On Improvement in the Adaptive Sliding-Mode Speed Observer

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Abstract: - This paper describes the further development concerning an adaptive sliding-mode speed observer employed as the rotor speed and resistance estimators for a three-phase squirrel-cage induction motor (IM). The novel observer is improved from the previous work by a way of on-line calculating the errors of rotor flux estimations and then feeding them to the PI adaptive laws. For that reason, these errors vanish in steady state. Moreover, stability of the proposed observer is guaranteed and verified via the Lyapunov function and its derivative. Finally, the performance between the novel and the previous observers is compared fairly.

Key-Words: - Adaptive sliding-mode observer; Error of rotor flux estimation; Induction motor; Lyapunov stability; PI adaptive law; Speed observer; State observer.

1 Introduction

Apart from the strategies of field-oriented and direct torque controls (vector control and DTC) for an AC induction motor drive, a wide variety of speed-sensorless controls has been devised in order to eliminate any mechanical speed sensor or rotary transducer out of feedback loop. Several benefits of the sensorless drive are better reliability, less maintenance requirements, stronger immunity against noise, the absence of a shaft extension for mounting a position or speed sensor (i.e. reduced size of the drive machine), fewer numbers of the signal cables as well as cheaper expenditure for only utilizing electric current and voltage sensors [1].

Nowadays the sensorless controls based on a state observer with certain self-adjustment mechanisms are more sophisticated [2]. Two or three independent estimators, whose output variables are compared with the same detectable ones, constitute the state observer. Their difference, analogous to the error in closed-loop control systems, is taken to manipulate state variables in the observer so that the error is minimized.

Recently, the rotor speed observer containing adaptive sliding-mode technique and including core-loss compensation was invented in order to on-line estimate the speed signal and reconstruct both stator currents and missing rotor fluxes as the state variables from merely terminal voltages and currents of machine [3]–[4]. The speed observer possessed logical statements as well as one or more PI adaptive laws which one estimates the rotor speed and others update some parameters of IM. Notwithstanding, its logical statements ensure only that the error of stator current estimation tends to become zero. Various adaptive sliding-mode observers excluding core-loss consideration were claimed that their error of rotor flux estimation tends to become zero after they have already entered sliding-mode situation [5]–[7]. An alternative algorithm based on the conventional model of an IM with an auxiliary variable was also established for estimating both the rotor flux and speed [8]–[9]. Without some clarification of stability, the simple equation was able to compute the rotor speed.

There is a possibility of enhancing the former [3]–[4] such that both errors of stator current and rotor flux estimations converge to zero without flux measurement and regardless of whether its operation is in sliding-mode. Thus far, this paper will elaborate on the novel adaptive sliding-mode speed observer. Its material is organized into the following sections. At first, the brief review on dynamic model of an induction motor is outlined and then the innovative concept of the proposed scheme is explained delicately. Finally, the performance of both the novel observer and its counterpart is assessed and compared together through simulation results.

2 Dynamic Model of an AC Induction Motor

An induction motor adopted herein is the type of a three-phase, y-connected, squirrel-cage motor which stator windings are identically distributed with 120° displacement. Under the α–β stator reference frame, when the electromagnetic model of the motor with linear magnetic circuits taking core-loss into account is written by splitting it into stator and rotor windings, this yields [10]

$$\left(di_s/dt\right) = A_{11}i_s + A_{12}\psi_r + B_1 v_s + D_1\psi_r \ , \qquad (1)$$

$$\left(d\psi_r/dt\right) = A_{21}i_s + A_{22}\psi_r + D_2\psi_r \ , \qquad (2)$$

where $v_s = [\, v_{s\alpha} \quad v_{s\beta}\,]^T$ means a stator voltage vector or input vector (volts), $i_s = [\, i_{s\alpha} \quad i_{s\beta}\,]^T$ represents a stator current vector or output vector (A), $\psi_r = [\, \psi_{r\alpha} \quad \psi_{r\beta}\,]^T$ denotes a rotor flux linkage vector (Wb), and α , β stand for the orthogonal components of a vector with respect to the fixed stator coordinates. Besides,

$$A_{11} = a_{r11}I = -\frac{1}{\sigma L_s}\left(R_s + \frac{M^2R_r}{L_r^2}\right)I \ ,$$

$$A_{12} = \frac{1}{\varepsilon}\left(\frac{R_r}{L_r}I - \omega_r J\right) \ , \ A_{21} = \frac{MR_r}{L_r}I \ ,$$

$$A_{22} = -\varepsilon A_{12} \ , \ B_1 = \frac{1}{\sigma L_s}I \ , \ D_2 = -\frac{sR_m}{L_r}I \ ,$$

$$D_1 = -\frac{R_m\left(L_r - sM\right)}{\varepsilon ML_r}I \ , \ I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$J = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}, \ \sigma = 1 - \frac{M^2}{L_sL_r} > 0 \ ,$$

$$\varepsilon = \frac{\sigma L_sL_r}{M} > 0 \ , \ s = \frac{\omega_{sl}}{\omega_s} = \frac{\omega_s - \omega_r}{\omega_s} \ ,$$

$$\omega_m = \frac{2\omega_r}{p} \ , \ \omega_s = 2\pi f_s \ , \ R_m = k_c\omega_s^{1.6} \ ,$$

R_s is stator resistance (Ω), R_r means rotor resistance (Ω) that may vary with the motor temperature, L_s represents stator self-inductance (H), L_r denotes rotor self-inductance (H), M stands for mutual inductance (H), R_m is core-loss resistance (Ω), σ means total leakage factor, s represents the slip, ω_s denotes stator angular velocity (rad/sec), f_s stands for stator or supply frequency (Hz), ω_{sl} is angular velocity of slip (rad/sec), ω_r means electrical angular velocity of rotor or rotor speed (rad/sec), ω_m represents mechanical angular velocity of rotor or shaft speed (rad/sec), p denotes the number of poles and k_c stands for a constant value (*rated* $R_m \div$ *rated* $\omega_s^{1.6}$). In addition, the electromagnetic torque on the rotor periphery is generated via interaction of currents and flux, i.e.

$$T_e = \left(3p/4\right)\left(M/L_r\right)\left(i_{s\beta}\psi_{r\alpha} - i_{s\alpha}\psi_{r\beta}\right) \ . \qquad (3)$$

The effort of the above torque causes the motor shaft and its mechanical load to rotate dynamically. However, an amount of the electromagnetic torque is not involved in construction of the proposed observer because greater complexity and higher order are derived such that they prevent the progression to the desirable observer.

3 Novel Adaptive Sliding-Mode Speed Observer

In this section, the scheme of the previous observer is improved into part of the rotor winding. Prior to any modification, it is assumed that the shaft speed dynamic is much slower than the dynamic of the electromagnetic system, and the measurable quantities are only stator voltages and currents. Instead of the estimated stator current $\hat{i}_s$, one portion of the rotor winding is altered to become the monitored stator current i_s. So far, the main structure of the proposed observer is written as follows:

$$\left(d\hat{i}_s/dt\right) = \hat{A}_{11}\hat{i}_s + \hat{A}_{12}\hat{\psi}_r + \hat{D}_1\hat{\psi}_r + B_1v_s + U_o \ , \qquad (4)$$

$$\left(d\hat{\psi}_r/dt\right) = \hat{A}_{21}i_s + \hat{A}_{22}\hat{\psi}_r + \hat{D}_2\hat{\psi}_r \ , \qquad (5)$$

where ^ indicates the estimated values or vectors,

$$\hat{i}_s = \left[\hat{i}_{s\alpha} \quad \hat{i}_{s\beta}\right]^T \ , \ \hat{\psi}_r = \left[\hat{\psi}_{r\alpha} \quad \hat{\psi}_{r\beta}\right]^T \ ,$$

$$\hat{A}_{11} = \hat{a}_{r11}I = -\frac{1}{\sigma L_s}\left(R_s + \frac{M^2\hat{R}_r}{L_r^2}\right)I \ ,$$

