Computer Simulation and Method for Heart Rhythm Control Based on ECG Signal Reference Tracking

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Abstract: - In this work we use computer simulations to test a method for controlling heart rhythm behavior based on the electrocardiogram (ECG) signal tracking. The proposed control algorithm is based on two parameters, the controller proportional gain and the L_{∞} norm of tracking error signal. The objective is to take out the heart rhythm dynamics from a non desirable situation to a normal specified behavior, which is given by a signal generated by a reference system. Computer simulations are carried out using as process to control a mathematical model (MS1) composed with six differential delayed equations (DDE), which, depending on model parameters, provides different dynamical behaviors, from normal behavior to non desirable performances interpreted as cardio-pathologies. In order to generate an specified normal ECG reference signal, it is used MS1 with normal behavior parameters, and as alternative model, it is used a third order nonlinear dynamical system (MS2), which produces specific statistics such as the mean and standard deviation of the heart rate and frequency-domain characteristics of heart rate variability (HRV). By means of the proposed control law application, the heart rhythm is conduced to a neighborhood of the specified behavior and satisfactory results are obtained in numerical simulations with both reference models.

Key-Words: - control law, heartbeat control, simulation, chaotic behavior, synchronization.

1 Introduction

Computer simulation is an useful tool for systems analysis, such as physiological control systems [1,2,3]. Heartbeat rhythm control must be carried out in some pathologies, which can be detected by means of the electrocardiogram (ECG) [1,2,3]. The electrocardiogram is a time-varying signal reflecting the ionic current flow which causes the cardiac fibers to contract and subsequently relax. The surface ECG is obtained by recording the potential difference between two electrodes placed on the surface of the skin. A single normal cycle of the ECG represents the successive atrial depolarization/polarization and ventricular depolarization/polarization which occurs with every heartbeat. These can be approximately associated with the peaks and troughs of the ECG waveform labeled P, Q, R, S and T. Maximum peak of wave is named R-peak, and the RR-interval is the time between successive R-peaks (see Fig. 1). The inverse of RR-interval gives the instantaneous heart rate. The normal cardiac rhythm is generated by a specialized aggregate of cells in the right atrium called sino-atrial (SA) node, which is considered the normal pacemaker. In addition, there is another pacemaker, the atrio-ventricular (AV) node [1, 2, 3].

Controlling irregular and chaotic heartbeats is a key issue in cardiology, underlying the experimental and clinical use of artificial pacemakers. There are different strategies of control, based on either in the use of external sources of periodic or quasi-periodic signals, as well as the use of small perturbations to stabilize periodic orbits embedded in the chaotic dynamics [4,5,6,7,8,9,10]. Synchronization of two system can be seen as a particular problem of control, where the reference signal is generated by the drive system, and the controlled process corresponds to the response system. Control engineering techniques, as well as specific methods based on special properties of chaotic systems, have been applied to the tracking of two dynamical systems, or synchronization problem [4,5,6,7,8,9,10,11,12,13,14].



Fig. 1. ECG waveform: P, Q, R, S and T.

In this paper we set out the problem of the heartbeat control where the reference signal is generated following a response patron or set-point. To carry out the proposed method the ECG signal is used, and a mathematical model for heartbeat based on three Van der Pol (VdP) type oscillators with time delays in signals transmission is employed, which captures, at least in a qualitative form, the general behavior of the heart rhythm simulating normal behavior and some heart disease cases, such as ventricular flutter, sinus bradicardia and ventricular fibrillation.

The rest of this paper is organized as follows: in section two control and synchronization problems are set out. Dynamical models used in simulations are described in section three. Simulation results are presented in section four, and finally conclusions are resumed in section five.

2 Control and synchronization

Synchronization and control are two equivalent terms, that nevertheless are used in different contexts. The term control is mainly used in relation with automatic control systems in engineering [11,12,13,14], while the term synchronization is mainly used in the context of chaotic systems dynamics [4,5,6,7,8,9,10].

