Determination of Temperature Field Distribution and Rate of Heat Transfer by Means of Thermal-electrical Analogy in DC Machine

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Abstract: - In this paper is presented an analysis method of electrical circuits, more exactly nodal method, in order to compute the temperature field distribution necessary to the heating and ventilation systems of electrical machine. As analysis method, the modified nodal method was used which does not impose any restriction at the circuit structure, being a prevalence of the nodal method. For this reason, this method was used in the analysis programme of circuits called SPICE. In the software application called NAPACI, authors analyses the temperature field distribution and the rate of heat transfer which are necessary for a complete study to range of the temperature in steady state. This analysis supposes the knowledge and the localization of temperature losses values, determination of the rate values of heat transfer and their distribution in different sides of electrical machine. Comparing the results obtained by means of other two programmes NAPACI and SPICE, it can be observed that these results are identical.

Key-Words: DC machine, steady-state, temperature field, rate of heat transfer, thermal resistance, generated power, consumed power, NAPACI programme

1 Introduction

In order to design correctly an electrical machine it is necessary that the machine cooling to be efficient and to know the values and the temperature distribution in this (in the rotor). To determine the temperature distribution, a thermal equivalent circuit is used for the heat transfer of machine. The thermal equivalent circuit is analogous to electric circuit, in which heat is generated by "current sources", temperature being in analogy with voltage.

In electric machines, the design of heat transfers has a high relevance, since the thermal rise of the machine eventually affects the output power of the machine. Use of thermal computation of electric machines is for determining the heating, temperature excess or over temperatures, in some parts of electric machine in regard to the environment temperature, at some operating conditions of electric machine [1].

The problem of temperature rise is based on two aspects: in most motors, adequate with the phenomenon of convection of air, conduction through the fastening of the machines of high power density, also direct cooling methods can be applied. The winding of the machine is made of copper pipe through which the cooling fluid flows during the operation of the machine. The thermal field within electric machines can be analyzed with equations based on the classical theory from the heat transfer. At the second aspect is that the distribution of heat in different parts of the machine has to be considered. This represents a problem of heat diffusion, which is a complicated three dimensional problem with several difficult details such as the question of heat transfer from the copper conductor over the insulation to the stator frame. When the distribution of losses in different parts of the machine and the heat removal power are known, then the distribution of heat in machine can be computed. Also, the control problems to an electric machine as electric as thermal should be regarded in a different way to research how to make control systems specialist. In the presence of variations (due to temperature and/or saturation) the response can disrupts in a different operation point [2].

At electric machines, there is power losses produced of current in conductors, magnetic losses produced of the filed variations in iron of machine, mechanical losses produced of the movement of a machine part. Power conversion must be realized with maximum efficiency. Therefore, it is necessary

a magnetic field with another value in time and space and voltages induced maximum for the losses to be minimum. These losses are converted in heat inside the machine, heat which must be exhaust, so it is necessary an efficient cooling system [3], [4].
Several interactions occur in DC machines among thermal and electromagnetic phenomena, as indicated in [5]-[7].

2 Computing methods for heating system of electric machines

Modeling of heating systems of electric machines leads to obtaining some complex equivalent circuits of large sizes. Analysis of these circuits in steady state regime and in transient conditions requires efficient methods for their solution. In order to write the equations of the electric circuits for the purpose their simulation, the modified nodal method is one of the most used methods due to its flexibility.
Elaboration of an efficient method in order to analyze the nonlinear resistive electric circuits has a relevant importance which results from the substitution of dynamic circuit elements with discrete resistive models associated to an implicit integration algorithm [8].

2.1 Nodal method

In the nodal method, nodal points are defined at locations in the circuit where there are unknown voltages. The known independent voltage sources and current sources are the independent variables. At these points Kirchhoff's first law of circuit analysis is applied, namely, the sum of the currents at a nodal point is zero. The currents are then formulated in terms of the dependent voltages and the independent voltages/currents [9]. The linear equations for the voltages are solved and the currents in the various branches are derived from these voltages.
The structure of circuits where the nodal method is applied has the following circuit elements:
•Resistors, magnetically non-coupled coils and linear capacitors;
•Nonlinear resistors controlled in voltage – R_u;
•Nonlinear capacitors;
•Independent current sources;
•Independent voltage sources which do not form alone a branch;
•Current sources driven in voltage or nonlinear – $jc(e_C)$.
Since the resistive models of nonlinear coils contain current-driven nonlinear resistors, their presence in the circuit makes it impossible to apply the nodal method. Coils and capacitors contribution in order to formulate the nodal equations in transient conditions is essentially different in comparison with continuous current regime. After what the resistive model is built the nodal method can be applied by writing the system of nodal equations which is solved by means of Newton-Raphson algorithm [10]. Nodal equations represent a particular form of standard equation system which works with a low number of variables. This expresses in fact, Kirchhoff's Current Law in which the branch currents depend on the potentials at nodes [11]. The advantages of nodal method are evident due to the simplicity of systematic formulae of equation system and the reduction of some computations at solving some systems of linear equation.
Complication of the circuit by adding the new branches which involve an additional memory in the computing systems represents one of the main disadvantages.

