The Study of recursive-based Sliding Mode Adaptive Control of the Permanent Magnet Synchronous Motor

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Abstract: The urgent needs in many occasions have stimulated the study of the permanent magnet synchronous motor (PMSM). The model of the PMSM system has multi-variable, highly nonlinear, strong coupling character and the control performance of the PMSM drive system is also influenced by the uncertainties of the plant composed of unpredictable plant parameter variations, external load disturbances, and unmodelled nonlinear dynamics. In order to design a controller to meet with the desired requirement, based on Lyapunov stability theorem, the combined multiple recursive based sliding mode adaptive controller has been proposed and the physical parameters learn algorithm has been derived. This combined controller integrates the virtues of these three kinds of controllers, and it provides a solution to the servo position control of permanent magnet synchronous motor with the parameters of uncertainty and external load disturbance and also a reference criterion for the controller parameters selection. Theory derivation and experiment simulation results verify the effectiveness and feasibility of the controller. This method provides the foundation of the servo position control of the permanent magnet synchronous motor.

Key word: Permanent Magnet Synchronous Motor (PMSM); sliding mode control; recursive-based control; adaptive control; Lyapunov stability theorem; nonlinear dynamics

1. Introduction

Permanent magnet synchronous motors (PMSM) are of great interest, particularly for industrial applications in the low-medium power range, since it has superior features such as compact size, high torque/weight ratio, and high torque/inertia ratio [1]. Moreover, compared with induction motors, PMSM have the advantages of higher efficiency [2].

The typical construction of a PMSM consists of a three phase stator winding and a solid iron rotor with magnets attached to its surface or inserted into the rotor body. This construction results in a magnetic field fixed to the rotor position. Since such machines are not capable of directly starting from the mains, excitation by voltage source inverters (VSI) controlled by field orientation is required. Control techniques such as vector control [3] or direct torque control (DTC) [4] are standard for this type of drives. Permanent magnet synchronous motor control system mainly consists of two parts, the main drive circuit and the control circuit. The main drive circuit topology structure remains basically unchanged, while the study of the control system focuses on the control circuit and control strategies [5]. Based on the characteristics of the model of permanent magnet synchronous motor, many modern control methods and intelligent control methods have been applied to the permanent magnet synchronous motor. The state feedback linearization of nonlinear systems control theory has been introduced into the motor control in [6], and this method can change the original nonlinear system into the α -order integrated dynamic inversion system, which can decouple the system and deduced the linear relationship between the control input and the output, then the linear control theory can be utilized, however the complexity of the theory of the method has limited its extension usage. Sliding mode variable structure control [7] can adjust the structure of feedback control system, which can make its vector changes in the switching hyper-plane, and the bounded state vectors of the system are sliding in the switch plane, and the dynamic quality of the system is decided by the parameters of the switching surface system. Sliding mode controllers (SMCs) have been widely used for speed and position control since they provide a fast dynamic response. The adaptive control strategy has been used in the control of permanent magnet synchronous motor in [8], and the method can update the recognition parameters of system real-timely, modify the system control procedures, thus improving the system characters in time-varying operating conditions, however this algorithm needs a large amount of computing and high-speed data processor. Besides these nonlinear control methods mentioned above, the intelligent control methods, such as the artificial intelligence expert system [9], fuzzy control [10], and neural networks [11, 12] have also been utilized in the motor drive system, and the great progress has been achieved. Brock [13] used fuzzy logic controllers to adjust the gain of a controller in a sliding mode controller for speed and position control of PMSM. The control strategies in which recurrent fuzzy neural networks (FNN) were used to adjust the gain of the SMC for position control of a PMSM was used by Wai [14].

To achieve fast four-quadrant operation, smooth starting and acceleration, the field-oriented control method is used in the design of the PMSM drive. However, the control performance of the PMSM drive system is still influenced by the uncertainties of the plant, which usually are composed of unpredictable plant parameter variations, external load disturbances, and unmodelled nonlinear dynamics. In order to solve this problem, the recursive-based sliding mode adaptive control method has been used in controlling PMSM, which integrated the virtues of the sliding-mode control method, the recursive control strategies, as well as the adaptive control methods to settle the PMSM with the unpredictable plant parameter variations, external load disturbances. Based on the Lyapunove stability theorem, the controller can be designed and can be testified. As compared with the general PID controller, the control parameters of this method have practical significance and can be accessed easily. Although there are parameter changes in the model, the good tracking results can also be reached and the experiments simulations show that the PMSM can quickly reach the desired position.