Since the seminar paper by Pecora and Carroll [4,5,6] on synchronization of chaotic systems, many attention and applications have been dedicated on this issue. Two basic situations are typical, the first corresponds to the case when the drive system and the response system have the same mathematical model except that parameters are lightly different, and the second case outlines the situation when the drive system and the response system are different. experimental configuration The known as unidirectional coupling supposes two chaotic oscillators, which are assumed to be identical, or nearly identical. In this case, the drive system and the response system correspond to systems of the same nature, i.e. the two have the same physical structure (dynamical model) and the model parameters take very similar values. Other practical situations correspond to when the response system parameters are very different to the corresponding parameters of the drive, and when the nature of the two systems is different, i.e. the systems are structurally different. Depending on case, several types of synchronization are considered, such as: complete synchronization synchronization (IS), (CS) or identical lag synchronization (LS), phase synchronization (PS), rhythm synchronization (RS), frequency synchronization (FS) and generalized synchronization (GS) [4,5,6,7,8].

If an autonomous nonlinear dynamical system is considered, its dynamics can be expressed by a set of *n* ordinary differential equations

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \quad (1)$$

where its dynamical state, or state vector, is given by a *n*-dimensional vector $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{F}(\mathbf{x})$ is a vector field of the same dimension. In case of chaos behavior it is assumed that the system parameters and initial condition are such that the steady evolution of the system occurs in a chaotic attractor, $\mathcal{A} \in \mathbb{R}^n$. In case of the vector field \mathbf{F} depends explicitly on time, or if an external signal $\mathbf{u}(t)$ (scalar or vector) is used for control,

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, t)$$
$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{u}) \quad (2)$$

then the system is considered as non-autonomous. This occurs when control or synchronization between two systems are considered. A particular case corresponds to when the response system (indicated with prime) has the same structure than the drive system given in (1),

$$\dot{\mathbf{x}'} = \mathbf{F}(\mathbf{x}')$$

In case of unidirectional drive, the response system results modified to become a new system with its dynamics given by

$$\dot{\mathbf{x}'} = \mathbf{G}(\mathbf{x}, \mathbf{x}')$$

with G(x,x') verifying the condition G(x,x') = F(x), for x'=x. This means that a signal made of the variables of the drive system, x, acts on the response system, which does not act on the drive system. This coupling describes a variety of practical situations, and two particular schemes are used: 1) *continuous control method*, as for example if it is employed

$$\mathbf{G}(\mathbf{x}, \mathbf{x}) = \mathbf{F}(\mathbf{x}) + \mathbf{K}\mathbf{p}(\mathbf{x} - \mathbf{x})$$
 (2)

where \mathbf{Kp} is a constant square matrix, and 2) replacement of variables or *replacement method*, which has a particular implementation known as *subsystem decomposition* due to the fact that the vector field **F** is decomposed in two components

 $\mathbf{F} = [\mathbf{F}_{\mathbf{a}} \quad \mathbf{F}_{\mathbf{b}}]^{\mathrm{T}}$, and likewise the state vector is decompose as $\mathbf{x} = [\mathbf{x}_{\mathbf{a}} \quad \mathbf{x}_{\mathbf{b}}]^{\mathrm{T}}$. In this case, the dynamics of the whole system is given by: The drive or reference system:

$$\dot{\mathbf{x}}_a = \mathbf{F}_a(\mathbf{x}_a, \mathbf{x}_b)$$

 $\dot{\mathbf{x}}_b = \mathbf{F}_b(\mathbf{x}_a, \mathbf{x}_b)$

The response system:

Identical synchronization (IS) is achieved between the response system and the drive system when there are sets of initial condition, $\mathcal{X}_D \subset \mathbb{R}^n$ for the drive system and $\mathcal{X}_R \subset \mathbb{R}^n$ for the response system, such that for all $\mathbf{x}(0) \in \mathcal{X}_D$ and for all $\mathbf{x}'(0) \in \mathcal{X}_R$

$$\lim_{t \to \infty} \|\mathbf{x}'(t) - \mathbf{x}(t)\| = 0$$

where $\|.\|$ represents the Euclidean norm. This definition is still valid in case of using the replacement method named subsystem decomposition, if the initial condition of the response system is restricted to $\mathcal{X}_R \subset \mathbb{R}^{n_b}$, and it is used the difference $\|\mathbf{x}'_{\mathbf{b}}(t) - \mathbf{x}_{\mathbf{b}}(t)\|$ instead of $\|\mathbf{x}'(t) - \mathbf{x}(t)\|$ in the previous definition.