2.2. Modified nodal method

The modified nodal method does not require restrictions at the circuit structure which is analyzed. The difference between the nodal method and the modified nodal method is that the modified nodal method includes the currents of current-driven nonlinear coils and the currents of magnetically coupled linear coils as additional variables [8].
Non-coupled linear coils do not introduce additional variables.
The general formulation of the modified nodal equation at time moment t^{k+1} is:

$$\begin{bmatrix} G_{n-1,n-1}^{(k+1)} & B_{n-1,m}^{(k+1)} \\ A_{m,n-1}^{(k+1)} & R_{m,m}^{(k+1)} \end{bmatrix} \cdot \begin{bmatrix} v_{n-1}^{(k+1)} \\ i_{m}^{(k+1)} \end{bmatrix} = \begin{bmatrix} i_{sc,n-1}^{(k+1)} \\ e_{m}^{(k+1)} \end{bmatrix} \qquad (1)$$

where:
• Superscript k+1 represents the time moment at which is related to the system of equations.
• $B_{n-1,m}$ has dimensionless elements; this includes elements -1, 0, +1 and transfer factors in current of current-driven sources;
• $A_{m,\ n-1}$ has dimensionless elements; this includes elements -1, 0, +1 and transfer factors in voltage of voltage-driven sources driven;
• $R_{m,n}$ has elements with resistance dimension; this includes the transfer resistances of current-driven voltage sources and resistances of current-driven resistors;

• e_m has e.m.f. of ideal sources independent of voltage and off-load voltages resulted from linearity of resistor characteristics driven in current.
The modified nodal equation is rewritten at each time moment. We assume that the circuit has a number of nonlinear coils driven in flux, $X^{L\varphi}$ and a number of nonlinear capacitors driven in charge, X^{Cq}. Additional variables are the vectors φ with $X^{L\varphi}$ components and q with X^{Cq} components which are taken together with independent variables of these elements.
General form of the system matrix, as in Eq. (1) will be written with $X^{L\varphi}+X^{Cq}$ lines and as much as columns. These types of elements have a low interest from applications point of view [9], [11].
Contributions of circuit elements incompatible with the classical nodal method at formulae of modified nodal method are established according to:
• *linear coil* - the discrete model associated to Euler integrating algorithm, implicitly for a linear coil can be written as:

$$v_+^{(k+1)} - v_-^{(k+1)} = \frac{L}{h_{k+1}}(i^{(k+1)} - i^{(k)}) \tag{2}$$

where:
the current at the moment t_{k+1} can be express as*:*

$$i(t_{k+1}) = i^{(k+1)} = \frac{h_{k+1}}{L} v_+^{(k+1)} - \frac{h_{k+1}}{L} v_-^{(k+1)} + i^{(k)} \tag{3}$$

Analyzing Eq. (3), it is observed what contribution has a linear coil at writing Eq. (1):

$$\begin{array}{cc} & \begin{array}{cc} v_+ & \quad v_- \end{array} \\ \begin{array}{c} v_+ \\ v_- \end{array} & \begin{bmatrix} \frac{h_{k+1}}{L} & -\frac{h_{k+1}}{L} \\ -\frac{h_{k+1}}{L} & \frac{h_{k+1}}{L} \end{bmatrix} \cdot \begin{bmatrix} v_+^{(k+1)} \\ v_-^{(k+1)} \end{bmatrix} = \begin{bmatrix} -i^{(k)} \\ i^{(k)} \end{bmatrix} \end{array} \tag{4}$$

Vector of the constant terms contains the current through the coil at previous time moment. This time moment represents an initial condition for the actual time moment. Therefore, the linear coil is a compatible element with nodal method and the additional variables not being necessary [11], [12].
• *linear capacitor* – the discrete associated to Euler integrating algorithm implicitly for a liner capacitor is described by Eqs. (5)- (7):

$$i_{k+1} = C\left(\frac{du}{dt}\right)_{k+1} = \frac{C}{h_{k+1}} u_{k+1} - \frac{C}{h_{k+1}} u_k = G_{k+1} u_{k+1} - j_{k+1} \tag{5}$$

where:

$$G_{k+1} = \frac{C}{h_{k+1}} \qquad j_{k+1} = G_{k+1} u_k \tag{6}$$

$$R_{k+1} = \frac{1}{G_{k+1}} = \frac{h_{k+1}}{C} \quad \text{si} \quad e_{k+1} = \frac{j_{k+1}}{G_{k+1}} = u_k \tag{7}$$

Writing Eq. (5) at the moment t_{k+1}, the following equation is obtained:

$$i^{(k+1)} = \frac{C}{h_{k+1}} v_+^{(k+1)} - \frac{C}{h_{k+1}} v_-^{(k+1)} - \frac{C}{h_{k+1}} u^{(k)} \tag{8}$$

By means of Eq. (8) result the contribution of linear condenser at writing the equation system (1):

$$\begin{array}{cc} & \begin{array}{cc} v_+ & \quad v_- \end{array} \\ \begin{array}{c} v_+ \\ v_- \end{array} & \begin{bmatrix} \frac{C}{h_{k+1}} & -\frac{C}{h_{k+1}} \\ -\frac{C}{h_{k+1}} & \frac{C}{h_{k+1}} \end{bmatrix} \cdot \begin{bmatrix} v_+^{(k+1)} \\ v_-^{(k+1)} \end{bmatrix} = \begin{bmatrix} \frac{C}{h_{k+1}} u^{(k)} \\ -\frac{C}{h_{k+1}} u^{(k)} \end{bmatrix} \end{array} \tag{9}$$

We observe that the additional variables are not necessary, the linear capacitor being a compatible element with the nodal method. The constant term vector has the terminal voltage at the previous time moment and represents the initial condition for the actual time moment [12].