This paper introduces a combined control method for position servo control of PMSM drive with internal and external interference. The organization of this paper is as follows: First, the model of the PM synchronous servo motor is introduced; then, the synthesis of the recursive-based sliding mode adaptive controller is described in detail with theoretical proof; finally, the servo position controller proposed is implemented to control the rotor position of a PM synchronous motor for the tracking of sinusoidal commands with plant parameter variations, external load disturbances in simulation; the simulation experimental results are given to demonstrate the effectiveness of the proposed control scheme.

2. Machine model of the field-oriented

PMSM

The PMSM is composed of three phase's stator windings and permanent magnets mounted on the rotor surface (surface mounted PMSM) or buried inside the rotor (interior PMSM). The electrical equations of the PM synchronous motor can be described in the rotor rotating reference frame, written in the (dq) rotor flux reference frame [15,16].

The mathematic model of PMSM is based on the following assumptions:

(1) Neglecting the saturation of armature;

(2) Neglecting the wastages of eddy and magnetic hysterics;

(3) There is no rotor damp resistance.

The relations of voltage, torque and flux of PMSM are described as follows:

$$\begin{bmatrix} \dot{i}_{q} \\ \dot{i}_{d} \end{bmatrix} = \begin{bmatrix} -\frac{L}{R} & -\omega_{r} \\ \omega_{r} & -\frac{L}{R} \end{bmatrix} \begin{bmatrix} i_{q} \\ i_{d} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 & 0 \\ 0 & \frac{1}{L} & -\frac{\omega_{r}}{L} \end{bmatrix} \begin{bmatrix} u_{q} \\ u_{d} \\ \phi \end{bmatrix}$$
(1)

where i_d and i_q are the *d* and *q* axis stator currents, *R* and *L* are the stator phase resistance and inductance respectively; the d-axis self inductance and the q-axis self inductance are all equal to *L*; ω_r is the rotor electrical speed; u_d and u_q are the stator voltages expressed in the *dq* reference frame and ϕ is the flux established by rotor permanent magnets; *P* is the number of magnetic pole. Equation (1) describes electrical dynamics and is nonlinear since it involves the products of state variables.

$$\omega_s = P\omega_r \tag{2}$$

 ω_s is inverter frequency [17].

The electromagnetic torque is given by

$$T_e = 3P \Big[\phi i_q + (L_d - L_q) i_d i_q \Big]$$
⁽³⁾

If $i_d = 0$, the electromagnetic torque T_e is

proportional to i_q . This description is similar to the torque generated in a DC motor with independent field excitation. This feature can simplify the controller design of the PMSM.

The equation of the motor dynamics is

$$T_e = T_L + B\omega_r + J\dot{\omega}_r \tag{4}$$

 T_L stands for external load torque. *B* represents the damping coefficient and *J* is the moment of inertia of the rotor.

Thus, the mechanical dynamic can be rewritten as

$$\frac{d\omega_r}{dt} = -\frac{B}{J}\omega_r + \frac{3P\left[\phi i_q + (L_d - L_q)i_d i_q\right]}{2J} - \frac{T_L}{J}$$
(5)

The equation (5) shows that the electromagnetic torque is the product of state variables and it is nonlinear. The equations (1) and (5) constitute the whole control model of the PMSM. The state-space model of the interior PMSM is similar to the surface mounted PMSM only that the model of the surface mounted PMSM does not have the product of the currents d-axis and q-axis in the electromagnetic torque. However the control method of these two kinds of motor is similar, for simple, the surface mounted PMSM is chosen as the control object in this paper.