Generalized synchronization (GS) is considered when the equations of the response system are different from the drive system. In this case a condition for GS is given by

$$\lim_{t\to\infty} \|\mathbf{x}'(t) - \mathbf{\Psi}(\mathbf{x}(t))\| = 0$$

where $\Psi(\mathbf{x}(t))$ is a vector function depending on **x**.

From the point of view of control engineering, a control law as given in (2) corresponds to linear state feedback with proportional gain matrix Kp. If state vector variables are not available, then a measurement variable (scalar magnitude y) can be used,

$$y = H(\mathbf{x})$$

It can be found in literature many design techniques based on a model of the process to control, such as optimal control, predictive control or robust control among others methods [11,12,13,14].

In this paper we employ a control strategy for heartbeat with pathological behavior. In first place we use a reference signal generated by a reference or drive system with the same structure than the response system (MS1 in section three) and the model parameters take very similar values (it can be associated with identical synchronization problem). In second place, the reference signal is generated by a system with different structure (MS2 in section three) or the response system has the same structure than the drive system (MS1) but its parameters are very different; and therefore it can be interpreted as a generalized synchronization problem. In both cases we outline the synchronization between systems as a control problem using the continuous control method, where the control law is based on the measurement of the ECG signal.

3. Mathematical models

The cardiac conduction system is considered to be a network of self-excitatory pacemakers, with sinoatrial (SA) node having the highest intrinsic rate, and where the SA node is the dominant pacemaker of the heart. Others pacemakers with slower excitation frequencies are located in the atrioventricular (AV) node and the His-Purkinje system (HP). A candidate for simulations is a mathematical model with correspondence to the physiology of the heart conduction system [15,16,17], where two-coupled nonlinear oscillators are used:

$$\begin{aligned} \dot{x}_1 &= \frac{1}{C_{SA}} x_2 \\ \dot{x}_2 &= -\frac{1}{L_{SA}} [x_1 + f_1(x_2) + R(x_2 + x_4)] \\ \dot{x}_3 &= \frac{1}{C_{AV}} x_4 \\ \dot{x}_4 &= -\frac{1}{L_{AV}} [x_3 + f_2(x_4) + R(x_2 + x_4)] \end{aligned}$$

where the parameters C_{SA} , L_{SA} , C_{AV} , L_{AV} , R are related with physiological properties of the heart conduction system, and can be obtained experimentally. The functions f_1 and f_2 are voltage sources depending on currents x_1 and x_2 respectively.

Other approach to characterize the cardiac pacemaker is based on the Van de Pol (VdP) ordinary differential equation (ODE), which is frequently used in theoretical models for modeling relaxation oscillators. The general expression of the VdP ODE has the form

$$\ddot{y} + a(1 - by^2)\dot{y} + cy = f(t)$$

where a, b, and c are parameters and f(t) is an external forcing signal.

The VdP system is a useful phenomenological model, due to it displays characteristic behaviors observed in physiological systems such as limit cycles, complex periodicity, synchronization and chaotic dynamics [5,8]. Although in this case no direct biophysical relation is taken with the dynamical variables of the VdP equation, it may be related the dynamical variable with the action potentials in the heart cells. Basically, an action potential is generated when the cell membrane is excited high enough to reach the threshold potential and activate the ion channels. Activated channels allow ionic currents to flow into or out the cell, thus changing its potential and resulting in the generation on an action potential [18]. A modified VdP equation used in practice for improved modeling of cardiac pacemakers is given by [15]

$$\ddot{y} + \alpha(y - v_1)(y - v_2)\dot{y} + y(y + d)(y + e)/(ed) = f(t)$$

where parameters (α, v_1, v_2, d, e) are obtained experimentally.

In order to describe the interaction between the rhythms generated by the SA and AV nodes, a set of two modified VdP equations are used [19], written in the general form of a pair of Lienard equations as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= K(x_1 - w_1)(x_1 - w_2)x_2 - b_1x_1 + c_1(x_3 - x_1) \\ &+ a_1\sin(\omega_1 t) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= K(x_3 - w_1)(x_3 - w_2)x_4 - b_2x_3 + c_2(x_1 - x_3) \end{aligned}$$

where the pairs of variables (x_1, x_2) and (x_3, x_4) refer to the SA and AV nodes respectively, and model parameters $(K, w_1, w_2, b_1, b_2, a_1, c_1, c_2)$ are obtained experimentally.