2.2 Thermal-electrical analogy

As found in the referenced literature the thermal-electrical analogy is useful in analysis of several steady heat transfer problems from property measurement to modeling. The analogy has some limitations including the non-linearities which there are between voltage and current at extremely high and low values. In the literature, there are some special cases where the analogy doesn't quite fit the actual physics of a problem. These special cases suppose a range of large or small values of current and voltage as well as heat transfer problems at the nano-scale level. Therefore, modifications can be made to still find the relationship helpful in the analysis [13]-[15]. Temperature is the driving force or potential for heat flow. The flow of heat over a "heat flow path" should then be governed by thermal potential difference across the path and the resistance of it. This suggests that heat flow is analogous to electric current flow [15]. In the development of his law for electrical circuits, Ohm performed experiments that modeled Fourier's law of heat conduction. An analogy between Ohm's law for electrical circuits and Fourier's law of heat conduction can be observed. Considering steady flows, the electric current can be written:

$$I = \frac{\Delta V}{R_e} = \frac{V_1 - V_2}{R_e} \quad (10)$$

where:

$R_e = \dfrac{\delta}{\sigma_e \cdot S}$ is the electric resistance, [Ω]

V_1-V_2 represents the voltage difference across the resistance, [V]

σ_e is the electrical conductivity, $\left[\dfrac{S}{m}\right]$.

Fourier's law can be written as:

$$\dot{Q}_{cond} = \frac{\Delta T}{R_{t,cond}} \quad (11)$$

where:

$\dot{Q}_{cond}$ is the rate of conduction heat transfer, [W].

ΔT is the temperature difference between the surfaces of a plane wall, [K].

$R_{t,cond} = \dfrac{\delta}{k \cdot S}$ represents the thermal resistance called conductive or internal resistance of a plane wall, $\left[\dfrac{K}{W}\right]$.

k is the thermal conductivity, $\left[\dfrac{W}{(m \cdot K)}\right]$.

δ is the wall thickness, [m].

S is the wall surface, [m^2].

Thus, the rate of heat transfer through a layer is analogue to the electric current, the thermal resistance is analogue to electrical resistance, and the temperature difference is analogue to voltage difference across the layer (Fig.1), [13].

The thermal resistance of a medium depends on the geometry and the thermal properties of the medium. Since heat is understood to transfer through lattice vibrations between adjacent atoms, and electricity is conducted by free valence electrons of atoms, it may seem intuitive that heat and electrical conduction are analogous [14].

Consider convection heat transfer from a solid surface area S and temperature T_p to a fluid whose temperature far from a surface is T_∞ (Fig.2).

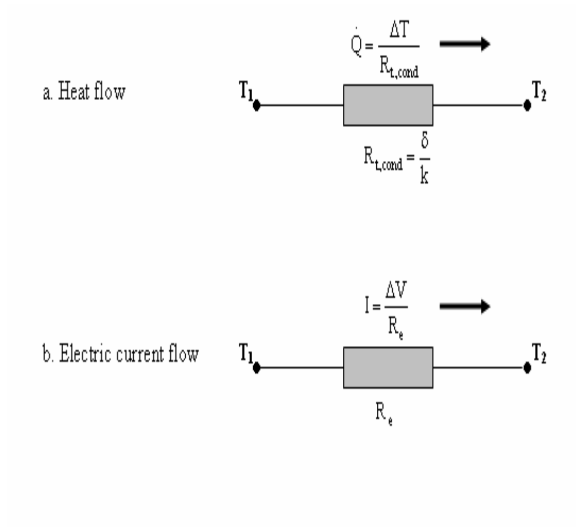


Fig.1 Analogy between thermal and electrical resistance concepts (*Source*: Y. A. Cenghel, R. H. Turner, [13])

Newton's law for thermal convection is:

$$\dot{Q}_{conv} = \frac{T_p - T_\infty}{R_{t,conv}} = \frac{\Delta T}{R_{t,conv}} \quad (12)$$

where:

$R_{conv} = \dfrac{1}{h \cdot S}$ is the thermal resistance of the surface against heat convection or the convective resistance of the surface $\left[\dfrac{K}{W}\right]$;

h is the convection heat transfer coefficient, $\left[\dfrac{W}{(m^2 \cdot K)}\right]$.