In order to facilitate the controller design, the model of the PMSM can be described in the form of the state space equation below with

$$\begin{cases} x_1 = \theta \\ x_2 = \omega \\ x_3 = i_q \\ x_4 = i_d \end{cases}$$

Then

$$\begin{cases} \dot{x}_{1} = x_{2} \\ \dot{x}_{2} = -\alpha_{1}x_{2} + \alpha_{2}x_{3} - T_{t} \\ \dot{x}_{3} = -\alpha_{3}x_{3} - \alpha_{4}x_{2}x_{4} - \alpha_{5}x_{2} + \alpha_{8}u_{qd} \\ \dot{x}_{4} = -\alpha_{6}x_{4} + \alpha_{7}x_{2}x_{3} + \alpha_{8}u_{dd} \\ y = x_{1} \end{cases}$$
(6)

where

$$\alpha_1 = \frac{B}{J}$$

 $\alpha_2 = \frac{3P\phi_f}{2J}$
 $\alpha_3 = \alpha_6 = \frac{R}{L}$
 $\alpha_4 = \alpha_7 = P$
 $\alpha_5 = \frac{\phi_f P}{L}$

 $u_{qd} = u_q$ *u* is the system input, and *y* is the output of the

system. The state variables of the system x_1 , x_2 , x_3 can be

measured by the angle sensor, angular rate sensor and current meter measurements respectively, moreover they are bounded.

Discussion: Some parameters of the permanent-magnet synchronous motor are changing with temperature and the external loads are changing over time when it is in operation. If the controller designed is based on fixed parameters, the robustness of the controller to the external interference and internal parameters will be very weak. To enhance the robust of the control system, based on this model, the adaptive control method is introduced into recursive-based sliding mode control method. The adaptive controller is designed based on the real-time information of the system, thus it can adapt to internal and external interference and can improve the robustness of controller.

3. The synthesis recursive-based sliding

mode adaptive controller design of the

PMSM

For the permanent magnet synchronous motor position servo system, the controller can make the system meet:

$$\lim_{t \to \infty} (y - y_d) = 0 \tag{7}$$

Where y_d is the desired output and it is assumed to

be known, and its first-order, second-order and third-order derivatives all existed.

In order to design the controller to meet requirement of the equation (7), the model of PMSM can be divided into two parts: the first part is the composition of the first three equations of (6) in section 2, which is used to determine the desired control voltage u_{qd} ; the last equation of (6) is utilized to design the controller to determine the desired control voltage u_{dd} .

3.1 The Sliding mode control

The basic idea underlying a sliding mode controller (SMC) is that the motion of the states of the system is constrained to a predetermined path in the state space. In a SMC, the predetermined path is called a sliding surface or sliding motion to provide ideal sliding motion. Theoretically, the sliding motion is smooth if the switching frequency of a system is infinite. However, in practice, the switching frequency of a system is finite, thus chattering comes out along the sliding surface [29].

Select x_1 of the original system (6) as the system

output and the system state variables can treated as the intermediate state variables based on the recursive control design method, and then the sub-system made up of the first two equations of the original system (6) can be regarded as the second-order system. Therefore, sliding mode control method can be used to design the system

controller. The expected value x_{3d} of the state

variables x_3 can be derived and can be used as the desired

control input of the next sub-system.

Define the position tracking error of the system as:

$$e = y - y_d = x_1 - y_d \tag{8}$$

Select the sliding surface as follows:

$$z_1 = ce_1 + \dot{e}_1$$
 ($c > 0$) (9)

With derivative of (9) and the system model (6), the equation (10) can be reached:

$$\begin{aligned} \dot{z}_{1} &= c(\dot{x}_{1} - \dot{y}_{d}) + (\dot{x}_{2} - \ddot{y}_{d}) \\ &= c(x_{2} - \dot{y}_{d}) + (-\hat{\alpha}_{1}x_{2} + \hat{\alpha}_{2}x_{3} - (10)) \\ \hat{T}_{t} &- \ddot{y}_{d}) + (-\tilde{\alpha}_{1}x_{2} + \tilde{\alpha}_{2}x_{3} - \tilde{T}_{t}) \end{aligned}$$

Where:

$$\begin{cases} \tilde{\alpha}_1 = \alpha_1 - \hat{\alpha}_1 \\ \tilde{\alpha}_2 = \alpha_2 - \hat{\alpha}_2 \\ \tilde{T}_t = T_t - \hat{T}_t \end{cases}$$

 \widehat{lpha}_1 , \widehat{lpha}_2 and \widehat{T}_t are the estimation values

of α_1 , α_2 and T_t respectively.