$$\hat{A}_{12} = \frac{1}{\varepsilon}\left(\frac{\hat{R}_r}{L_r}I - \hat{\omega}_r J\right) \ , \ \hat{A}_{21} = \frac{M\hat{R}_r}{L_r}I \ ,$$

$$\hat{A}_{22} = -\varepsilon\hat{A}_{12} \ , \ \hat{D}_2 = -\frac{\hat{s}R_m}{L_r}I \ , \ \hat{\omega}_m = \frac{2\hat{\omega}_r}{p} \ ,$$

$$\hat{D}_1 = -\frac{R_m\left(L_r - \hat{s}M\right)}{\varepsilon ML_r}I \ , \ \hat{s} = \frac{\hat{\omega}_{sl}}{\omega_s} = \frac{\omega_s - \hat{\omega}_r}{\omega_s} \ ,$$

and U_o is the correction vector imposed to compel the estimation error towards zero. For the purpose of provision, it is feasible to write the mismatches between the estimated and the actual state-variables as well as between the estimated and the actual parameters in the following:

$$e_i = \begin{bmatrix} e_{i\alpha} \\ e_{i\beta} \end{bmatrix} = \begin{bmatrix} i_{s\alpha} - \hat{i}_{s\alpha} \\ i_{s\beta} - \hat{i}_{s\beta} \end{bmatrix} , \tag{6}$$

$$e_\psi = \begin{bmatrix} e_{\psi\alpha} \\ e_{\psi\beta} \end{bmatrix} = \begin{bmatrix} \psi_{r\alpha} - \hat{\psi}_{r\alpha} \\ \psi_{r\beta} - \hat{\psi}_{r\beta} \end{bmatrix} , \tag{7}$$

$$\Delta R_r = R_r - \hat{R}_r , \tag{8}$$

$$\Delta\omega_r = \omega_r - \hat{\omega}_r . \tag{9}$$

When the proposed observer is compared with the induction motor as a plant, two error equations can be unfolded as

$$\begin{aligned} \dot{e}_i &= A_{11}e_i + \left(A_{12} + D_1\right)e_\psi + \Delta A_{11}\hat{i}_s \\ &\quad + \left(\Delta A_{12} + \Delta D_1\right)\hat{\psi}_r - U_o , \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{e}_\psi &= \left(A_{22} + D_2\right)e_\psi + \Delta A_{21}i_s \\ &\quad + \left(\Delta A_{22} + \Delta D_2\right)\hat{\psi}_r , \end{aligned} \tag{11}$$

where $\Delta A_{21} = \dfrac{M}{L_r}\Delta R_r I$, $\Delta A_{21} = -\varepsilon \Delta A_{11}$,

$$\Delta A_{11} = \left(a_{r11} - \hat{a}_{r11}\right)I = -\frac{M^2}{\sigma L_s L_r^2}\Delta R_r I ,$$

$$\Delta A_{11} = -\frac{M}{\sigma L_s L_r}\Delta A_{21} = -\frac{1}{\varepsilon}\Delta A_{21} ,$$

$$\Delta A_{12} + \Delta D_1 = \frac{1}{\varepsilon}\left\{\frac{1}{L_r}\left[\Delta R_r - \frac{R_m}{\omega_s}\Delta\omega_r\right]I - \Delta\omega_r J\right\} ,$$

and

$$\Delta A_{22} + \Delta D_2 = -\frac{1}{L_r}\left(\Delta R_r - \frac{R_m}{\omega_s}\Delta\omega_r\right)I + \Delta\omega_r J ,$$

$$\Delta A_{22} + \Delta D_2 = -\varepsilon\left(\Delta A_{12} + \Delta D_1\right) .$$

Now the integral of the stator current error vector and its surface vector are defined in the following, respectively [11]:

$$\dot{z}_i = -e_i , \tag{12}$$

$$S_i = e_i - K z_i , \tag{13}$$

where $z_i = \begin{bmatrix} z_{i\alpha} \\ z_{i\beta} \end{bmatrix}$, $S_i = \begin{bmatrix} s_{i\alpha} \\ s_{i\beta} \end{bmatrix}$, and $K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$. K is the surface gain matrix that is positive definite. From Eq. (10), Eq. (12), and Eq. (13), the time derivative of the surface vector is written as

$$\begin{aligned} \dot{S}_i &= \left(A_{11} + K\right)e_i + \left(A_{12} + D_1\right)e_\psi \\ &\quad + \Delta A_{11}\hat{i}_s + \left(\Delta A_{12} + \Delta D_1\right)\hat{\psi}_r - U_o , \end{aligned} \tag{14}$$

where $(A_{12} + D_1)e_\psi$ becomes an unknown term since $\psi_{r\alpha}$, $\psi_{r\beta}$ are inaccessible and ω_r , s are not measured deliberately by physical transducers. This term is expressed as

$$\begin{aligned} &\left(A_{12} + D_1\right)e_\psi = \begin{bmatrix} f_1 & f_2 \end{bmatrix}^T \\ &= \begin{bmatrix} \dfrac{1}{\varepsilon}\left\{\dfrac{R_r}{L_r} - \dfrac{R_m\left(L_r - sM\right)}{ML_r}\right\}e_{\psi\alpha} + \dfrac{\omega_r}{\varepsilon}e_{\psi\beta} \\ \dfrac{1}{\varepsilon}\left\{\dfrac{R_r}{L_r} - \dfrac{R_m\left(L_r - sM\right)}{ML_r}\right\}e_{\psi\beta} - \dfrac{\omega_r}{\varepsilon}e_{\psi\alpha} \end{bmatrix} . \end{aligned} \tag{15}$$

Furthermore, the correction vector U_o is available from the combination of three terms as

$$U_o = \Phi_1 e_i + \Phi_2 K z_i + \Lambda , \tag{16}$$

in which $\Phi_1 = \begin{bmatrix} \phi_{1\alpha} & 0 \\ 0 & \phi_{1\beta} \end{bmatrix}$, $\Phi_2 = \begin{bmatrix} \phi_{2\alpha} & 0 \\ 0 & \phi_{2\beta} \end{bmatrix}$, and $\Lambda = \begin{bmatrix} \Lambda_\alpha & \Lambda_\beta \end{bmatrix}^T$ are the correction gain matrices. At this stage, both the error equations are arranged into suitable forms as follows:

$$\begin{aligned} &\left(A_{12} + D_1\right)e_\psi + \Delta A_{11}\hat{i}_s + \left(\Delta A_{12} + \Delta D_1\right)\hat{\psi}_r \\ &\qquad = \dot{e}_i - A_{11}e_i + U_o , \end{aligned} \tag{17}$$

$$\begin{aligned} \dot{e}_\psi &= \left(A_{22} + D_2\right)e_\psi + \Delta A_{21}\hat{i}_s \\ &\quad + \left(\Delta A_{22} + \Delta D_2\right)\hat{\psi}_r + \Delta A_{21}e_i . \end{aligned} \tag{18}$$

Then, the relation between these two errors can be written into a single form as

$$\dot{e}_\psi + \frac{R_m}{M}e_\psi = -\varepsilon\left\{\dot{e}_i - A_{11}e_i + U_o\right\} + \Delta A_{21}e_i , \tag{19}$$

where $D_2 = -\varepsilon D_1 - \dfrac{R_m}{M}I$ and

$$A_{22}+D_2 = -\varepsilon\left(A_{12}+D_1\right)-\frac{R_m}{M}I \ .$$

Next, a few terms in the right side of the above equation are given with more detail below:

$$\varepsilon A_{11}e_i+\Delta A_{21}e_i = -\frac{L_r R_s}{M}e_i - A_{21}e_i+\Delta A_{21}e_i \ , \qquad (20)$$

where

$$\varepsilon A_{11} = -\frac{L_r}{M}\left(R_s+\frac{M^2}{L_r^2}R_r\right)I = -\frac{L_r R_s}{M}I-\frac{MR_r}{L_r}I \ .$$