Then wall is surrounded by a gas, the radiation effects which were ignored, can be significant and may need to be considered. The rate of radiation heat transfer between a surface of emissivity ε and area S at temperature T_p and the surrounding surfaces at some average temperature T_{surr} can be expressed as:

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot S \cdot \left(T_p^4 - T_{surr}^4\right) = h_{rad} \cdot S \cdot \left(T_p - T_{surr}\right) \Rightarrow$$
$$\dot{Q}_{rad} = \frac{T_p - T_{surr}}{R_{rad}} \quad (13)$$

where:

$R_{rad} = \frac{1}{h_{rad} \cdot S}$ is the thermal resistance of a surface against radiation or radiation resistance, $\left[\frac{K}{W}\right]$;

$\sigma = 5.67 \cdot 10^{-8}$ is the Stefan Boltzmann constant, $\left[\frac{W}{(m^2 \cdot K^4)}\right]$

$h_{rad} = \varepsilon \cdot \sigma\left(T_p^2 + T_{surr}^2\right)\cdot\left(T_p + T_{surr}\right)$ is the radiation heat transfer coefficient, $\left[\frac{W}{(m^2 \cdot K)}\right]$.

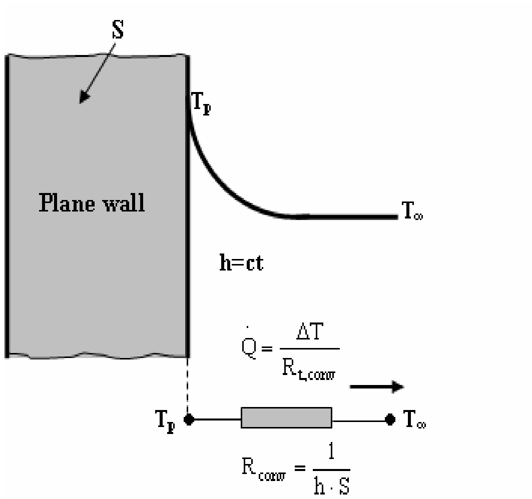


Fig.2 Schematic for the convective resistance at a surface (*Source*: Y. A. Cenghel, R. H. Turner, [13])

In Fig.3 the convection and radiation resistances are parallel to each other causing some complication in the thermal resistance network.

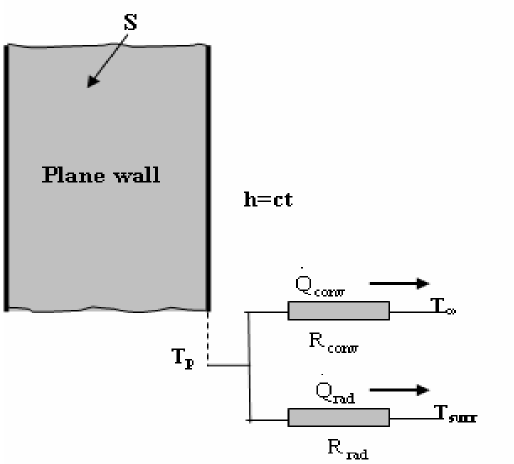


Fig.3 Schematic for convection and radiation resistances at a surface (*Source*: Y. A. Cenghel, R. H. Turner, [13])

When $T_{surr} \approx T_\infty$, the radiation effect can properly be accounted for by replacing *h* in the convection resistance relation by:

$$h_{combined} = h_{conv} + h_{rad} \quad (14)$$

where:
$h_{combined}$ the combined heat transfer coefficient, $\left[\frac{W}{(m^2 \cdot K)}\right]$ [13], [15].

3 DC machine description

In Fig.4 is shown the transversal section by a core pole of a direct current machine for which is considered a constant temperature in whole coil mass taking into account the medium value of temperature.

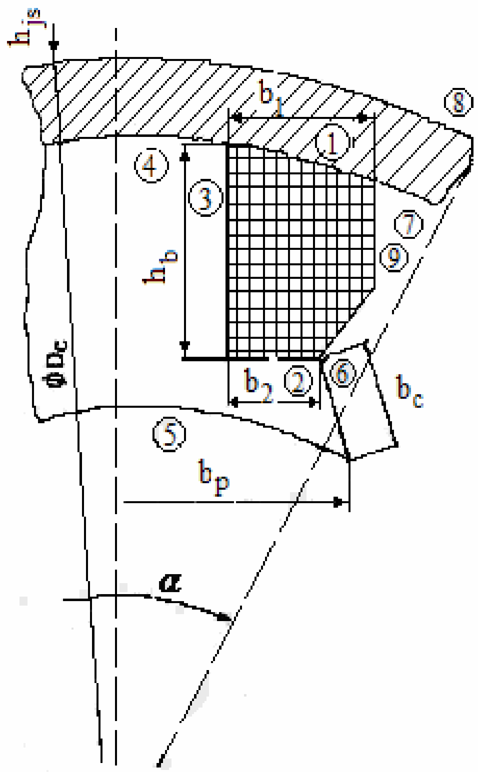


Fig.4 Cross section by a coil pole of a direct-current machine (*Source*: I. Cioc, N.Bichir, N.Cristea, [1])