According to the theory of state feedback linearization, the intermediate control variable can be selected as follows:

$$x_{3d} = \frac{1}{\alpha_2} [\hat{\alpha}_1 x_2 + \hat{T}_t + \ddot{y}_d - c(x_2 - \dot{y}_d) - k_1 z_1]$$
(11)

Where k_1 is constant and $k_1 > 0$.

3.2 The recursive adaptive controller design

Adaptive control is a valid method to overcome system uncertainties, especially uncertainties derived from uncertain parameters; hence several kinds of nonlinear adaptive control schemes have been proposed in hydraulic control systems such as feedback linearization adaptive control [18], sliding mode adaptive control [19], and nonlinear adaptive control based on back stepping techniques[20-22]. The nonlinear adaptive control laws not only solved the control problem coming from uncertain nonlinear system successfully in some conditions [23-25] but also demonstrated that nonlinear control schemes can achieve better performance than conventional linear controllers [26-28].

For x_{3d} is the intermediate control variable and the

expected value of the state variable x_3 , there exists some error in between, then define the error as follows:

$$z_2 = e_2 = x_3 - x_{3d} \tag{12}$$

Then, the derivative of the sliding surface (9) can be described as:

$$\dot{z}_{1} = -k_{1}z_{1} + \hat{\alpha}_{2}z_{2} - \tilde{\alpha}_{1}x_{2} + \tilde{\alpha}_{2}x_{3} - \tilde{T}_{t}$$
(13)

Then, the differential of the equation (10) can be expressed as:

$$\dot{z}_{2} = -\alpha_{3}x_{3} - \alpha_{4}x_{2}x_{4} - \alpha_{5}x_{2} + \alpha_{8}u_{qd} - \dot{x}_{3d}$$

$$= -\hat{\alpha}_{3}x_{3} - \hat{\alpha}_{4}x_{2}x_{4} - \hat{\alpha}_{5}x_{2} + \hat{\alpha}_{8}u_{qd} - \qquad(14)$$

$$\tilde{\alpha}_{3}x_{3} - \tilde{\alpha}_{4}x_{2}x_{4} - \tilde{\alpha}_{5}x_{2} + \tilde{\alpha}_{8}u_{qd} - \dot{x}_{3d}$$
Where
$$\begin{cases} \tilde{\alpha}_{3} = \alpha_{3} - \hat{\alpha}_{3} \\ \tilde{\alpha}_{4} = \alpha_{4} - \hat{\alpha}_{4} \\ \tilde{\alpha}_{5} = \alpha_{5} - \hat{\alpha}_{5} \\ \tilde{\alpha}_{8} = \alpha_{8} - \hat{\alpha}_{8} \end{cases}$$

 $\hat{lpha}_{_3}$, $\hat{lpha}_{_4}$, $\hat{lpha}_{_5}$ and $\hat{lpha}_{_8}$ are the estimation values

of α_3 , α_4 , α_5 and α_8 respectively.

Then \dot{x}_{3d} can be reached from the equation (11):

$$\dot{x}_{3d} = \frac{1}{\hat{\alpha}_{2}} [\hat{\alpha}_{1} \dot{x}_{2} + \dot{\hat{\alpha}}_{1} x_{2} + \dot{\hat{T}}_{t} + \ddot{y}_{d} - c(\dot{x}_{2} - \ddot{y}_{d}) - k_{1} \dot{z}_{1}] - \frac{\dot{\hat{\alpha}}_{2}}{\hat{\alpha}_{2}} x_{3d}$$
(15)

Moreover

$$\dot{z}_1 = c(\dot{x}_1 - \dot{y}_d) + (\dot{x}_2 - \ddot{y}_d)$$
 (16)

Then

$$\dot{x}_{3d} = \frac{1}{\hat{\alpha}_{2}} [(\hat{\alpha}_{1} - c - k_{1})\dot{x}_{2} + \dot{\hat{\alpha}}_{1}x_{2} + \dot{\hat{T}}_{t} + (c + k_{1})\ddot{y}_{d} + \ddot{y}_{d} - k_{1}c(x_{2} - \dot{y}_{d}) - \dot{\hat{\alpha}}_{2}x_{3d}]$$
(17)
$$= \dot{\hat{x}}_{3d} + \frac{\hat{\alpha}_{1} - c - k_{1}}{\hat{\alpha}_{2}} (-\tilde{\alpha}_{1}x_{2} + \tilde{\alpha}_{2}x_{3} - \tilde{T}_{t})$$