Usually, two oscillators are considered representing the SA and AV nodes, however, it is observed that these two oscillators are not enough to reproduce the ECG signal. This motivates the inclusion of a third oscillator that represents the pulse propagation through the ventricles, and it is included to take into account the His-Purkinje (HP) complex. If time delays in signals transmission are considered, a sixth order system of delay differential equations (DDE) is obtained. This model has been adapted form from [20]. It has been included a control signal $\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$. In original equations the coupling terms are summing instead of subtracting. Two scaling factors have been also included (β_T, β_G) in order to achieve correspondence between our simulations and the time responses given by authors in [20].

In order to verify the correct simulation of a sixth order system of delay differential equations (DDE) we have employed several computations tools. We have compared results obtained solving delay differential equations with dde23 [21] for Matlab and using Simulink [22]. Also, we have implemented our own method in C language, with the additional objective for next works of simulating the system in real time with hardware in the loop (HILS). The following DDE have been used (Model Structure 1, MS1):

$$\begin{split} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_{SA} x_2 (x_1 - w_{SA1}) (x_1 - w_{SA2}) \\ &- x_1 (x_1 + d_{SA}) (x_1 + e_{SA}) + \rho_{SA} \sin(\omega_{SA} t) \\ &- K_{SA-AV} (x_1 - x_3^{T_{dSA-AV}}) \\ &- K_{SA-HP} (x_1 - x_5^{T_{dSA-HP}}) + u_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -a_{AV} x_4 (x_3 - w_{AV1}) (x_3 - w_{AV2}) \\ &- x_3 (x_3 + d_{AV}) (x_3 + e_{AV}) + \rho_{AV} \sin(\omega_{AV} t) \\ &- K_{AV-SA} (x_3 - x_1^{T_{dAV-SA}}) \\ &- K_{AV-HP} (x_3 - x_5^{T_{dAV-HP}}) + u_2 \\ \dot{x}_5 &= x_6 \\ \dot{x}_6 &= -a_{HP} x_6 (x_5 - w_{HP1}) (x_5 - w_{HP2}) \\ &- x_5 (x_5 + d_{HP}) (x_5 + e_{HP}) + \rho_{HP} \sin(\omega_{HP} t) \\ &- K_{HP-SA} (x_5 - x_1^{T_{dHP-SA}}) \\ &- K_{HP-AV} (x_5 - x_3^{T_{dHP-AV}}) + u_3 \end{split}$$

where $x_i^{T_d} \equiv x_i(t - T_d)$, and T_d represents the transport time delay. With this mathematical model (named as MS1 from now on) composed by three coupled oscillators, the ECG signal is built from the composition of signals as follows,

$$ECG = (\alpha_0 + \alpha_1 x_1 + \alpha_3 x_3 + \alpha_5 x_5)\beta_G$$

where β_G is a magnitude scaling factor that we have added for this work for the ECG signal.

In simulations carried out, as parameters values the suggested by [18,19,20] have been used, except some modifications that are indicated in each case, such as ECG magnitude scale factor (β_G) and time scaling factor (β_T). As it is proposed in reference [20], parameters values suggested by [18,19] are used as reference values of the SA node oscillator for the normal ECG, and the other parameters are adjusted in order to qualitatively match real ECG signals. It is beyond the scope of this model (MS1) an optimal determination of the system parameters. As it is indicated in [20], the parameters choice are done in an interactive ad-hoc way, with the objective to understand the heart rhythms by a dynamical point of view, and therefore the interest is essentially in the qualitative system response. With these considerations, the following parameters have been used for simulation in case of the normal heart rhythm functioning (for MS1 structure):

$$\begin{aligned} a_{SA} &= 3, \ w_{SA1} = 0.2, \ w_{SA2} = -1.9, \ d_{SA} = 3 \\ e_{SA} &= 4.5, \ a_{AV} = 3, \ w_{AV1} = 0.1, \ w_{AV2} = -0.1 \\ d_{AV} &= 3, \ e_{AV} = 3, \ a_{HP} = 5, \ w_{HP1} = 1 \\ w_{HP2} &= -1, \ d_{HP} = 3, \ e_{HP} = 7, \ \rho_{SA} = 0 \\ \rho_{AV} &= 0, \ \rho_{HP} = 0, \ T_{dSA-AV} = T_{dAV-HP} = 0 \\ T_{dAV-SA} &= 0.8, \ T_{dHP-AV} = 0.1 \\ T_{dSA-HP} &= 0, \ K_{SA-HP} = 0, \ K_{AV-SA} = 5 \\ K_{AV-HP} &= 0, \ K_{HP-SA} = 0, \ K_{HP-AV} = 20 \\ \omega_{SA} &= \omega_{AV} = \omega_{HP} = 70 \end{aligned}$$

