## Locally Optimal Fuzzy Control of a Heat Exchanger

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*Abstract:* - The paper presents a fuzzy control based on parallel distributed fuzzy controllers for a heat exchanger. Each subcontroller is LQR designed and provides local optimal solutions. First, a Takagi-Sugeno fuzzy model is employed to represent a system. The stability of the system with the proposed fuzzy controllers is discussed. The simulation results are compared with classical PID control and illustrate the validity and applicability of the presented approach.

*Key-Words:* - Takagi-Sugeno fuzzy model, Lyapunov function, LMI problem, shell heat exchanger, Takagi-Sugeno parallel distributed fuzzy LQR, PID control.

## **1** Introduction

Fuzzy controllers have found popularity in many practical situations. Many complex plants have been controlled very well using fuzzy controllers without any difficult analysis common in classical control design. Fuzzy controllers are general nonlinear ones and their benefits are well-known [15]. In spite of these advantageous properties of fuzzy controllers, the main crisis of them was the absence of a formal method for proving the system's stability. However, after introducing the fuzzy plant modelling in [18], some methods for stable controller design have arisen.

The described approach is based on fuzzy modelling of a nonlinear plant as a sum of nonlinearweighted linear subsystems [28]. Following this approach, one can design a linear controller for each some subsystem and satisfying constraints expressible as Linear Matrix Inequalities (LMIs), stability of the whole system can be proved [11, 21, 25]. This is what is called Takagi-Sugeno (TS) fuzzy controller. The idea is similar to traditional gain scheduling method in which controlling gains change according to the state of the controlled system [1, 14]. The ability of converting linguistic descriptions into automatic control strategy makes it a practical and promising alternative to the classical control scheme for achieving control of complex nonlinear systems.

Many real systems can be represented by TS fuzzy models [10, 18, 21]. A TS fuzzy model approximates the system using simple models in each

subspace obtained from the decomposition of the input space. The dynamic TS models are easily obtained by linearization of the nonlinear plant around different operating points. After the TS fuzzy models obtaining, linear control methodology can be used to design local state feedback controllers for each linear model. Aggregation of the fuzzy rules results in a generally nonlinear model [26].

Stability and optimality are the most important requirements for any control system [11]. Most of the existed works are based on Takagi–Sugeno type fuzzy model combined with parallel distribution compensation concept [24] and apply Lyapunov's method to do stability analysis. Tanaka and coworkers reduced the stability analysis and control design problems to linear matrix inequality (LMI) problems [24, 26].

The state feedback gain design method is developed based on assigning a common positive definite matrix P, which was developed in [6, 21, 22, 28] for the Lyapunov stability sense. It is important to find a suitable P such that the stable feedback gains exist. Some theorems will be derived to solve the suitable P as well as the diagonal elements of P and can be assigned by the designers. After assigning a suitable common positive definite matrix P, one can obtain the feedback gains for each rule of the TS type fuzzy system.

A heat exchanger is a device in which energy is transferred from one fluid to another across a solid surface. Exchanger analysis and design therefore involve both, convection and conduction. The heat exchangers are widely used in many industrial power generation units, chemical, petrochemical, and petroleum industries. These types of heat exchangers are robust units that work for wide ranges of pressures, flows and temperatures [17].

## 2 **Problem Formulation**

Fuzzy modelling is a framework in which different modelling and identification methods are combined, providing a transparent interface with the designer or operator *and*. It is a flexible tool for nonlinear system modelling and control too. The rule-based character of fuzzy models allows for a model interpretation in a way that is similar to the one humans use to describe reality [2].

Using fuzzy systems it is possible to define very general nonlinearities. In order to be able to derive any analytical useful results it is necessary to constrain the classes of nonlinearities that one consider. The class of systems that has achieved most attention is linear and affine Takagi-Sugeno systems on state-space form. For these systems both stability and synthesis results are available based on Lyapunov theory.