Where

$$\dot{x}_{3d} = \frac{1}{\hat{\alpha}_2} [(\hat{\alpha}_1 - c - k_1)(-\hat{\alpha}_1 x_2 + \hat{\alpha}_2 x_3) \\ -\hat{T}_t) + \dot{\alpha}_1 x_2 + \dot{T}_t + (c + k_1) \ddot{y}_d + (18) \\ \ddot{y}_d - k_1 c (x_2 - \dot{y}_d) - \dot{\alpha}_2 x_{3d}]$$

Here, the equation (14) can be described as:

$$\dot{z}_{2} = -\alpha_{3}x_{3} - \alpha_{4}x_{2}x_{4} - \alpha_{5}x_{2} + \alpha_{8}u_{qd}$$

$$-\tilde{\alpha}_{3}x_{3} - \tilde{\alpha}_{4}x_{2}x_{4} - \tilde{\alpha}_{5}x_{2} + \tilde{\alpha}_{8}u_{qd} - \dot{x}_{3d} \qquad (19)$$

$$-\frac{\hat{\alpha}_{1} - c - k_{1}}{\hat{\alpha}_{2}} (-\tilde{\alpha}_{1}x_{2} + \tilde{\alpha}_{2}x_{3} - \tilde{T}_{t})$$

From the derivation above, the ultimate control value can be selected as:

$$u_{qd} = \frac{\hat{\alpha}_{3}x_{3} + \hat{\alpha}_{4}x_{2}x_{4} + \hat{\alpha}_{5}x_{2} + \hat{x}_{3d} - k_{2}z_{2}}{\hat{\alpha}_{8}}$$
(20)

Where k_2 is constant and $k_2 > 0$.

Then, insert equation (20) into the equation (19), the equation (21) can be reached:

$$\dot{z}_{2} = -\tilde{\alpha}_{3}x_{3} - \tilde{\alpha}_{4}x_{2}x_{4} - \tilde{\alpha}_{5}x_{2} + \\ \tilde{\alpha}_{8}u_{qd} - \frac{\hat{\alpha}_{1} - c - k_{1}}{\hat{\alpha}_{2}}(-\tilde{\alpha}_{1}x_{2} + \\ \tilde{\alpha}_{2}x_{3} - \tilde{T}_{t}) - k_{2}z_{2}$$

$$(21)$$

3.3 The Error based Controller Design

For the controller design of the fourth model of the system (6), define the error as follows:

$$e_3 = x_4 - x_{4d} \tag{22}$$

Then, considering the changes of the system parameters, the differential of the error can be described as follows:

$$\dot{e}_3 = -\hat{\alpha}_6 x_4 + \hat{\alpha}_7 x_2 x_3 + \hat{\alpha}_8 u_{dd} - \dot{x}_{4d} -\tilde{\alpha}_6 x_4 + \tilde{\alpha}_7 x_2 x_3 + \tilde{\alpha}_8 u_{dd}$$
(23)

Where

$$\begin{cases} \tilde{\alpha}_6 = \alpha_6 - \hat{\alpha}_6 \\ \tilde{\alpha}_7 = \alpha_7 - \hat{\alpha}_7 \end{cases}$$

 $\hat{\alpha}_6$ and $\hat{\alpha}_7$ are the estimation values of

$$\alpha_6$$
 and α_7 respectively.

Then, the actual control value of the system can be selected as:

$$u_{dd} = \frac{\hat{\alpha}_{6}x_{4} - \hat{\alpha}_{7}x_{2}x_{3} - k_{3}e_{3} + \dot{x}_{4d}}{\hat{\alpha}_{8}}$$
(24)

Thus, the differential of the error equation (23) can be described as:

$$\dot{e}_{3} = -\tilde{\alpha}_{6}x_{4} + \tilde{\alpha}_{7}x_{2}x_{3} + \tilde{\alpha}_{8}u_{dd} - k_{3}e_{3}$$
(25)

3.4 Stability Condition and adaptive laws

Define Lyapunov function as follows:

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2}\sum_{i=1}^5 \rho_i \tilde{\alpha}_i^2 + \frac{1}{2}\rho_8 \tilde{\alpha}_8^2 + \frac{1}{2}\rho_6 \tilde{T}_t^2$$
(26)

Where $\rho_i > 0$, $i = 1, \dots, 6, 8$ are adaptive gains.