The temperature drop is practically zero as in pole mass as in machine core, for core side which contributes to the cooling of magnetizing coil and corresponds to the angle $\alpha = \frac{\pi}{2p}$, where α is the angle between the symmetry axes between a main

pole and neighbor auxiliary poles [8].
Real thermal scheme of heat transfer to a coil from a pole of the electrical machine is shown in Fig.5 where are indicated Joule losses from one coil, P_{cub} and all thermal resistances:
•thermal resistance from coil to core, R_1 corresponding to the field *1* as shown in Fig.4;
•thermal resistance from coil to pole by pole piece, R_2 which corresponds the field *2* as shown in Fig.4;
•thermal resistance from coil to pole by pole body, R_3 corresponding the field *3* as shown in Fig.4;
•thermal resistance from pole to core, R_4 taking into account that core-pole junction is about 0.1mm corresponding to the field *4* as shown in Fig.4;
•thermal resistance from pole to air through air gap, R_5 corresponding to the field *5* as shown in Fig.4;
•thermal resistance from pole to the air between poles, R_6 corresponding the field *6* as shown in Fig.4;
•thermal resistance from core to the air inside the electrical machine, R_7 corresponding the field *7* as shown in Fig.4;
•thermal resistance from core to environment, R_8 corresponding the field *8* as shown in Fig.4;
•thermal resistance from coil to the air, R_9 corresponding the field *9*, as shown in Fig.4, [8], [10], [3], [4], [16], [1].

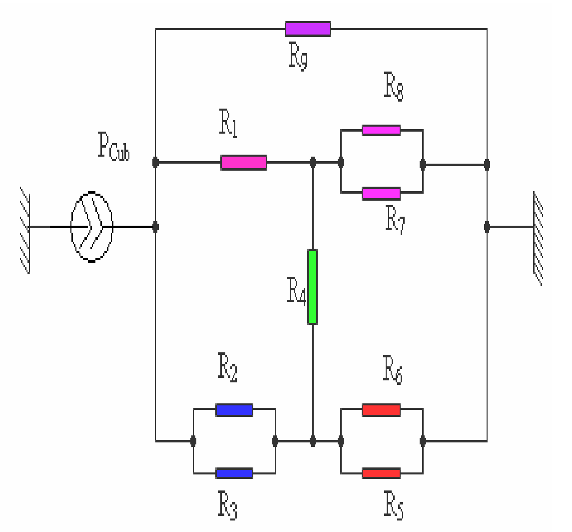


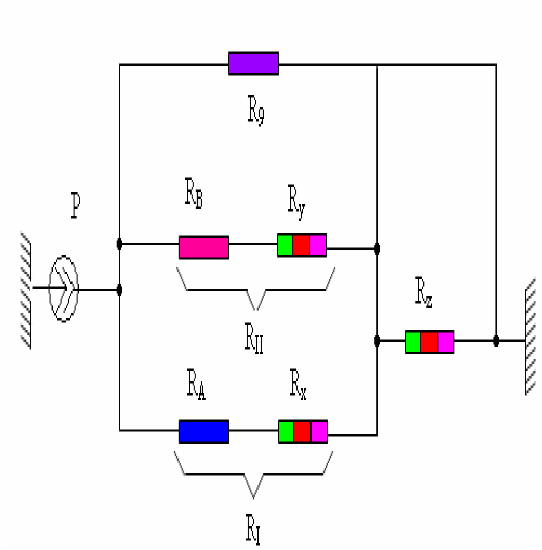
Fig.5 Schematic for the heat transfer from a coil to a pole at a direct current machine

3.1 Computation of thermal resistances

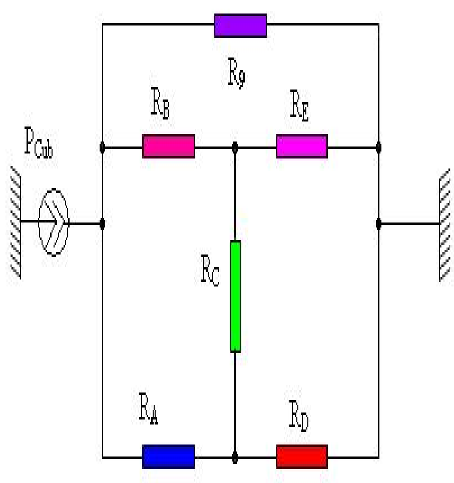
In order to find the equivalent total thermal resistance, we need the intermediary thermal schemes as shown in Fig.6. Method of solving for the thermal networks is identical with electric circuits, according to the connection between the thermal resistances (series, parallel or mixed circuit).