Quadratic Lyapunov functions are wery powerful if they can be found. In many cases it is very difficult to find a common global Lyapunov function. The feasible solution is to use a piecewise quadratic Lyapunov function that is tailored to fit the cell partition of the system [8]. The search for piecewise quadratic Lyapunov function can also be formulated as an LMI-problem.

### 2.1 Fuzzy Modelling

Processes are in general complex, nonlinear, with time delays. Conventional system modelling methods are not easy to use for nonlinear processes because it is difficult to describe properly all their nonlinearities.

Fuzzy modelling methods are attractive, because they can be developed from real process data with or without expert knowledge. The nonlinearity can be handled efficiently, and the results presented as fuzzy rules are informative. Many different approaches to fuzzy identification have been proposed [2].

Fuzzy models can be considered as logical models, which use If-Then rules to establish qualitative relationships among variables in the model. In general case fuzzy model has these main components: fuzzification of inputs, inference mechanism with rule base that relates inputs to outputs and defuzzification of the output fuzzy set for crisp output calculation [20], see Figure 1.



Fig. 1: Fuzzy system

- Fuzzification maps the crisp values of the preprocessed inputs of the model into fuzzy sets, represented by membership functions. The degree of membership of a single crisp variable to a single fuzzy set is evaluated using membership function and can get the values from an interval [0, 1]. Each input variable in most cases is described by more than three fuzzy sets. Because of simple calculations, triangular membership functions are usually used in fuzzy systems. Other types like trapezoidal, S-type membership Gaussian. functions sometimes have the advantages over triangular but the choice depends on the application.
- The relationship between input and output variables are described in a rule base, composed of If - Then form rules. Usually fuzzy systems are synthesized using two types of rules that differ in the consequent (Then part) proposition form: Mamdani, or standard and Takagi-Sugeno, or functional.
- Inference mechanism calculates the degree to which each rule for a given fuzzified input by considering the rules. A rule fires when the degree of membership of the If part is higher than 0.
- A defuzzifier compiles the information provided by each of the rules and makes a decision from this basis. There are different methods for the calculation of crisp output of fuzzy system like Centroid average CA, Center of gravity COG, Maximum center average MCA, Mean of maximum MM, Smallest of maximum SM, etc.

## 2.2 Neuro-fuzzy Systems

Neuro-fuzzy systems can be viewed upon as a combination of fuzzy systems and artificial neural networks. The fuzzy inference system is implemented in the framework of these adaptive networks. This provides the possibility to use backpropagation learning rules, commonly used to train these nets. Several approaches have been developed. The Anfis approach defines a Takagi-Sugeno fuzzy inference system through a neural network approach by defining 5 layers [7]. Anfis is shown schematically in Figure 2 for a system with two input, two rules and one output.



- layer 1: fuzzification of the input values due to membership degrees
- layer 2: aggregation of membership degrees due to an appropriate *t*-norm applied in the premise parts
- layer 3: evaluation of the basis functions by normalization of aggregated membership degrees
- layer 4: weighting of basis functions with linear (Takagi-Sugeno system) or constant (Sugeno system) consequent functions
- layer 5: evaluation of output values by summing the incoming signals.

#### 2.3 Fuzzy Plant Model

The continuous fuzzy dynamic model, proposed by [18] is described by fuzzy *if-then* rules. It can be seen as a combination of linguistic modelling and mathematical regression, in the sense that the antecedents describe fuzzy regions in the input space in which consequent functions are valid. The *i*<sup>th</sup> rule is of the following form [19].