With the derivation of (23), the equation (24) can be reached:

$$\dot{V} = z_1 \dot{z}_1 + z_2 \dot{z}_2 + e_3 \dot{e}_3 - \rho_1 \tilde{\alpha}_1 \hat{\alpha}_1 - \rho_2 \tilde{\alpha}_2 \hat{\alpha}_2 - \rho_3 \tilde{\alpha}_3 \hat{\alpha}_3 - \rho_4 \tilde{\alpha}_4 \hat{\alpha}_4 - \rho_5 \tilde{\alpha}_5 \hat{\alpha}_5 \qquad (27)$$
$$-\rho_8 \tilde{\alpha}_8 \hat{\alpha}_8 - \rho_6 \tilde{T}_t \hat{T}_t$$

Insert the equation (6), (13), (21) and (25) into the equation (27), the equation (28) can be got:

$$\dot{V} = -k_{1}z_{1}^{2} - k_{2}z_{2}^{2} - k_{3}e_{3}^{2} + \hat{\alpha}_{2}z_{1}z_{2} - \tilde{\alpha}_{1}(z_{1}x_{2} - \frac{(\hat{\alpha}_{1} - c - k_{1})x_{2}z_{2}}{\hat{\alpha}_{2}} + \rho_{1}\dot{\alpha}_{1}) + \tilde{\alpha}_{2}(z_{1}x_{3} - \frac{(\hat{\alpha}_{1} - c - k_{1})x_{3}z_{2}}{\hat{\alpha}_{2}} + \rho_{2}\dot{\alpha}_{2}) - \tilde{\alpha}_{3}(z_{2}x_{3} + e_{3}x_{4} + \rho_{3}\dot{\alpha}_{3}) - \tilde{\alpha}_{4}(z_{2}x_{2}x_{4} - (28))$$

$$e_{3}x_{2}x_{3} + \rho_{4}\dot{\alpha}_{4}) - \tilde{\alpha}_{5}(z_{2}x_{2} + \rho_{5}\dot{\alpha}_{5}) + \tilde{T}_{t}(-z_{1} + \frac{(\hat{\alpha}_{1} - c - k_{1})z_{2}}{\hat{\alpha}_{2}} - \rho_{6}\dot{T}_{t}) + \tilde{\alpha}_{8}(z_{2}u_{qd} + e_{3}u_{dd} - \rho_{8}\dot{\alpha}_{8})$$

As the parameter of α_4 is magnetic pole number of

the permanent magnet synchronous motor, this parameter will not change during the operation, thus, the change

rate of α_4 can be taken as zero. Thus the adaptive laws of

the system can be obtained as follows:

$$\begin{aligned} \dot{\hat{\alpha}}_{1} &= -\frac{1}{\rho_{1}} \left(z_{1}x_{2} - \frac{(\hat{\alpha}_{1} - c - k_{1})x_{2}z_{2}}{\hat{\alpha}_{2}} \right) \\ \dot{\hat{\alpha}}_{2} &= \frac{1}{\rho_{2}} \left(z_{1}x_{3} - \frac{(\hat{\alpha}_{1} - c - k_{1})x_{3}z_{2}}{\hat{\alpha}_{2}} \right) \\ \dot{\hat{\alpha}}_{3} &= -\frac{1}{\rho_{3}} \left(z_{2}x_{3} + e_{3}x_{4} \right) \\ \dot{\hat{\alpha}}_{5} &= -\frac{1}{\rho_{5}} z_{2}x_{2} \\ \dot{\hat{T}}_{t} &= \frac{1}{\rho_{6}} \left(-z_{1} + \frac{(\hat{\alpha}_{1} - c - k_{1})z_{2}}{\hat{\alpha}_{2}} \right) \\ \dot{\hat{\alpha}}_{8} &= \frac{1}{\rho_{8}} \left(z_{2}u_{qd} + e_{3}u_{dd} \right) \end{aligned}$$
(29)