After the substitution from such clearer expression into Eq. (19), the error of rotor flux estimation can be on-line computed by

$$\dot e_\psi+\frac{R_m}{M}e_\psi = -\varepsilon U_o-\varepsilon\,\dot e_i -\frac{L_r R_s}{M}e_i-\frac{M}{L_r}\hat R_r e_i \ . \qquad (21)$$

As a result, this equation is not only helpful and significant for the scheme but it also makes the proposed observer different from the previous one [3]–[4]. In order to determine the correction gains and create two parameter updating laws, the candidate Lyapunov function is selected as follows:

$$V = \frac{1}{2}S_i^T S_i+\frac{1}{2\varepsilon}e_\psi^T e_\psi +\frac{1}{2\varepsilon L_r k_{ri}}\left(\Delta R_r+k_{rp}\Theta_R\right)^2 +\frac{1}{2\varepsilon k_{\omega i}}\left(\Delta\omega_r-k_{\omega p}\Theta_\omega\right)^2 \ \geq \ 0 \ , \qquad (22)$$

where $\Theta_R = \Theta_R\left(S_i, e_\psi, \hat\psi_r, \hat i_s, i_s\right)$,

$$\Theta_R = \left(S_i-e_\psi\right)^T\hat\psi_r - M\left(S_i^T\hat i_s-e_\psi^T i_s\right) ,$$

$$\Theta_\omega = \Theta_\omega\left(S_i, e_\psi, \hat\psi_r\right) ,$$

$$\Theta_\omega = \left(S_i-e_\psi\right)^T\left(J+\frac{R_m}{\omega_s L_r}I\right)\hat\psi_r \ ,$$

k_{rp}, $k_{\omega p}$ are the positive proportional gains, and k_{ri}, $k_{\omega i}$ are the positive integral gains.

Beneath the assumption that R_r and ω_r are almost constant in comparison with system dynamic of state variables, then the procedure due to differentiating the function V along time as well as substitution together with particular simplification results in the time derivative of Lyapunov function gradually below:

$$\begin{aligned}\dot V = {} & S_i^T\dot S_i+\frac{1}{\varepsilon}e_\psi^T\dot e_\psi+\frac{\Delta R_r}{\varepsilon L_r}\left(\frac{\Delta\dot R_r}{k_{ri}}+\frac{k_{rp}}{k_{ri}}\dot\Theta_R\right)\\ & +\frac{k_{rp}}{\varepsilon L_r}\Theta_R\left(\frac{\Delta\dot R_r}{k_{ri}}+\frac{k_{rp}}{k_{ri}}\dot\Theta_R\right)\\ & +\frac{\Delta\omega_r}{\varepsilon}\left(\frac{\Delta\dot\omega_r}{k_{\omega i}}-\frac{k_{\omega p}}{k_{\omega i}}\dot\Theta_\omega\right)\\ & -\frac{k_{\omega p}}{\varepsilon}\Theta_\omega\left(\frac{\Delta\dot\omega_r}{k_{\omega i}}-\frac{k_{\omega p}}{k_{\omega i}}\dot\Theta_\omega\right) .\end{aligned} \qquad (23)$$

At once, it is necessary to impart the important equation from Eq. (11) and Eq. (14) into

$$\begin{aligned}& S_i^T\dot S_i+\frac{1}{\varepsilon}e_\psi^T\dot e_\psi\\ & = S_i^T\left\{\left(A_{11}+K\right)e_i+\left(A_{12}+D_1\right)e_\psi-U_o\right\}\\ & \quad +\frac{1}{\varepsilon}e_\psi^T\left(A_{22}+D_2\right)e_\psi+S_i^T\Delta A_{11}\hat i_s\\ & \quad +\frac{1}{\varepsilon}e_\psi^T\Delta A_{21}i_s+S_i^T\left(\Delta A_{12}+\Delta D_1\right)\hat\psi_r\\ & \quad +\frac{1}{\varepsilon}e_\psi^T\left(\Delta A_{22}+\Delta D_2\right)\hat\psi_r \ .\end{aligned} \qquad (24)$$

Then, it is straightforward to rewrite the equation just mentioned as

$$\begin{aligned}& S_i^T\dot S_i+\frac{1}{\varepsilon}e_\psi^T\dot e_\psi\\ & = S_i^T\left\{\left(A_{11}+K\right)e_i+\left(A_{12}+D_1\right)e_\psi-U_o\right\}\\ & \quad +\frac{1}{\varepsilon}e_\psi^T\left(A_{22}+D_2\right)e_\psi\\ & \quad -\frac{\Delta\omega_r}{\varepsilon}\left(S_i-e_\psi\right)^T\left(J+\frac{R_m}{\omega_s L_r}I\right)\hat\psi_r\\ & \quad +\frac{\Delta R_r}{\varepsilon L_r}\left\{\left(S_i-e_\psi\right)^T\hat\psi_r-M\left(S_i^T\hat i_s-e_\psi^T i_s\right)\right\} ,\end{aligned} \qquad (25)$$

$$\begin{aligned}& S_i^T\dot S_i+\frac{1}{\varepsilon}e_\psi^T\dot e_\psi\\ & = S_i^T\left\{\left(A_{11}+K\right)e_i+\left(A_{12}+D_1\right)e_\psi-U_o\right\}\\ & \quad -\frac{R_r+sR_m}{\varepsilon L_r}e_\psi^T e_\psi+\frac{\Delta R_r}{\varepsilon L_r}\Theta_R-\frac{\Delta\omega_r}{\varepsilon}\Theta_\omega \ .\end{aligned} \qquad (26)$$

The substitution from Eq. (26) into Eq. (23) achieves the time derivative of Lyapunov function as

$$\begin{aligned}\dot{V} = & -\frac{R_r + sR_m}{\varepsilon L_r} e_\psi^T e_\psi \\ & + S_i^T \left\{ \left(A_{11} + K\right) e_i + \left(A_{12} + D_1\right) e_\psi - U_o \right\} \\ & + \frac{\Delta R_r}{\varepsilon L_r} \left(\frac{\Delta \dot{R}_r}{k_{ri}} + \frac{k_{rp}}{k_{ri}} \dot{\Theta}_R + \Theta_R \right) \\ & + \frac{k_{rp}}{\varepsilon L_r} \Theta_R \left(\frac{\Delta \dot{R}_r}{k_{ri}} + \frac{k_{rp}}{k_{ri}} \dot{\Theta}_R \right) \\ & + \frac{\Delta \omega_r}{\varepsilon} \left(\frac{\Delta \dot{\omega}_r}{k_{\omega i}} - \frac{k_{\omega p}}{k_{\omega i}} \dot{\Theta}_\omega - \Theta_\omega \right) \\ & - \frac{k_{\omega p}}{\varepsilon} \Theta_\omega \left(\frac{\Delta \dot{\omega}_r}{k_{\omega i}} - \frac{k_{\omega p}}{k_{\omega i}} \dot{\Theta}_\omega \right) .\end{aligned} \tag{27}$$