 $\alpha_0 = 1, \ \alpha_1 = 0.1, \ \alpha_3 = 0.05, \ \alpha_5 = 0.4$

with scaling factor for the ECG signal $\beta_G=(1.15/1.5)$, and a time scaling factor $\beta_T = 16$. As initial condition, it is used the following state vector:

$$\mathbf{x}_0 = \begin{bmatrix} 0 & 0.7 & 0 & 0.2 & 0 & 0.7 \end{bmatrix}^T$$

When a time scaling factor (β_T) is used, the result over the system equations and parameters (delay time and frequency) are the following:

$$\dot{\mathbf{x}} = \beta_T \mathbf{F}(\mathbf{x}, \mathbf{u})$$
$$T'_d = \frac{T_d}{\beta_T}, \quad \omega' = \beta_T \, \omega$$

Taking into account these modifications with respect to the model given by [20], the simulations are carried out.

The model MS1 is used in order to simulate some heart pathologies identified from ECG, such as: 1) *ventricular flutter* (eliminating the coupling between first and second oscillators, $K_{AV-SA} = 0$, a chaoticlike response is obtained), 2) *sinus bradycardia* (obtained with $K_{HP-AV} = 0$, $\beta_T = 8$) corresponds to regular behavior with lower oscillation frequency, 3) *ventricular fibrillation* (chaotic-like signal is suggested) is obtained with

 $e_{SA} = 6$, $\rho_{SA} = 1$, $\rho_{AV} = 1$, $\rho_{HP} = 20$

In our study we use this model (MS1) as process to control when an anomalous heart rhythm behavior is given. MS1 is also used for generating normal heartbeat and in this case MS1 is employed as reference system.

Second reference signal model for simulations (MS2)

In order to dispose another method for generating a reference signal, corresponding to a different dynamical system to MS1, we have employed the mathematical model given by [23]. This model structure (MS2) generates typical (normal) human

ECG, signal with a-priori specified characteristics of heart rate variability. Equations of MS2 are given by:

$$\begin{aligned} \dot{x} &= \alpha x - \omega y \\ \dot{y} &= \alpha y + \omega x \\ \dot{z} &= -\sum_{i \in \{P,Q,R,S,T\}} a_i \Delta \theta_i \exp\left(-\frac{-\Delta \theta_i^2}{2b_i^2}\right) - (z - z_0) \end{aligned}$$

where

$$\alpha = 1 - \sqrt{x^2 + y^2}$$
$$\Delta \theta_i = \operatorname{rem}((\theta - \theta_i), 2\pi), \ \theta = \operatorname{atan2}(y, x)$$
$$z_0 = A \sin(2\pi f_2 t), \ A = 0.15 \text{ mV}$$

and f_2 is the respiratory frequency. The aim of this model is to provide a standard realistic ECG signal (*z* in the previous ODEs system) with known characteristics, which can be generated with specific statistics such as the mean and standard deviation of the heart rate and frequency-domain characteristics of heart rate variability (HRV). Although authors [23] do not study possible chaotic performance of the system for determined parameters values, it would be interesting this study in next works for synchronization and control analysis of chaotic systems.

In this paper we have used this model (MS2) as reference system, which generates a signal, ECGr, to follow by the ECG of the response system or process to control. In this form, the heart rhythm dynamics is modified by feedback. Structure of MS2 is completely different from MS1, and the tracking problem with MS2 as reference and MS1 as drive system is considered.

	Р	Q	R	S	Т
t_i (sec)	-0.2	-0.05	0	0.05	0.3
θ_i (rad)	$-\pi/3$	$\pi/12$	0	$\pi/12$	$\pi/2$
a_i	1.2	-5	30	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4

Table 1. Parameters for the ECG (z signal) model specified a priori.