Plant Rule *i*:

if 
$$z_1(t)$$
 is  $M_1^l$  and ... and  $z_s(t)$  is  $M_s^l$  then

$$\dot{x}(t) = A_i x(t) + B_i u(t) y(t) = C_i x(t) \qquad i=1,.., N$$
(1)

where  $x(t) = [x_1(t), x_2(t), ..., x_n(t)]^T \in \mathbb{R}^n$  is the state vector,  $u(t) = [u_1(t), u_2(t), ..., u_m(t)]^T \in \mathbb{R}^m$  is the control input,  $y(t) = [y_1(t), y_2(t), ..., y_p(t)]^T \in \mathbb{R}^p$  is the controlled output,  $M_j^i$  are fuzzy sets,  $z(t) = [z_1(t), z_2(t), ..., z_s(t)]$  are the premise parameters,  $A_i \in \mathbb{R}^{nxn}$  is the state transition matrix,  $B_i \in \mathbb{R}^{nxm}$  is input matrix,  $C_i \in \mathbb{R}^{pxn}$  is output matrix. Let us use product as *t*- norm operator of the antecedent part of rules and the center of mass method for defuzzification. The final output of the fuzzy system is inferred as follows [16, 19]:

$$\dot{x}(t) = \frac{\sum_{i=1}^{N} \mu_i(z(t)) (A_i x(t) + B_i u(t))}{\sum_{i=1}^{N} \mu_i(z(t))} =$$

$$= \sum_{i=1}^{N} h_i(z(t)) (A_i x(t) + B_i u(t))$$

$$y(t) = \frac{\sum_{i=1}^{N} \mu_i(z(t)) (C_i x(t))}{\sum_{i=1}^{N} \mu_i(z(t))} =$$

$$= \sum_{i=1}^{N} h_i(z(t)) (C_i x(t))$$
(3)

where

$$h_{i}(z(t)) = \frac{\mu_{i}z(t)}{\sum_{i=1}^{N} \mu_{i}z(t)}$$
(4)

$$\mu_i(z(t)) = \prod_{j=1}^s M_i^j z_j(t)$$
(5)

$$\sum_{i=1}^{N} h_i(z(t)) = 1$$
 (6)

# 2.4 Parallel Distributed Compensation (PDC)

The main idea of the PDC controller design is to derive each control rule from the corresponding rule of the TS fuzzy model so as to compensate for it. The resulting overall fuzzy controller, which is non-linear in general, is a fuzzy mixture of individual linear controllers, knowing that the fuzzy controller shares the same fuzzy sets with the fuzzy system [26].

The design concept of PDC is simple and natural. Other nonlinear control techniques require special and rather involved knowledge. This is an advantage of the PDC.

Having TS plant model, it can be used parallel distributed compensation control defined as follows:

Control Rule *j*:  
if 
$$z_{l}(t)$$
 is  $M_{1}^{i}$  and ... and  $z_{s}(t)$  is  $M_{s}^{i}$  then  
 $u(t) = -K_{j}x(t) \quad j=1,..,N$ 
(7)

Hence, the fuzzy controller is given

$$u(t) = -\sum_{j=1}^{N} h_j(z(t)) K_j x(t)$$
(8)

in which *Kj* are state feedback gains. We can see it as local gains of gain scheduling design which overall control signal is made from combining each local control signal with different weights according to the closeness to the each rule's region.

The closed loop system can be expressed by combining (2) and (7) as following system

$$\dot{x}(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} h_i(z(t)) h_j(z(t)) \Big[ A_j - B_i K_j \Big] x(t) = \\ = \sum_{j=1}^{N} h_i^2(z(t)) G_{ij} x(t) + \\ + \sum_{i=1}^{N} \sum_{j=1}^{N} h_i(z(t)) h_j(z(t)) \Big[ \frac{G_{ij} + G_{ji}}{2} \Big] x(t)$$
(9)

where  $G_{ij} = A_i - B_i K_j$ .

# 2.5 Stability Analysis Using Lyapunov Method

After defining the model, the conditions are found under which the system is stable.