Then, it is preferable to rewrite the time derivative of Lyapunov function as

$$\begin{aligned}\dot{V} = & -\frac{R_r + sR_m}{\varepsilon L_r} \left(e_{\psi\alpha}^2 + e_{\psi\beta}^2 \right) \\ & - \left\{ \left| \phi_{1\alpha} \right| \left| s_{i\alpha} e_{i\alpha} \right| - \left(a_{r11} + k_1 \right) s_{i\alpha} e_{i\alpha} \right\} \\ & - \left\{ \left| \phi_{1\beta} \right| \left| s_{i\beta} e_{i\beta} \right| - \left(a_{r11} + k_2 \right) s_{i\beta} e_{i\beta} \right\} \\ & - \left(\left| \Lambda_\alpha \right| \left| s_{i\alpha} \right| - f_1 s_{i\alpha} \right) \\ & - \left(\left| \Lambda_\beta \right| \left| s_{i\beta} \right| - f_2 s_{i\beta} \right) \\ & - k_1 \left| \phi_{2\alpha} \right| \left| s_{i\alpha} z_{i\alpha} \right| - k_2 \left| \phi_{2\beta} \right| \left| s_{i\beta} z_{i\beta} \right| \\ & - \frac{k_{rp}}{\varepsilon L_r} \Theta_R^2 - \frac{k_{\omega p}}{\varepsilon} \Theta_\omega^2 \le 0 ,\end{aligned} \tag{28}$$

where $s > -\frac{R_r}{R_m}$. As long as an induction motor attached to its load with alignment is normally running forwards at a speed or during acceleration, the slip value s remains positive. Nevertheless, whenever the motor shaft is rotating more and much more slowly during regenerative braking, the slip s becomes negative value. Usually, such a braking method takes place within only a short period of time. Thus, the slip value s is nearly always positive. Via the Lyapunov's stability theorem, the conditions on $V \ge 0$ and $\dot{V} \le 0$ must be obeyed to guarantee the stability of the proposed speed observer [12]. When the $\dot{V}$ is constrained to be strictly semi-negative, the sufficient conditions for fulfilling the inequality (28) are

$$\begin{aligned} & |\phi_{1\alpha}| |s_{i\alpha} e_{i\alpha}| - (a_{r11} + k_1) s_{i\alpha} e_{i\alpha} \ge 0 , \\ & |\phi_{1\beta}| |s_{i\beta} e_{i\beta}| - (a_{r11} + k_2) s_{i\beta} e_{i\beta} \ge 0 , \\ & k_1 |\phi_{2\alpha}| |s_{i\alpha} z_{i\alpha}| \ge 0 , \\ & k_2 |\phi_{2\beta}| |s_{i\beta} z_{i\beta}| \ge 0 , \\ & |\Lambda_\alpha| |s_{i\alpha}| - f_1 s_{i\alpha} \ge 0 , \\ & |\Lambda_\beta| |s_{i\beta}| - f_2 s_{i\beta} \ge 0 . \end{aligned} \tag{29}$$

Really, these conditions can be found from an obligation of $S_i^T \left\{ \left(A_{11} + K\right) e_i + \left(A_{12} + D_1\right) e_\psi - U_o \right\} \le 0$ and they are rewritten as

$$\begin{aligned} & (a_{r11} + k_1 - \phi_{1\alpha}) s_{i\alpha} e_{i\alpha} \le 0 , \\ & (a_{r11} + k_2 - \phi_{1\beta}) s_{i\beta} e_{i\beta} \le 0 , \\ & -\phi_{2\alpha} k_1 s_{i\alpha} z_{i\alpha} \le 0 , \\ & -\phi_{2\beta} k_2 s_{i\beta} z_{i\beta} \le 0 , \\ & (f_1 - \Lambda_\alpha) s_{i\alpha} \le 0 , \\ & (f_2 - \Lambda_\beta) s_{i\beta} \le 0 . \end{aligned} \tag{30}$$

In order to assure the convergence of S_i towards zero, the expansion of the inequality (30) deals with the following logical statements:

$$\begin{array}{llll} \text{if} & s_{i\alpha} e_{i\alpha} > 0 & \text{then} & \phi_{1\alpha} > |a_{r11} + k_1| , \\ \text{if} & s_{i\alpha} e_{i\alpha} < 0 & \text{then} & \phi_{1\alpha} < -|a_{r11} + k_1| , \\ \text{if} & s_{i\beta} e_{i\beta} > 0 & \text{then} & \phi_{1\beta} > |a_{r11} + k_2| , \\ \text{if} & s_{i\beta} e_{i\beta} < 0 & \text{then} & \phi_{1\beta} < -|a_{r11} + k_2| , \\ \text{if} & s_{i\alpha} z_{i\alpha} > 0 & \text{then} & \phi_{2\alpha} > 0 , \\ \text{if} & s_{i\alpha} z_{i\alpha} < 0 & \text{then} & \phi_{2\alpha} < 0 , \\ \text{if} & s_{i\beta} z_{i\beta} > 0 & \text{then} & \phi_{2\beta} > 0 , \\ \text{if} & s_{i\beta} z_{i\beta} < 0 & \text{then} & \phi_{2\beta} < 0 , \\ \text{if} & s_{i\alpha} > 0 & \text{then} & \Lambda_\alpha > |f_1| , \\ \text{if} & s_{i\alpha} < 0 & \text{then} & \Lambda_\alpha < -|f_1| , \\ \text{if} & s_{i\beta} > 0 & \text{then} & \Lambda_\beta > |f_2| , \\ \text{if} & s_{i\beta} < 0 & \text{then} & \Lambda_\beta < -|f_2| . \end{array} \tag{31}$$

Practically, all correction gains are chosen via trials and errors until $s_{i\alpha}$, $s_{i\beta}$, $z_{i\alpha}$, $z_{i\beta}$, $e_{i\alpha}$, and $e_{i\beta}$ diminish towards zero within finite time. Afterwards, the inequality (28) still requires the other conditions satisfying it. These conditions are $\frac{\Delta \dot{R}_r}{k_{ri}} + \frac{k_{rp}}{k_{ri}} \dot{\Theta}_R + \Theta_R = 0$ and

$\frac{\Delta\dot{\omega}_r}{k_{\omega i}} - \frac{k_{\omega p}}{k_{\omega i}}\dot{\Theta}_\omega - \Theta_\omega = 0$. Thereby, they lead to two adaptive laws in a typical proportional-integral (PI) manner below:

$$\hat{R}_r = \hat{R}_{r0} + k_{rp}\Theta_R + k_{ri}\int_0^t \Theta_R(\tau)\,d\tau \ , \tag{32}$$

$$\hat{\omega}_r = \hat{\omega}_{r0} - k_{\omega p}\Theta_\omega - k_{\omega i}\int_0^t \Theta_\omega(\tau)\,d\tau \ , \tag{33}$$

where $\dot{\omega}_r \approx 0$, $\dot{R}_r \approx 0$, $\hat{R}_{r0}$ and $\hat{\omega}_{r0}$ are the initial estimates of the corresponding $\hat{R}_r$ and $\hat{\omega}_r$. Hence, the last equation handles the on-line estimation of rotor speed. It is noticeable that such two PI adaptive laws obtain the error of rotor flux estimation as one element of them. Therefore, the speed estimation also partially depends on this element. According to the conditions on $V \geq 0$ and $\dot{V} \leq 0$, V is a decreasing function of t (i.e. $V(t) \leq V(0)$). When time elapses adequately long ($t \to \infty$), $S_i \to 0$, $e_i \to 0$, $e_\psi \to 0$, $\Theta_R \to 0$, and $\Theta_\omega \to 0$ as well as ΔR_r and $\Delta\omega_r$ converge to their corresponding steady-state values [12]. Meanwhile, the operation of the observer reaches the sliding surface and then goes into the so-called sliding-mode. Thus, the proposed speed observer is stable in general. The block diagram of the overall scheme is shown in Fig. 1.

