and error for ANFIS for the 3<sup>rd</sup> estimation algorithm test is presented in Fig. 24.



Fig. 24 FIS testing output and error for the 3<sup>rd</sup> algorithm

The compared outputs and error for ANFIS for the 4th estimation algorithm test is presented in Fig. 25.



Fig. 25 FIS testing output and error for the 4<sup>th</sup> algorithm

In both cases, for the 3<sup>rd</sup> and 4<sup>th</sup> algorithms small errors at tests were obtain. The comparison characteristics are the same, according to the experiments.

# 7 Conclusion

The paper presents four algorithms for estimation of state variables in distributed parameter systems of parabolic and hyperbolic cases. Also, a method for monitoring distributed parameter systems based on these algorithms, sensor networks and ANFIS for non-linear system identification is presented. The sensor network is seen as a distributed sensor. The algorithms are two based on regression using the values provided by the adjacent nodes of the sensor network and the other two are based on autoregressive relation with the values from anterior time moments of the same node.

The method described the way how to use all these concepts for fault detection and diagnosis in distributed parameter systems, using the measured values provided by the sensor and the estimated values computed by the ANFIS estimator, calculating an error and detecting the fault based on a decision taken after a threshold comparison.

Four case studies for all four algorithms are presented for parabolic type and for hyperbolic type of equations. A comparison between the algorithms is made. Good approximations were obtained.

Developing of the algorithms and the method are taken in consideration in the future, in other applications, considering all the capabilities of the sensor nodes to measure physical variables. This approach allows treatment of large and complex systems with many variables by learning and extrapolation. Estimations methods may be applied in the case of discovery of malicious nodes in wireless sensor networks. An interesting application could be the monitoring of earth environment at low and high altitudes, based on new types of sensor networks specialized for this purpose.

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