*Theorem 1*[18]. The continuous uncontrolled (u=0) fuzzy system of (1) - (3) is globally quadratically stable if there exists a common positive definite matrix  $P=P^{T}$  such that

$$A_i^T P + P A_i < 0, i = 1, \dots, N$$
 (10)

This is equivalent to saying that one must find a single function  $V(x) = x^T P x$  §as a candidate for Lyapunov function. Such a Lyapunov function is also called energy function. If the derivative of the Lyapunov function is always negative, then the system must be asymptotically stable.

In this sense it is easy to obtain the following result using *Theorem 1*: The fuzzy system (2), (3) with fuzzy control of (9) is globally stable if there exists  $P=P^{T}$  such that

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^{T}P+P\left(\frac{G_{ij}+G_{ji}}{2}\right)<0, \quad i=1,...,N \\ j=1,...,N$$
(11)

Finding a common *P* can be considered as linear matrix inequality (LMI) problem. Matlab LMI toolbox presents simple appliance for solving this problem [9].

#### 2.6 Locally Optimal Control Design

Since the local fuzzy system (i.e., fuzzy subsystem) is linear, its quadratic optimization problem is the same as the general linear quadratic (LQ) issue [1, 5, 9, 15]. Therefore, solving the optimal control problem for fuzzy subsystem can be achieved by simply generalizing the classical theorem from the deterministic case to fuzzy case.

$$J = \int_{0}^{\infty} \left( x^{T}(t) Q x(t) + u^{T}(t) R u(t) \right) dt$$
 (12)

where Q is a real symmetric positive semidefinite weighting matrix and R is a real symmetric positive definite weighting matrix. Solution of the optimization problem, i. e. minimization of J for any  $x_0$  satisfies the feedback control law

$$u(t) = -Kx(t) \tag{13}$$

where  $K = R^{-1}B^T P$ .

The optimal gain is K in which P is a symmetric positive semidefinite solution of the matrix Ricatti equation [3, 12]

$$PA + A^{T}P + Q - PBR^{-1}B^{T}P = 0$$
 (14)

If the matrix  $(A - BR^{-1}B^T P)$  is stable, i.e. (A - BK) is stable, the closed-loop system is stable.

### **3** Simulations and Results

#### **3.1 Shell Heat Exchangers**

Consider two heat exchangers shown in Fig. 3.

The measured and controlled output is temperature from second exchanger. The control objective is to keep the temperature of the output stream close to a desired value 353 K. The control signal is input volumetric flow rate of the heated liquid. Assume ideal liquid mixing and zero heat losses. We neglect accumulation ability of exchangers walls. Hold-ups of exchanger as well as flow rates and liquid specific heat capacity are constant.



Fig. 3: Two shell heat exchangers in series

Under these assumptions the mathematical model of the exchangers is given as

$$\frac{dT_1}{dt} = \frac{q}{V_1} \left( T_0 - T_1 \right) + \frac{A_1 k}{V_1 \rho C_p} \left( T_p - T_1 \right), \ T_1(0)$$
(15)

$$\frac{dT_2}{dt} = \frac{q}{V_2} (T_1 - T_2) + \frac{A_2 k}{V_2 \rho C_p} (T_p - T_2), \quad T_2(0)$$
(16)

where  $T_1$  is temperature in the first exchanger,  $T_2$  is temperature in the second exchanger,  $T_0$  is liquid temperature in the inlet stream of the first tank, q is volumetric flow rate of liquid,  $\rho$  is liquid density,  $V_1$ ,  $V_2$  are liquid volumes,  $A_1, A_2$  are heat transfer areas, kis heat transfer coefficient,  $C_p$  is specific heat capacity. The superscript *s* denotes the steady-state values in the main operating point.

Parameters and inputs of the exchangers are enumerated in Table 1.