4 Simulation Results and Discussions

Simulation task is a potential way in testing the capability of the proposed scheme. Numerical solutions are acquired to confirm the effectiveness that is anticipated. It is interesting to find out performances of the novel and previous observers [3]–[4]. Owing to not equal numbers of the adaptive laws in each observer, the rotor speed is only on-line estimated whereas the other parameters are not varied from their actual values. Thereby, it allows a distinct comparison between these two schemes. Another ability of the proposed observer is to render simultaneous estimations of speed and rotor resistance. Throughout all simulations the key nominal parameters per phase of the 3 kW squirrel-cage motor are: $R_s = 2.15\ \Omega$, $R_r = 2.33\ \Omega$, $L_s = L_r = 0.21$ H, $M = 0.2025$ H, $R_m = 4.48\ \Omega$, $p = 4$, $J_m = 0.008$ kg·m^2, $f_s^{rated} = 50$ Hz, and $\omega_m^{rated} = 1420$ rpm where J_m is the motor moment of inertia. The motor coupled to a load with inertia $J_L = 0.084$ kg·m^2 becomes the motor-load system whose the viscous friction coefficient is $B_t = 0.0697$ N·m·s/rad. A load torque against rotation is supposed to be constant at 5 N·m. According to direct-on-line starting, at the initial instant of time ($t = 0$) the motor earlier de-energized at standstill is joined directly to a 220 V, 50 Hz three-phase ac supply. All initial conditions of state variables of both the motor-load system and either the novel observer or the previous one are put to zero. The initial value of rotor speed estimation is also set to zero, i.e. $\hat{\omega}_{r0} = 0$. The surface gains k_1, k_2 for both the observers are chosen to be 5. The possible correction gains for them are: $|\phi_{1\alpha}| = |\phi_{1\beta}| = 290$, $|\phi_{2\alpha}| = |\phi_{2\beta}| = 1$, and $|\Lambda_\alpha| = |\Lambda_\beta| = 10$ while the PI gains in the adaptive law of them are adjusted to $k_{\omega p} = 10$ and $k_{\omega i} = 6000$.

In transient period, there are some errors of rotor speed estimations when the shaft of the motor-load system starts rotating by a rise in the actual speed and the two observers begin computing their state variables as shown in Fig. 2 and Fig. 3. These errors rather resemble each other but the error due to the estimation in the proposed scheme seems smaller. Such both estimations can track and then meet the actual speed within the transient state before the trend of the actual one is towards a constant. However, the previous scheme gives less error in rotor flux estimations during the same time as shown in Fig. 4 and Fig. 5. In steady state, the novel observer provides quite slighter errors in both rotor speed and flux estimations than those that the previous one does as shown from Fig. 6 to Fig. 11. In other words, the Lyapunov function and its derivative through the Eq. (21) make the rotor flux estimation become more accurate. Although the trajectories being related to the stator current errors and their integrals have abruptly left the origin at the commencement of computation as shown in Fig. 12 and Fig. 13, they move around inside some local regions and then are confined into a vicinity of a straight line with its slope of $(1/k_1) = (1/k_2) = 0.2$ in steady state as shown in Fig. 14 and Fig. 15. The straight line is known as an ideal sliding line or a sliding surface. Yet, the trajectories fluctuate within the tiny neighborhood of the ideal line because the fixed-stepsize used under numerical integration of simulation is 0.0001 sec. Caused by $s_{i\alpha} \to 0$ and $s_{i\beta} \to 0$, they even move up along this straight line with narrower oscillation, then they approach and come onto the origin. Eventually, they stay at this point

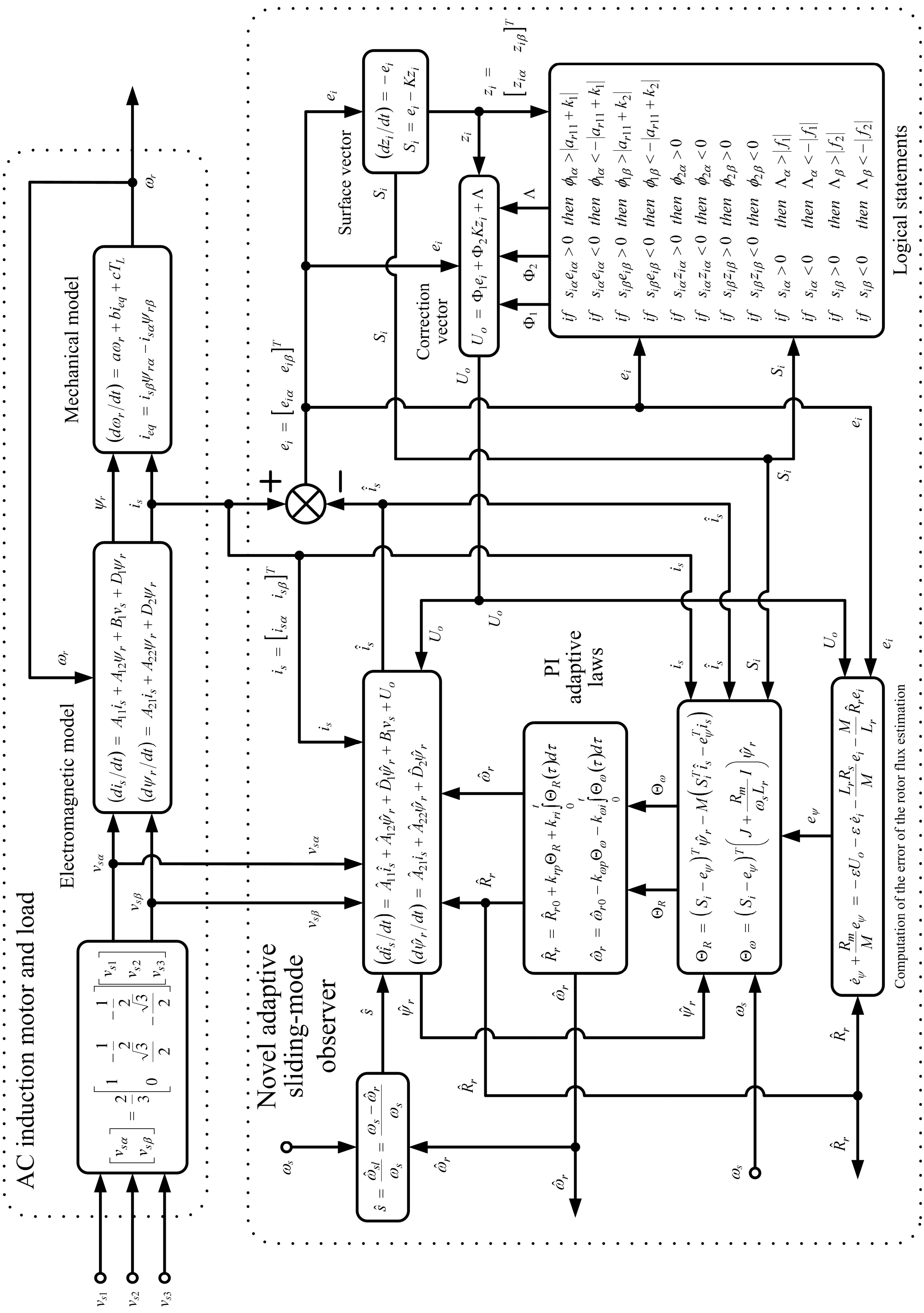

Fig. 1 the configuration of the proposed adaptive sliding-mode speed observer

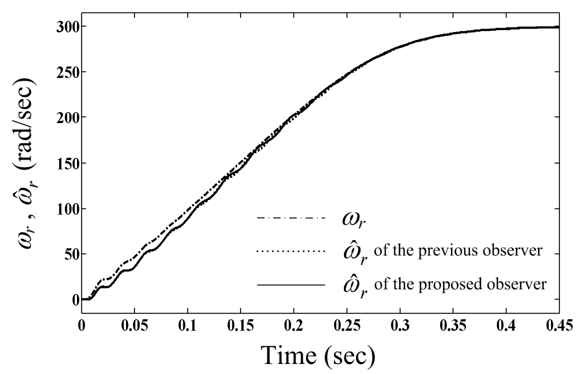

Fig. 2 comparisons between the actual speed (ω_r) and the estimated ones ($\hat{\omega}_r$) obtained from the proposed observer and the previous one [3]–[4]

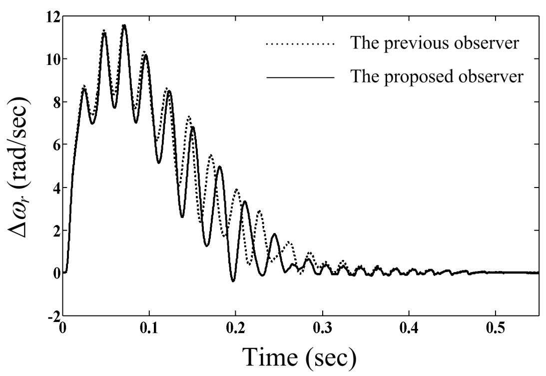

Fig. 3 comparison between the rotor speed estimation errors ($\Delta\omega_r$) derived from the proposed observer and the previous one