Thermal resistances for determination of the magnetizing coil heating from Fig.5 correspond to the following equations:

$$R_A = \frac{R_2 R_3}{R_2 + R_3},$$
$$R_B = R_1$$
$$R_C = R_4 \tag{15}$$
$$R_D = \frac{R_5 R_6}{R_5 + R_6}$$
$$R_E = \frac{R_7 R_8}{R_7 + R_8}$$



a. Real scheme



b. Equivalent scheme
Fig.6 The intermediary thermal schemes

After transformation of the triangle R_C, R_D and R_E from Fig.6a in star R_x, R_y and R_z, equations for thermal resistances are:

$$R_x = \frac{R_C R_D}{R_C + R_D + R_E}$$
$$R_y = \frac{R_C R_E}{R_C + R_D + R_E} \qquad (16)$$
$$R_z = \frac{R_D R_E}{R_C + R_D + R_E}$$
$$R_I = R_A + R_x,\ R_{II} = R_B + R_y$$

According to Eqs. (16) and the equivalent scheme as shown in Fig.6b, the total equivalent resistance can be determined as follows:

$$R_T = \frac{\left(\frac{R_I R_{II}}{R_I + R_{II}} + R_z\right) R_9}{\frac{R_I R_{II}}{R_I + R_{II}} + R_z + R_9} \qquad (17)$$

After the total equivalent resistance R_T and by means of Eq.11 the temperature drop is:

$$\Delta T_{cub} = \dot{Q} \cdot R_T \qquad (18)$$

Therefore, it can be computed the heating of magnetizing coil in regard to the cooling air from machine.

The intermediary thermal schemes in order to find the heating of magnetizing coil are shown in Fig.6.

Computation of thermal resistances from real scheme is based on the computation of the following resistances above mentioned. These resistances depend on:

•*Thermal resistance from coil to core, R_1* is computed according to the total thickness (insulation thickness of coil, insulation thickness on core, thickness of varnish film), equivalent thermal conductivity (when the coil is insulated, thermal conductivity for insulation on core, thermal conductivity for varnish film) and the surface by which the heat transfer is made considering only lateral sides.

•*Thermal resistance from coil to pole by pole, R_2* is computed as thermal resistance from coil to the machine core for the same insulation but taking into consideration the insulating framework and the heat transfer surface.

•*Thermal resistance from coil to pole by pole body, R_3* is computed according to the insulation thickness of coil only then the coil is insulated, thickness of the impregnating film, thermal conductivity of coil insulation, thermal conductivity of insulation in regard to the pole body, thermal conductivity of varnish film.

•*Thermal resistance from pole to core, R_4* is computed depending on the parasite air-gap thickness to the junction pole-core which is about 0.1mm, thermal conductivity for all machines which have the stator which are not impregnated after winding, and for machines having the stator which is impregnated is computed according to the surface.

•*Thermal resistance from pole core towards air gap, R_5* is computed depending on the surface according to the length and the width of field pole, respectively, thermal resistance to the heat transfer from the pole core towards air-gap.

•*Thermal resistance at heat transfer from the field pole to air between poles*, R_6 is computed depending on the convection heat transfer coefficient and the surface according to width of polar body and length of polar core.

•*Thermal resistance from core to the air inside the electrical machine, R_7* is computed like the thermal resistance between poles by filed poles, and the heat transfer surface depends on the inner diameter of core and the number of pole pairs and the length of core.

•*Thermal resistance from core to environment, R_8* is computed according to the convective heat transfer coefficient, the surface where the inner diameter of core, the number of pole pairs and core length.

•*Thermal resistance from coil to the cooling air from machine, R_9* depends on the thickness and thermal conductivity of coil insulation then the coil is insulated, the convective heat transfer coefficient and the hear transfer surface [1].

3.2 Processing of data by means of NAPACI

Authors have used the NAPACI Programme (Nodal Analysis Programme of Analogue Circuits) which is written in C++ and uses for the numerical integration of ordinary differential equations, the first order generalized regressive method.

Dynamic circuit elements are substituted with discrete resistive circuits associated to this numerical method, and for this analysis of electric circuits, the NAPACI programme uses the modified nodal method.

Circuit description is made by a file of *netlist* type with *extension .nln* having the following structure:

•Number of branches;

•Number of nodes.

It follows a set of *l* lines, where *l* is the number of circuit branches, which describe the circuit branches.

NAPACI creates two output files:

•In the first file, the voltages and the currents of branches are written, as well as the input power at

the terminals of branches for the first ten time steps and the last nine time steps.
All these data are necessary for verifying if the integrating process is right, more exactly, at each time step the total power received to circuit at the terminals of branches is zero.
•The second file called *result.dat* gives the values of currents and voltages of circuit branches at each integrating step.

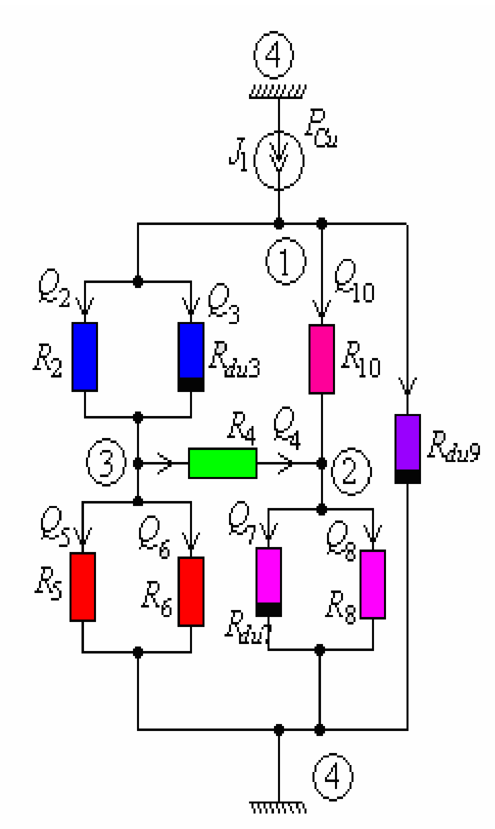


Fig.7 Equivalent thermal scheme for the heat transfer from a coil to a pole at a direct-current machine