$q = 1 m^3 min^{-1}$	$k = 72 \text{ kJ m}^{-2} \text{min}^{-1} \text{K}^{-1}$
$V_1 = 5 m^3$	$\rho = 900 \text{ kg m}^{-3}$
$V_2 = 5 m^3$	$T_0^{s} = 293 \text{ K}$
$C_p = 3.84 \text{ kJ kg}^{-1}\text{K}^{-1}$	$T_{p}^{s} = 373 \text{ K}$
$A_1 = 16 m^2$	$T_1^{s} = 313 \text{ K}$
$A_2 = 16 \text{ m}^2$	$T_2^s = 328 \text{ K}$

Table 1: Parameters and inputs of heat exchangers

#### 3.2 Takagi-Sugeno Fuzzy Model and LQR Control

The controlled system was approximated by nine fuzzy models [2, 9]

if x is 
$$M_1^i$$
 and u is  $M_2^i$  then  
 $\dot{x}(t) = A_i x(t) + B_i u(t) \ i = 1,..., 9$  (17)

The bell curve membership functions for the premise variables *x* and *u* in each rule are adopted:

$$f(x;[a,b,c]) = \left( \left( 1 + \left| \frac{(x-c)}{a} \right| \right)^{2b} \right)^{-1}$$
(18)

The antecedent parameters a, b and c for bell shaped membership functions are listed in the Table 2 and membership functions are shown in Figures 4, 5. The consequent parameters are given in Table 3 and the resulting plot of the output surface of a described fuzzy inference system is presented in Figure 6.

Table 2: Bell curve membership functionsparameters

	x			u		
$a_i$	$b_i$	$\mathcal{C}_i$	$a_i$	$b_i$	$\mathcal{C}_i$	
6	2	55	0.13	2	0.34	
6	2	67	0.14	2	0.62	
6	2	79	0.14	2	0.91	

Table 3: Consequent parameters

$A_i$	$B_i$
0.73	-13.41
0.27	-4.94
-0.33	9.44
0.01	0.22
0.13	-3.04
-0.37	11.11
-0.25	0.72
2.33	-74.17
2.05	-59.04



Fig. 4: Bell curve membership functions for input x



Fig. 5: Bell curve membership functions for input *u* 



Fig. 6: Output surface of a fuzzy inference system x' = f(x, u)

Rule viewer that simulates the entire fuzzy inference process is shown in Figure 7. Figure 8 shows the structure of Anfis.



Fig. 7: Fuzzy inference system



Fig. 8: Structure of Anfis

After obtaining  $A_i$ ,  $B_i$ , gains  $K_j$  were calculated of each subsystem using LQR design and then tested for stability using the closed-loop eigenvalues e=eig(A-BK). The eigenvalues of matrix (A-BK) are called the regulator poles. If matrix *K* is chosen properly, the matrix (*A-BK*) can be made an asymptotically stable matrix, and for all  $x(0)\neq 0$  it is possible to make x(t) approach 0 as t approaches infinity. Figure 9 illustrates the fuzzy model-based fuzzy control design methodology [24].

The optimal gain K of each fuzzy subsystem if measured and controlled output is only temperature from second exchanger, the Riccati equation solution P and the closed-loop eigenvalues e are listed in Table 4 for example Q=100, R=0.1.



Fig. 9: Fuzzy model-based fuzzy control design

Figure 10 shows optimal trajectory  $T_2$  of the heat exchangers.



The problem was solved using LMI optimization toolbox in Matlab software package [5].

Table 4:	Gains K, Riccati equation solution P,
	eigenvalues e

Q=100, R=0.1			
K	E		
-31.6577	0.2362	-424.0621	
-31.6773	0.6412	-156.2167	
-31.6775	0.3346	-298.5192	
31.5878	14.3533	-6.9570	

31.5774	1.0416	-96.1333
-31.6656	0.2843	-351.3292
31.5904	4.3441	-22.7698
31.2775	0.0427	-2345
-31.6542	0.0536	-1867

The results for different performance measures are compared in Table 5.