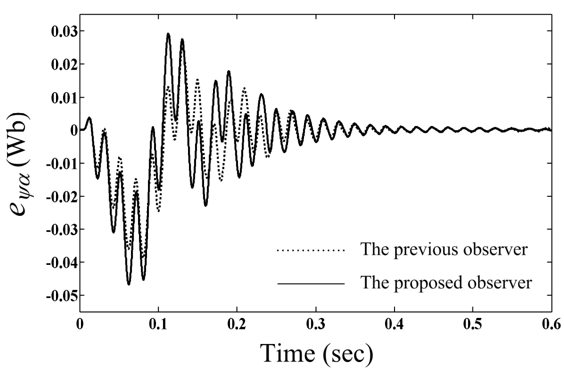

Fig. 4 comparison between the rotor flux estimation errors ($e_{\psi\alpha}$) achieved from the proposed observer and the previous one

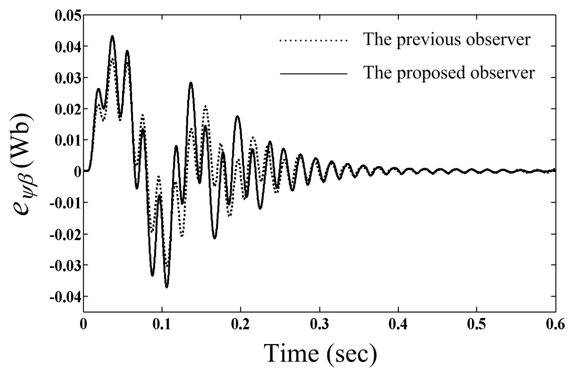

Fig. 5 comparison between the rotor flux estimation errors ($e_{\psi\beta}$) obtained from the proposed observer and the previous one

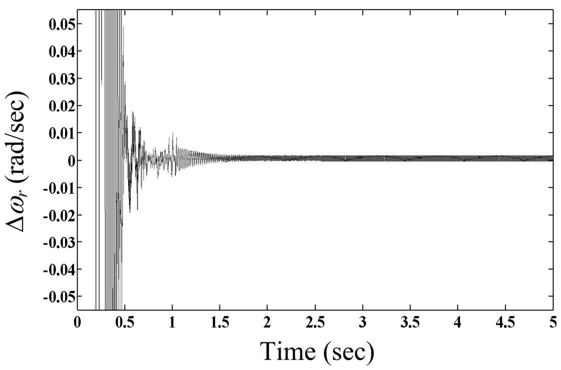

Fig. 6 the rotor speed estimation error ($\Delta\omega_r$) derived from the proposed observer in steady state

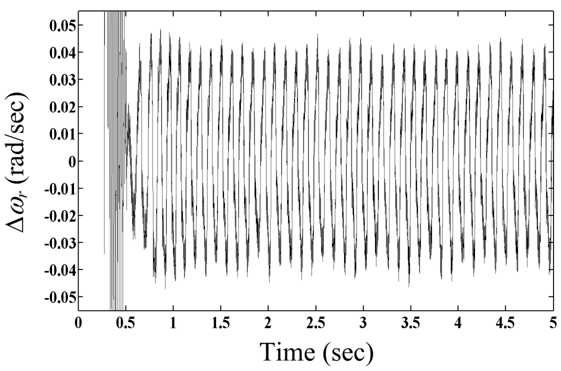

Fig. 7 the rotor speed estimation error ($\Delta\omega_r$) achieved from the previous observer in steady state

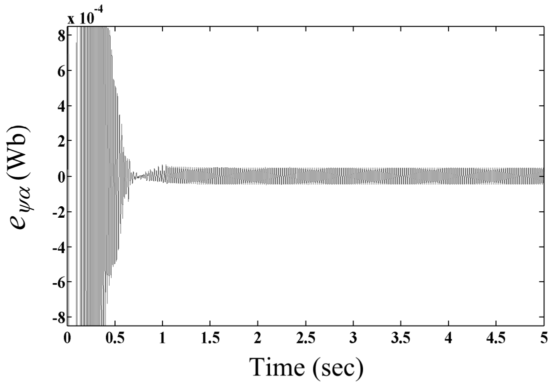

Fig. 8 the rotor flux estimation error ($e_{\psi\alpha}$) obtained from the proposed speed observer in steady state

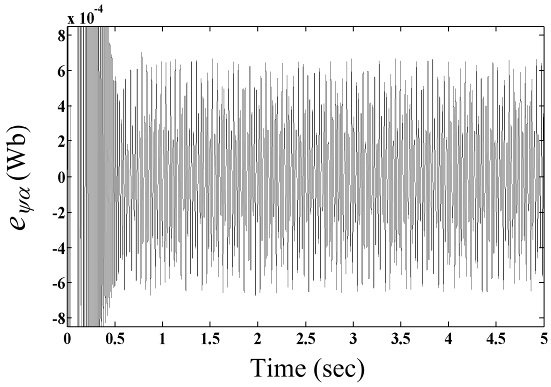

Fig. 9 the rotor flux estimation error ($e_{\psi\alpha}$) derived from the previous speed observer in steady state

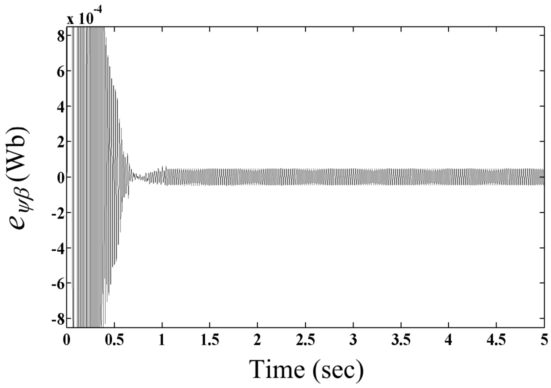

Fig. 10 the rotor flux estimation error ($e_{\psi\beta}$) achieved from the proposed speed observer in steady state

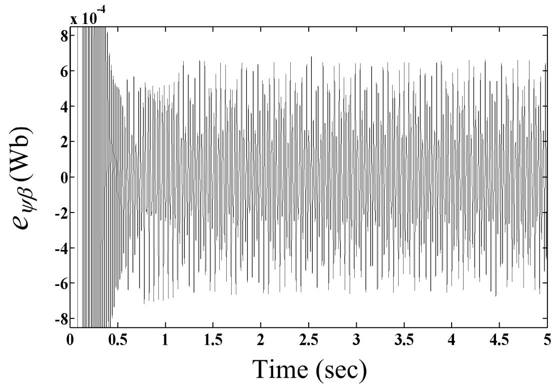

Fig. 11 the rotor flux estimation error ($e_{\psi\beta}$) obtained from the previous speed observer in steady state

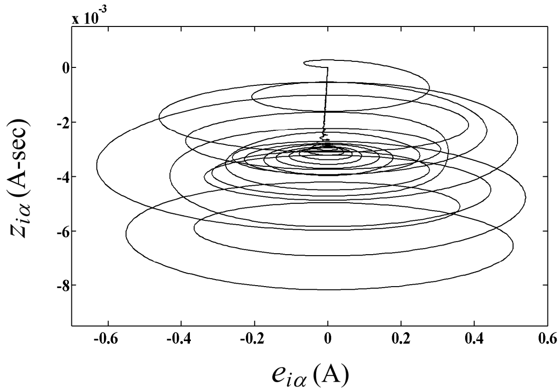

Fig. 12 plotting the stator current estimation error ($e_{i\alpha}$) against its integral ($z_{i\alpha}$) derived from the proposed observer