4 Simulation results

As process to control, or response system, it is used a model with structure MS1, which will be characterized, in general, with a non desirable behavior or pathology; but the case of normal behavior will be also considered. As reference system it is employed a model with structure MS1 (with initial condition different to the response system one, and whose behavior will be normal or anomalous, according to the case) or a model with structure MS2 to generate an specified reference signal. Four types of control problems or simulation experiments (SE) are considered: SE1, SE2, SE3 y SE4.

Simulation experiment 1 (SE1). The objective is to take out the heart rhythm dynamics from a non desirable situation (associated to a cardio-pathology) to a normal specified behavior, which is given by the reference system. This is interpreted as a tracking problem of a variable set-point signal. The reference system is MS1 with heartbeat normal behavior, and the process to be controlled corresponds also to MS1, but with $\mathbf{u} = \mathbf{0}$ and different parameters values for three dynamical situations: a) ventricular flutter, b) sinus bradycardia and c) ventricular fibrillation.

Simulation experiment 2 (SE2). The reference system is MS2 with normal heartbeat properties specified a priori: heart rate mean of 70 beat per minute (bpm) and heart rate standard deviation of 1 bpm. For that, it is used the function *ecgsyn.m* for Matlab given by [21,22]. As process to be controlled is used MS1 in four different dynamical situations: a) ventricular flutter, b) sinus bradycardia, c) ventricular fibrillation, d) normal.

Simulation experiment 3 (SE3). In this case the problem of *synchronization of two chaotic systems* is considered, this is to say: the drive (reference) system has a like-wise chaotic behavior (ventricular fibrillation or ventricular flutter) and the response system has also an irregular behavior which seems to be chaotic (ventricular fibrillation or ventricular fibrillation or ventricular fibrillation or ventricular fibrillation. System). Both systems have the structure of MS1.

Simulation experiment 4 (SE4). The response system presents a normal functioning (with MS1 structure) and drive system (also with MS1 structure) has a like-wise chaotic behavior (ventricular flutter). A chaotification problem is established in this case.

As control law, we have considered two options: 1) MIMO or multivariable control, where the controller has three inputs and three outputs; and 2) SISO or scalar control. In case of MIMO controller, the control vector is given by

$$[u_1 \ u_2 \ u_3]^T = \text{diag} (K_{p1} \ K_{p2} \ K_{p3}) \mathbf{e}$$
$$\mathbf{e} = [x_{r1} - x_1 \ x_{r3} - x_3 \ x_{r5} - x_5]^T$$

where $\mathbf{x}_{\mathbf{r}} = [x_{ri}]^T$, $i = 1, 2, \dots, 6$, is the state vector of the reference system, and $\mathbf{x} = [x_i]^T$, $i = 1, 2, \dots, 6$, is the state vector of the process to control or response system. For SISO control, we propose to use as set-point (SP), or reference signal, the electrocardiogram signal (ECGr) generated by the reference system (with structure MS1 or MS2), and as process variable (PV), or controlled variable (CV), the signal the ECG produced by the simulated heart to be controlled (with structure MS1). We have studied by simulation different options for applying a control signal given by (see MS1):

$$u_i = K_p(ECGr - ECG), \quad i = 1, 2, 3$$

and we have concluded that if u_3 is applied as an input signal acting on equation of \dot{x}_{6} , as it appears in MS1, better results are obtained than other options when scalar control signal is adopted.

During simulation analysis, we have observed that in case of MIMO control, similar results to SISO control are obtained. Nevertheless, SISO control is easier to implement, and therefore, finally the control vector components in the process to control, or response system, with structure MS1 are given by:

$$u_1 = 0, \ u_2 = 0, \ u_3 = K_p(ECGr - ECG)$$

The controller proportional gain, K_p , is adjusted to achieve a tracking error signal sufficiently small

$$\|e(t)\|_{\infty} < \gamma, \quad t \ge t_0$$

where t_0 is taken one half second after to connect the controller, the tracking error is given by e(t) = ECGr(t) - ECG(t), and the L_{∞} norm of a scalar signal e(t) is defined as

$$\|e(t)\|_{\infty} = \sup_{t \in [t_0,\infty)} |e(t)|$$

Adjusted values of γ and Kp are used as tuning parameters for obtaining more precise tracking. Simulation data presented in Fig. 2 to 8 have been carried out using: i) Kp = 800 and $\gamma = 0.1$ in case of the reference signal is generated by MS1; and ii) Kp = 2500 and $\gamma = 0.1$ in case of the reference signal is generated by MS2. For simulation results obtained in Fig. 9 it is used Kp = 3000, and for Fig. 10 and 11 it is used Kp= 6000. These values have been obtained experimentally in an iterative simulation procedure.