The comparison of LQR controllers was made using *iae* and *ise* criteria described as follows:

$$iae = \int_{0}^{T} |e| dt$$
 (19)

$$ise = \int_{0}^{T} e^{2} dt \qquad (20)$$

The simulation results were compared in the case without disturbances and when disturbances affect the controlled process. Disturbances were represented by temperature changes from 373 K to 353 K at t=25 min, from 353 K to 383 K at t=75 min and from 383 K to 368 K at t=125 min.

Table 5: Performance comparison between fuzzyLQR controllers

performance measure	set-point tracking	disturbance rejection
Q=1*I(2,2) R=1	K=-0.1803 iae = 0.41 e3 ise = 2.67 e3	K=-0.1803 iae = 0.46 e3 ise = 2.54 e3
Q=100*I(2,2) R=1	K=-1.1858 iae = 0.19e3 ise = 1.91e3	K=-1.1858 iae = 0.18e3 ise = 1.56e3
Q=40*I(2,2) R=40	K=-0.1803 iae =1.41e3 ise = 2.67 e3	K=-0.1803 iae = 0.46e3 ise = 2.54 e3
Q=100*I(2,2) R=0.1	K=-3.5888 iae = 0.16 e3 ise = 1.88 e3	K=-3.5888 iae = 0.14 e3 ise = 1.54 e3
Q=1000*I(2,2) R=1	K=-3.5888 iae = 0.16 e3 ise = 1.88 e3	K==-3.5888 iae = 0.14 e3 ise = 1.54 e3

The closed-loop system consisting of the controlled heat exchangers and controller calculated from (8) cannot guarantee the zero steady-state control error. To solve this issue, pure integrator is needed in the controller. The results for different performance measures in this case are compared in Table 6.

Table	6:	Performance	e con	parison	between	fuzzy
LQR (	con	trollers with	a pure	e integra	tor	

performance measure	set-point tracking	disturbance rejection
Q=1*I(2,2) R=1	K=-0.1803 iae = 0.34 e3 ise = 3.28 e3	K=-0.1803 iae = 0.30 e3 ise = 2.83 e3
Q=100*I(2,2) R=1	K=-1.1858 iae = 0.34 e3 ise = 3.28 e3	K=-1.1858 iae = 0.27 e3 ise = 2.73 e3
Q=40*I(2,2) R=40	K=-0.1803 iae = 1.41 e3 ise = 2.67 e3	K=-0.1803 iae = 1.46e3 ise = 2.54 e3
Q=100*I(2,2) R=0.1	K=-3.5888 iae = 0.31 e3 ise = 3.17 e3	K=-3.5888 iae = 0.27 e3 ise = 2.73 e3
Q=1000*I(2,2) R=1	K=-3.5888 iae = 0.31 e3 ise = 3.17 e3	K = = -3.5888 iae = 0.27 e3 ise = 2.73 e3

#### **3.3 PID** Control

The PID control algorithm is used for the control of almost all loops in the process industries, and is also the basis for many advanced control algorithms and strategies. In order to use a controller, it must first be tuned to the system. This tuning synchronizes the controller with the controlled variable, thus allowing the process to be

kept at its desired operating condition. Standard methods for tuning controllers and criteria for judging the loop tuning have been used for many years.

For feedback controller tuning, the approximate model of a system with complex dynamics can have the form of a first-order-plus-time-delay transfer function (21). The process is characterised by a steady-state gain K, an effective time constant T and an effective time delay D.

$$G_P(s) = \frac{K}{Ts+1}e^{-Ds}$$
(21)

The transfer function describing the controlled heat exchangers was identified from step response data in the form (21) with parameters: K = -38.57, T = 11.3 min, D = 2 min. These parameters were used for feedback controller tuning. The feedback PID controllers were tuned by various methods [13]. Two controllers were used for comparison: PID controller (22) tuned using Rivera-Morari method with parameters  $K_C = -0.1063$ ,  $T_I = 12.3$ ,  $T_D = 0.91$  and PID controller tuned using Ziegler-Nichols method with parameters  $K_C = -0.17$ ,  $T_I = 4$ ,  $T_D = 1$ . The transfer function of the used PID controller is following

$$G_C(s) = K_C \left( 1 + \frac{1}{T_I s} + T_D s \right)$$
(22)