Therefore, in Fig.7 is shown the equivalent thermal scheme for Fig.6 in order to find the heating of magnetizing coil.
In this software application, it is analyzed the temperature field distribution and the rate of heat transfer which are necessary for a complete study to range of the temperature in steady state.
Input file *exem41.nln*, for the main scheme of the heat transfer from a coil to a pole at a direct-current machine is presented in Table 1 where the input data are presented in order to obtain the temperature drop and the rate of heat transfer.
The output file called *exem41.out* indicates the output data that means the temperature drop, the rate of heat transfer and losses (generated power and consumed power) presented as well in Table 1.

Table 1. Simulation results for Input file *exem41.nln* and output file *exem41.out*

Unknown quantities of system	
V1 V2 V3	
Equation System	
+ (+G2+G3+G9+G10)*V1 + (-G10)*V2 + (-G2-G3)*V3 = +J1	
+ (-G10)*V1 + (+G4+G7+G8+G10)*V2 + (-G4)*V3 = 0	
+ (-G2-G3)*V1 + (-G4)*V2 + (+G2+G3+G4+G5+G6)*V3 = 0	
Potentials at nodes	
V1=73.767560	
V2=28.283608	
V3=28.637780	
V4=0	
Currents and voltages of branches	
U1=-109.246744	I1=52.357000
U2=76.156573	I2=5.655891
U3=76.156573	I3=9.411174
U4=-8.292309	I4=-5.208737
U5=33.090170	I5=9.703862
U6=33.090170	I6=10.571939
U7=41.382479	I7=8.173216
U8=41.382479	I8=13.872772
U9=109.246744	I9=10.035210
U10=67.864265	I10=27.254725
Power balance	
Generated power : 5719.831754	
Consumed power : 5719.831754	

In Table 2 by means of NAPACI programme are presented input and output data of circuit, considering all thermal resistances as being linear.

Table 2. Input and output data for NAPACI programme

Structure of input file:
10
4
4 1 j1 j=52.357
1 3 R2 r=13.465
1 3 R3 r=3.13
3 2 R4 r=1.592
3 4 R5 r=3.41
3 4 R6 r=3.13
2 4 R7 r=3.14
2 4 R8 r=2.983
1 4 R9 r=4.52
1 2 R10 r=2.49
Results:

Equation system
+ (+G2+G3+G9+G10)*V1 + (-G10)*V2 + (-G2-G3)*V3 = +J1
+ (-G10)*V1 + (+G4+G7+G8+G10)*V2 + (-G4)*V3 = 0
+ (-G2-G3)*V1 + (-G4)*V2 + (+G2+G3+G4+G5+G6)*V3 = 0

Potentials at nodes
V1=73.767560
V2=28.283608
V3=28.637780
V4=0

Currents and voltages of branches	
U1=-73.767560	I1=52.357000
U2=45.129780	I2=3.351636
U3=45.129780	I3=14.418460
U4=0.354172	I4=0.222470
U5=28.637780	I5=8.398176
U6=28.637780	I6=9.149450
U7=28.283608	I7=9.007519
U8=28.283608	I8=9.481598
U9=73.767560	I9=16.320257
U10=45.483952	I10=18.266647

Power balance
Generated power : 3862.248132
Consumed power : 3862.248132

Analysis using SPICE programme is difficult because SPICE does not accept any nonlinearity.
Using SPICE programme for the same linear circuit of DC machine, the input data and the obtained results are shown in Table 3.

Table 3. Input file for SPICE programme and results of simulation

Input file: input.cir

```
I1 0 1 dc 52.357
R2 1 3 R2 r=13.465
R3 1 3 3.13
R4 3 2 1.592
R5 3 4 3.41
R6 3 4 3.13
R7 2 4 3.14
R8 2 4 2.983
R9 1 4 4.52
R10 1 2 2.49
.dc I1 52.357 52.357 52.357
.print dc V(1) V(2) V(3)
+ I(R2) I(R3) I(R4) I(R5)
+ I(R6) I(R7) I(R8) I(R9) I(R10)
.end
```

Results for linear circuit:

```
I1 0 1 dc 52.357
R2 1 3 13.465
R3 1 3 3.13
R4 3 2 1.592
R5 3 0 3.41
R6 3 0 3.13
R7 2 0 3.14
R8 2 0 2.983
R9 1 0 4.52
R10 1 2 2.49
.dc I1 52.357 52.357 52.357
.print dc V(1) V(2) V(3)
+ I(R2) I(R3) I(R4) I(R5)
+ I(R6) I(R7) I(R8) I(R9) I(R10)
.end
```

```
DC Transfer curves temperature =   27.000 DEG C

     I1          V(1)         V(2)         V(3)
                 I(R2)        I(R3)

 5.236E+01   7.377E+01   2.828E+01   2.864E+01
             3.352E+00   1.442E+01
```

```
DC Transfer curves temperature =   27.000 DEG C

     I1          I(R4)        I(R5)        I(R6)
                 I(R7)        I(R8)

 5.236E+01   2.225E-01   8.398E+00   9.149E+00
             9.008E+00   9.482E+00
```

```
DC Transfer curves temperature =   27.000 DEG C

        I1            I(R9)        I(R10)

     5.236E+01   1.632E+01   1.827E+01
```