In order to simulate "ventricular flutter" (irregular characteristic and higher frequency rhythm when compared with the normal ECG), it is employed $K_{AV-SA} = 0$ (elimination of coupling between first and second oscillators, which corresponds to communication interruption in the heart electric system). A chaotic-like response is obtained in this case, although to verify chaotic nature specific data analysis must be carried out for chaos testing, such as the maximal Lyapunov exponent or other techniques to distinguish between regular and chaotic dynamics in deterministic time series data [5,24,25,26].

In Fig. 2 it is shown as controller avoids the irregular behavior with chaotic aspect (*ventricular flutter*) when it is connected or activated. The reference signal (ECGr) is generated with MS1 and for the response system is used MS1 but with other parameters. In this case the situation corresponds to two systems with the same structure but with different parameters.

In case of Fig. 3, the reference signal (ECGr) is generated by MS2 and the ECG of the response system shows ventricular flutter before controller is activated in t = 5 seconds. In this case, the drive system has different structure than the response system; nevertheless, good tracking is obtained as it can be seen in Fig. 4.

In Fig. 4, the tracking errors obtained when MS1 and MS2 are respectively used as reference system, and for drive system it is used MS1 with parameters values corresponding to ventricular flutter. In both cases, it is satisfied the design specification $||e(t)||_{\infty} < 0.1$, as it can be seen in Fig. 4.

In order to obtain "sinus bradycardia" (regular behavior presenting a lower frequency rhythm), two parameters are changed, $\alpha = 8$ and $K_{HP-AV} = 11$. Fig. 5 and 6 show simulation results when MS1 and MS2 are respectively used as reference system. The ECG signal of the response system shows sinus bradycardia before the controller is activated in t = 5 seconds. Due to controller action, the response system follows the reference signal with $||e(t)||_{\infty} < 0.1$.

Ventricular fibrillation is other undesirable anomaly (irregular response which seems to be chaotic). Simulation of *ventricular fibrillation* is carried out modifying the following parameters:

 $e_{SA} = 6, \ \rho_{SA} = 1, \ \rho_{AV} = 1, \ \rho_{HP} = 20$

Fig. 7 and 8 show simulation results when MS1 and MS2 are respectively used as reference system. The ECG signal of the response system shows fibrillation ventricular before controller is activated in t = 5 seconds. In this case is also obtained that the tracking error satisfied the restriction $||e(t)||_{\infty} < 0.1$.

Other simulation experiment considers that MS1 has dynamics corresponding to ventricular flutter oscillation (like-wise chaotic). MS1 is used as reference system (ECGr) and in this case its initial state vector is given by

 $\mathbf{x}_0 = [0.1 \ 0.71 \ 0.1 \ 0.21 \ 0.1 \ 0.71]^T$

As response system MS1 is used, whose ECG signal also shows ventricular flutter, but in this case it is obtained with initial state vector lightly different to the response system one,

$$\mathbf{x}_0 = [0 \ 0.7 \ 0 \ 0.2 \ 0 \ 0.7]^T$$

In this case the problem corresponds to synchronization of two systems with the same structure but different initial conditions, whose behaviors have chaotic aspect. In Fig. 8 the error signal between the reference signal (ECGr) and the response signal (ECG) is shown. In this case controller parameters are $K_p = 3000$, $\gamma = 0.1$.

The graphic given in Fig. 10 corresponds to the following situation: the reference signal (ECGr) is generated with MS1 in ventricular oscillation (likewise chaotic), and the ECG signal shows ventricular fibrillation (with chaotic aspect too). As it can be seen in Fig. 9, the tracking error signal satisfied the specification $||e(t)||_{\infty} < \gamma$ after the controller is activated in t = 10 sec. In this case, the controller parameters are $K_p = 6000$, $\gamma = 0.1$.