The step changes of the reference  $y_r$  were generated and the fuzzy LQR and PID controllers were compared. Figure 11 presents the comparison of the simulation results obtained by LQR TS controller, and PID controllers tuned using Rivera-Morari and Ziegler-Nichols methods. Figure 12 presents the comparison control inputs generated by above mentioned controllers.

Figure 13 presents the simulation results of the fuzzy LQR and PID control of the heat exchanger in the case when disturbances affect the controlled process. The comparison of the controllers output is shown in Figure 14.



Fig. 11. Comparison of the temperature of the <u>output</u> stream from second heat exchanger: reference trajectory ( \_\_), fuzzy LQR ( \_\_\_), fuzzy LQR with integrator ( \_\_.), PID controllers: Rivera-Morari (....), Ziegler-Nichols (- -)



Fig. 12. Comparison of the control inputs



Fig. 13. Control responses in the presence of disturbances: PID controllers: reference trajectory ( \_\_\_\_\_), fuzzy LQR ( \_\_\_\_\_), fuzzy LQR with integrator ( \_\_\_\_\_), PID controllers: Rivera-Morari (....), Ziegler-Nichols (- - )



Fig. 14: Comparison of control inputs in the presence of disturbances

The described controllers were compared using *iae* and *ise* criteria. The *iae* and *ise* values are given in Table 7.

Used fuzzy LQR controller is simple, and it offers the smallest values *iae* and *ise* or equal to linear LQR. The disadvantage of the LQR controllers is, that using these controllers can lead to nonzero steady-state errors, but without overshoots and undershoots practically. In the case of the fuzzy LQR controller with integrator using steady-state control error is equal zero but the control responses show any overshoots and undershoots.

Table 7: Comparison of the simulation results by integrated absolute error *iae* and integrated square error *ise* 

set-point tracking					
control method <i>iae ise</i>					
fuzzy LQR: K =-3.5888	0.16e3	1.88e3			
fuzzy LQR with integrator:	0.31e3	3.17e3			
K =-3.5888					
PID (Rivera-Morari)	0.36e3	4.09e3			
PID (Ziegler-Nichols)	0.52e3	5.79e3			

set-point tracking			
control method	iae	ise	
fuzzy LQR: K =-3.5888	0.14e3	1.54e3	
fuzzy LQR with integrator: K =-3.5888	0.27e3	2.73e3	
PID (Rivera-Morari)	0.40e3	3.64e3	
PID (Ziegler-Nichols)	0.50e3	5.48e4	

## 4 Conclusion

In this paper, a stable nonlinear fuzzy controller based on parallel distributed fuzzy controllers is proposed. Each subcontroller is LQR designed and provides local optimal solution. The Takagi-Sugeno fuzzy model is employed to approximate the nonlinear model of the controlled plant. Based on the fuzzy model, a fuzzy controller is developed to guarantee not only the stability of fuzzy model and fuzzy control system for the heat exchanger but also control the transient behaviour of the system. The design procedure is conceptually simple and natural. Moreover, the stability analysis and control design problems are reduced to LMI problems. Therefore, they can be solved very efficiently in practice by convex programming techniques for LMIs. Simulation results shows that the proposed control approach is robust and exhibits a superior performance to that of established traditional control methods.

Comparison of the LQR simulation results with classical PID control demonstrates the effectiveness and superiority of the proposed approach.

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#### Acknowledgments

The authors gratefully acknowledge the contribution of the Scientific Grant Agency of the Slovak Republic under the grants 1/0071/09, 1/0537/10 and the Slovak Research and Development Agency under the project APVV-0029-07.