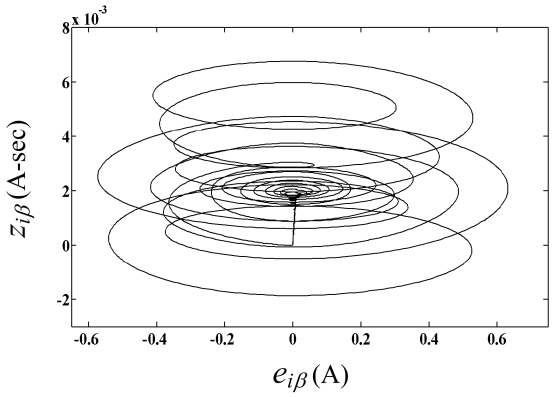

Fig. 13 plotting the stator current estimation error ($e_{i\beta}$) against its integral ($z_{i\beta}$) achieved from the proposed observer

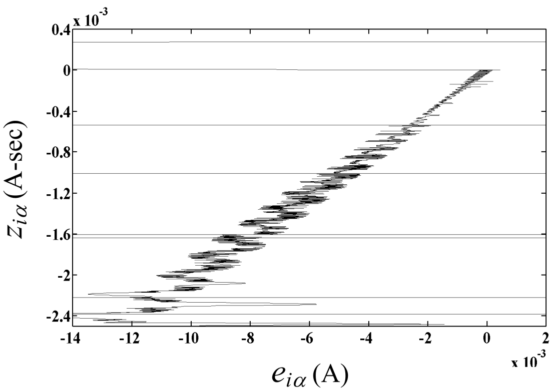

Fig. 14 plotting the stator current estimation error ($e_{i\alpha}$) against its integral ($z_{i\alpha}$) obtained from the proposed observer in steady state

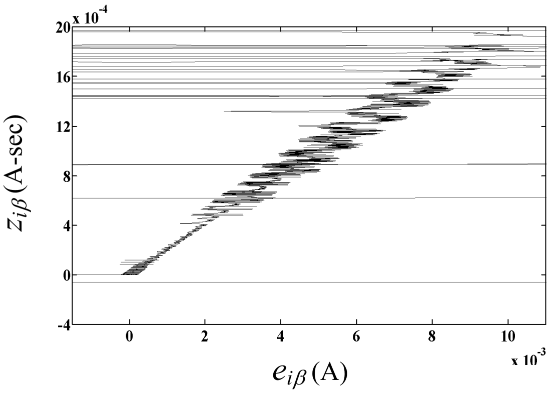

Fig. 15 plotting the stator current estimation error ($e_{i\beta}$) against its integral ($z_{i\beta}$) derived from the proposed observer in steady state

and are no longer away until the operation of the observer changes. Such an origin is called the equilibrium point. The trajectories of the proposed scheme are only plotted because these and the others of the previous one are alike.

When the novel observer treats concurrently the on-line estimations of rotor resistance and speed with two adaptive laws via the Eq. (32) and Eq. (33), the rotor speed and flux estimations are more reasonably erroneous than the corresponding ones from the preceding test in transient period as shown from Fig. 16 to Fig. 18. The initial value of rotor resistance estimation is set to half the nominal rotor resistance, i.e. $\hat{R}_{r0}$ = 1.165 Ω while the another PI gains in one of the two adaptive laws are tuned to k_{rp} = 0.06 and k_{ri} = 1.24. At the instant of initiation the estimated rotor resistance jumps up suddenly to a value relatively larger than its initial one as shown in Fig. 19. This value is updated consecutively further until it is increased to its proper value with very little overshoot. In steady state the estimation of rotor resistance becomes consistent because it has a certain value, i.e. $\hat{R}_r(\infty) \approx$ 2.33 Ω as shown in Fig. 20 while the errors of the rotor flux and speed estimations are similar to them which are attained from the preceding test as shown from Fig. 21 to Fig. 23. Under the simulation with the smaller fixed-stepsize of 0.00001 sec., the rotor speed estimation has a certain and minute steady-state error while the rotor flux estimation gives the slighter steady-state error. The trajectories of (z_i , e_i) in transient period are bounded within some areas where the equilibrium point is near as shown in Fig. 24 and Fig. 25. In steady state they are attracted to the sliding line and then get closer and much closer to the origin along this ideal line as shown in Fig. 26 and Fig. 27. Lastly, they arrive at the origin and the movements of them rest. Due to the smaller fixed-stepsize the movements of the trajectories on the sliding line are somewhat smoother.

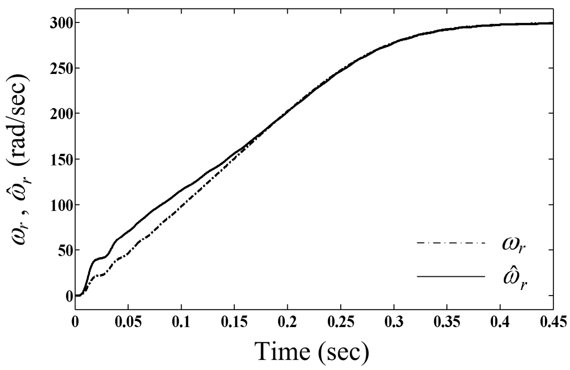

Fig. 16 comparison between the actual speed (ω_r) and the estimated one ($\hat{\omega}_r$) achieved from the proposed observer

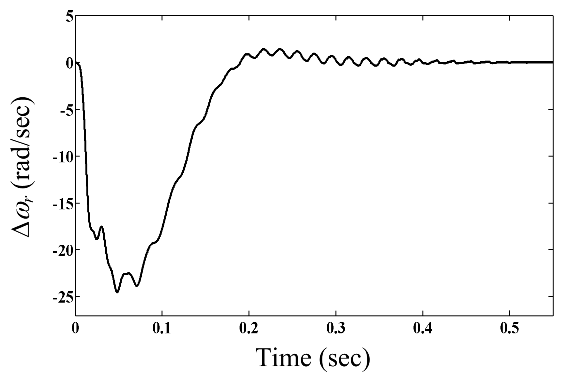

Fig. 17 the rotor speed estimation error ($\Delta\omega_r$) obtained from the proposed observer

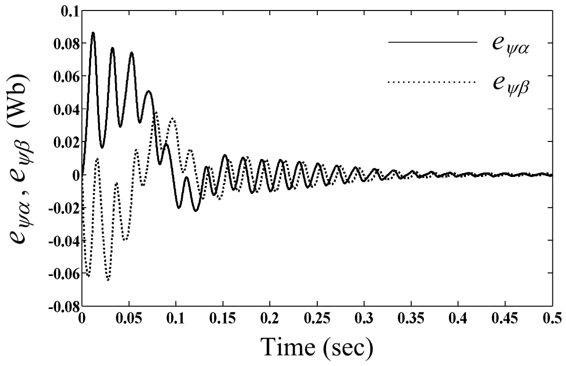

Fig. 18 the rotor flux estimation errors ($e_{\psi\alpha}$, $e_{\psi\beta}$) derived from the proposed speed observer

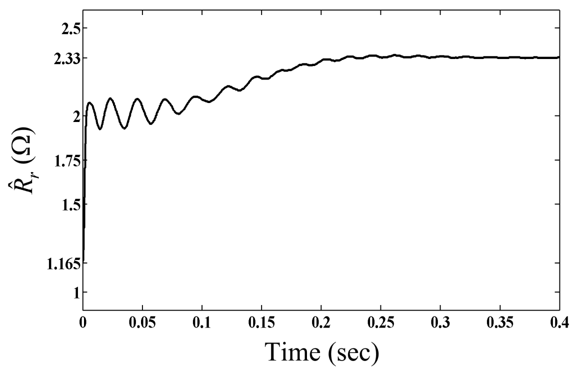

Fig. 19 the estimated rotor resistance ($\hat{R}_r$) achieved from the proposed speed observer