Fig. 11 shows a chaotification experiment. It can be seen the tracking error signal, before an after the controller is turned on. In this case, the reference signal (ECGr) has been generated with MS1 in ventricular flutter oscillation (like-wise chaotic), meanwhile the response system corresponds to MS2 in normal behavior. The ECG signal shows normal heartbeat before controller is activated in t = 5 sec, and after this instant a chaotification of the systems is achieved.

The results showed in Fig. 9 and 10 are used to demonstrate how it is carried out synchronization between two oscillators with chaotic behavior; and the experiment showed in Fig. 11 has been carried out to show how chaotification of the system response is obtained. In all cases, the controller parameter, K_p , is tuned in order to satisfy the control specification $||e(t)||_{\infty} < \gamma$, $\gamma = 0.1$



Fig. 2. Reference signal (ECGr) generated with MS1. The ECG signal shows ventricular flutter before controller is activated in t = 5 sec.

When we refer to chaos behavior in ECG signal, we mean chaotic aspect, or qualitative chaos, due to quantitative tests tests [5,24,25] for chaotic signal have been not carried out by authors [20], and it will be studied in next works with MS1 and MS2.



Fig. 3. Reference signal (ECGr) generated with MS2. The ECG (mV) signal shows ventricular flutter before controller is activated in t = 5 sec.



Fig. 4. Tracking error obtained with Kp=800 when the reference system is MS1, and with Kp=2500 when the reference signal is MS2, for $\gamma = 0.1$. Previous to t=5 sec., uncontrolled system has a ventricular flutter behavior.



Fig. 5. Reference signal (ECGr) generated with MS1. The ECG signal shows sinus bradycardia before controller is activated in t = 5 sec.



Fig. 6. Reference signal (ECGr) generated with MS2. The ECG (mV) signal shows sinus bradycardia before controller is activated in t = 5 sec.



Fig. 7. Reference signal (ECGr) generated with MS1. The ECG signal shows fibrillation ventricular before controller is activated in t = 5 sec.



Fig. 8. Reference signal (ECGr) generated with MS2. The ECG (mV) signal shows fibrillation ventricular before controller is activated in t = 5 sec.



Fig. 9. Tracking error signal, before an after controller is connected. Reference signal (ECGr) generated with MS1 in ventricular flutter oscillation (like=wise chaotic) with initial state vector [0.1 0.71 0.1 0.21 0.1 0.71]. The ECG signal also shows ventricular flutter (obtained with other initial state vector [0 0.7 0 0.2 0 0.7] before controller is activated in t = 10 sec. Controller parameters are Kp= $3000, \gamma = 0.1$.

5 Conclusions

A controller law based on the electrocardiogram (ECG) signal tracking is proposed to control heart rhythm dynamics. Two control parameters are considered: Kp and γ , where Kp is the controller gain and γ corresponds to the L_{∞} norm of the tracking error signal with respect to a reference signal

established as desired electrocardiogram (ECG). A mathematical model based on six differential equations with dead-times (DDE) is used for heart rhythm dynamics simulation (as process to control), and different non-desirable behaviors (cardiopathologies) are used for testing our control algorithm. The objective of taking out the heartbeat from anomalous behavior to normal rhythm is achieved as results from simulation studies carried out. Chaotification (from normal ECG signal to ECG with chaotic aspect) and synchronization processes systems characterized with chaotic ECG (two signals) are also evaluated in numerical simulations. In future works, real time simulations with hardware in the loop (HILS) will be carried out. Clinic applications viability and other control methods will be also considered.



Fig. 10. Tracking error signal, before an after controller connected. Reference signal (ECGr) generated with MS1 in ventricular flutter oscillation (like=wise chaotic). The ECG signal shows ventricular fibrillation before controller is activated in t = 10 sec. Controller parameters are Kp=6000, $\gamma = 0.1$.



Fig. 11. Tracking error signal, before an after controller is activated. Reference signal (ECGr) generated with MS1 in ventricular flutter oscillation (likewise chaotic). The ECG signal shows normal heartbeat before controller is activated in t = 5 sec (chaotification). Controller parameters are Kp=6000, $\gamma = 0.1$.

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Fig. 12. Simulink implementation of the model corresponding to structure MS1, with signal control $u = u_3$.