Therefore, in Table 1, Table 2 and Table 3 the electric sizes used are analogue with thermal sizes thus: voltage difference is analogue to the temperature difference and electric current is analogue to the rate of heat transfer through a layer.
The effectiveness of the results obtained is also confirmed by the fact that simulations from NAPACI and SPICE provide identical outcomes.

3.3 Discussions

Solving of thermal schemes was made in a similar way with respect to solving the electrical schemes taking into account the connections between the thermal resistances (series, parallel or mixed). Results obtained by simulation with NAPACI were compared with other results obtained with SPICE programme, observing that the differences between these results are negligible (practical the results are identical). This validates the NAPACI use in computation of the heating systems of electric machines. It was presented the systematic computing procedure of heat transfer in steady state, by means of equivalent electric schemes with concentrated losses, for a coil on a pole of the direct-current machine and for the distributed windings of the same machine. It was computed: thermal resistances for all heat transfer processes as well as the structure of equivalent thermal network.

4 Conclusions

The modified nodal method does not require restrictions of structure of the analyzed, and can be used as the linear circuit analysis as nonlinear circuit. At the modified nodal method, the additional variables are not necessary, and the vector of the constant terms contains the load at the last time moment which represents the initial condition for the actual time moment. The sizes are modified during the inner iterative cycle due to the pass from a segment to another of the voltage-load characteristic. In this paper, it is presented a computing systematic procedure of heat transfer in steady state by means of equivalent electric schemes with concentrated losses, for a coil on a pole of the direct – current machine. Therefore, the thermal resistances and the rates of heat transfer were computed the structure of equivalent thermal network being presented.

References:

[1] I. Cioc, N. Bichir, N. Cristea, *Electrical Machine, Design book,* vol. I, II, III, Scrisul Romanesc Publishing House, 1981

[2] M. Ionel, M. F. Stan, C.I. Salisteanu, O.M. Ionel, Advanced command techniques of electrical induction machines, *The 9th WSEAS International Conference on Power Systems (PS '09)*, Budapest, Hungary, September 3-5, 2009, pp.176-180, ISSN 1790-5117.

[3] C. Bala, *Electrical Machine*, Didactic and Pedagogic Publishing House, 1982.

[4] I. Boldea, *Transformators and electrical Machine,* Didactic and Pedagogic Publishing House, 1994-Hill.

[5] A. K. Daud, Performance Analysis of Two Phase Brushless DC Motor for Sensor less Operation, *WSEAS Transactions on Power Systems*, Vol. 1, 2006, pp. 802-809

[6] F. Della Torre, S. Leva, A. P. Morando, Use of An Equivalent Thermal Network for the Analysis of a Coupled Thermo-Magnetic problem, *WSEAS Transactions on Circuits and Systems*, Vol. 7, 2008, pp.658-667

[7] S. Sharifian Attarl, M. C. E. Yagoub, F. Mohammadi, Simulation of Electro-Thermal Effects in Device and Circuit, *WSEAS Transactions on Circuits and Systems*, Vol. 5, No. 7, July 2006, p. 926.

[8] O. Nedelcu, *Contributions to heat and ventilation modelling of the electrical machines*, Doctoral Dissertation, Bucharest, 2004.

[9] L. O. Chua, C. A. Desoer, E. S. Kuh, *Linear and nonlinear circuits*, McGraw-Hill Publishing House, 1987.

[10] O. Nedelcu, M. Iordache, D. Enescu, An efficient method for computing of the electric machine ventilation, *The sixth World Energy System Conference*, Torino, Italy, 2006, pp.251-257.

[11] M. Iordache, L. Dumitriu, *Analysis assisted by computer of nonlinear analogical circuits*, Politehnica Press Publishing House, 2004.

[12] L. Dumitriu, M. Iordache, A complex tool for symbolic analysis and design of linear analog circuit, *SMACTD' 2000*, Lisboa, Portugal, 2000, pp.2-10.

[13] Y. A. Cenghel, R. H. Turner, *Thermal-Fluid Science*, McGraw-Hill Publishing House, 2001.

[14] Y.A. Cenghel, *Heat Transfer an Engineering Approach*, 2nd ed. McGraw-Hill Publishing House, 2003 2nd ed. McGraw.

[15] S. Kakaç, Y. Yener, *Heat Conduction,* 3rd ed. Taylor and Francis Washington D.C. Publishing House, 1993.

[16] A.J. Sistino, *Essentials of electronic circuitry:analysis and design, 1996,* ISBN-10: 0824796934 , ISBN-13: 978-0824796938