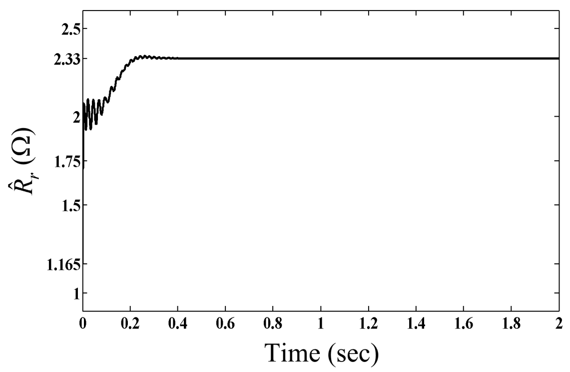

Fig. 20 the estimated rotor resistance ($\hat{R}_r$) obtained from the proposed speed observer in steady state

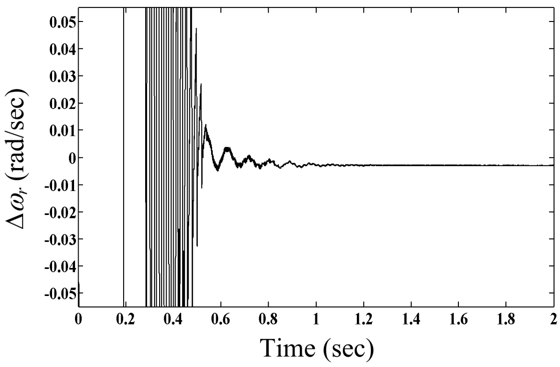

Fig. 21 the rotor speed estimation error ($\Delta\omega_r$) derived from the proposed observer in steady state

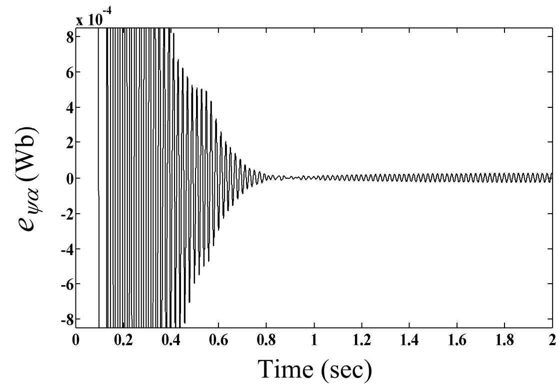

Fig. 22 the rotor flux estimation error ($e_{\psi\alpha}$) achieved from the proposed speed observer in steady state

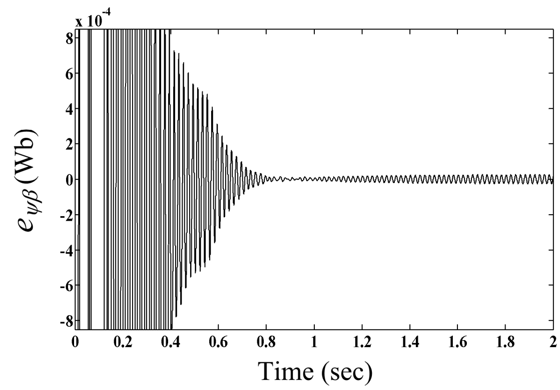

Fig. 23 the rotor flux estimation error ($e_{\psi\beta}$) obtained from the proposed speed observer in steady state

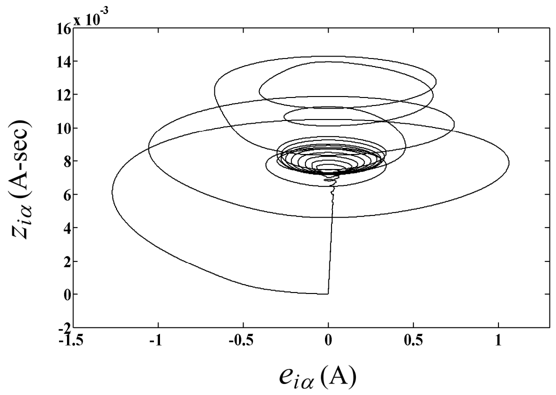

Fig. 24 plotting the stator current estimation error ($e_{i\alpha}$) against its integral ($z_{i\alpha}$) derived from the proposed observer

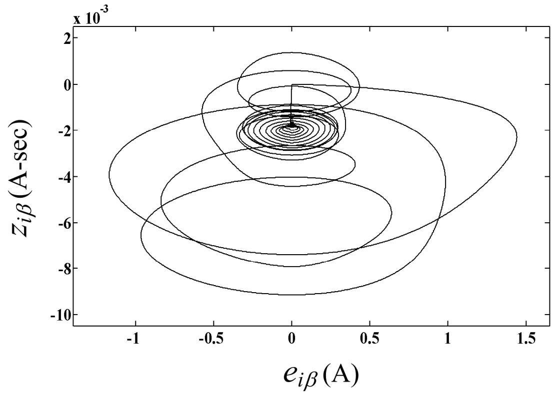

Fig. 25 plotting the stator current estimation error ($e_{i\beta}$) against its integral ($z_{i\beta}$) achieved from the proposed observer

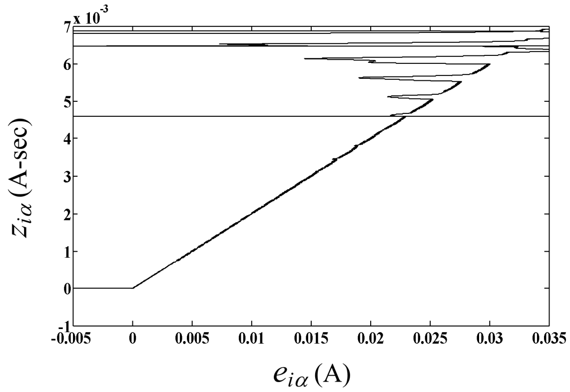

Fig. 26 plotting the stator current estimation error ($e_{i\alpha}$) against its integral ($z_{i\alpha}$) obtained from the proposed observer in steady state

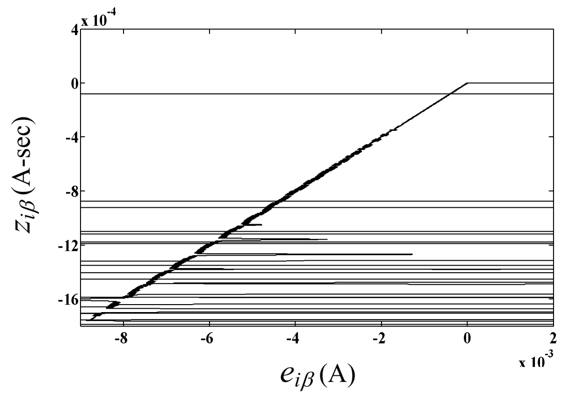

Fig. 27 plotting the stator current estimation error ($e_{i\beta}$) against its integral ($z_{i\beta}$) derived from the proposed observer in steady state

5 Conclusion

In this article, one part of the novel adaptive sliding-mode observer, which on-line estimates both the rotor resistance and speed of an induction motor, is formulated from the error equations of state-variable computations. This arrangement permits the errors of the rotor flux estimations to be also on-line calculated. When such errors become an element of the PI adaptive laws, they affect the estimated rotor resistance and speed so that the operation of the novel observer is stable. The positive Lyapunov function and its time derivative that is strictly semi-negative firmly guarantee the stability of the proposed observer and also the convergence of the rotor-flux estimation errors towards zero. Without the rotor resistance estimation, the novel observer offers much less steady-state errors in the rotor flux and speed estimations than those that the previous one does. Thus, the proposed scheme accomplishes superior performance in steady state. Moreover, the novel observer could estimate exactly the rotor resistance if the PI gains of its adaptive law in the Eq. (32) are tuned finely. In steady state the proposed scheme is subject to sliding-mode along the prescribed sliding-line until its operation reaches the equilibrium point. Therefore, anyone would expect a plan for a sensorless closed-loop speed-control system using the proposed observer.

Acknowledgement

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