Here, the equation (28) can be described as:

$$\dot{V} = -k_1 z_1^2 - k_2 z_2^2 - k_3 e_3^2 + \hat{\alpha}_2 z_1 z_2$$

- - -

For
$$\alpha_2 = \frac{3P\phi_f}{2J} > 0$$
 and α_2 is bounded. Then if

choosing $k_1 k_2 - \frac{\alpha_2^2}{4} > 0$, moreover $k_1, k_2, k_3 > 0$,

then $\dot{V} < 0$. And then V will be positive definite and \dot{V} will be negative definite. Thus this system will be Lyapunov asymptotic stability.

From the derivation above, with the definition sliding surface of equation (9) and the error function of the equation (22), the controller made up of equation (20) and equation (24) can be designed. With the selected adaptive control law of equation (29), when selecting

$$k_1 k_2 - \frac{\alpha_2^2}{4} > 0 \tag{30}$$

The whole system will be asymptotic stability.

4. Simulation Experiments

In this section, the proposed approach above has been carried out in PMSM to verify the performance of recursive-based sliding mode adaptive control scheme, using MATLAB/Simulink. The parameters of the PMSM are given in Tab. I. The simulation results are shown in Figs. 1-4.

Tab. 1 PMSM Parameters	
Machine parameters	value
Resistance of the stator windings(Ω)	2.875
Magnetic Pole number	4
Combined inertia of rotor and	0.001
load J (kg.m ²)	
Amplitude of the flux induced by the	0.175
permanent magnets of the rotor in the stator	
phases ϕ (Wb)	
q and d axis inductances $L_q = L_d$ (H)	0.0085
Combined viscous friction of rotor and	0
load B(N.m.s/rad)	

In table 1, except magnetic pole number, the remaining parameters values of the PMSM are all changed or unpredictable. The paper focuses on the study of the control system tracking features when the load moment of inertia J and the load torque T_m change, and then proves the correctness of the controller used in simulation experiment. The sinusoidal signal is selected as the reference input in simulation. During the operation, the moment of inertia $(0.5t)(kg \cdot m^2)$, which is made up of a constant value and a sinusoidal disturbance. And the load acted upon the motor is the superposition of a

step signal and a unit sinusoidal signal $1\sin(2\pi t)$.

With the derivation in last section, it can be noticed that in theory, the greater the controller parameters $k_{1,2,3}$ and *c* are, the faster the system response is and the smaller the tracking error will be. However, if these parameters are chosen too larger, this will make the system speed, voltage and other control variables beyond the allowable range. After a lot of simulation experiments, according to the physical parameters in Table 1 and the derived control parameters scopes in section 3,the control parameters can be chosen as $k_1 = 570$, $k_2 = 600$, $k_3 = 5$ and c = 30.

The simulation experiment shows that the control system has a good tracking performance, and all the system parameters also operate normally, the simulation results as shown below from figure 1 to figure 4.



Figure 1 the compare of the desired angle and simulation angle







Figure 3 the compare of the desired current of d-axis and the simulation current of d-axis



Figure 4 the control current of the q-axis in simulation

In Figure 1, the solid line is the simulation angle of the motor, while the dot line means the desired angle change process provided. It can be seen that the motor can trace the desired angle very good, only leaving small vibration error in figure 2. As the simulation is conducted in the situation where there exist uncertain in moment of inertia system and the unstable load, this error is small enough to be accepted. And still with the larger control parameters, this error will become smaller. In figure 3, for the desired current of the d-axis is zero, the simulation value of the d-axis current can approximate the zero very sharply. Moreover, from Figure 3 and Figure 4 it can be noticed that the current of the system permanent magnet synchronous motor are within the definite scope. From the simulation results above, it can be concluded that with changing physical parameters in PMSM and the changing external load, the combined recursive-based sliding mode adaptive controller can control the permanent magnet motor effectively.

Discussion From the above simulation results it can be seen that the combined control method ie.recursive-based sliding mode adaptive control, compared with common the vector control, has the following advantages:

(1) In the process of system design, the adjusted parameters are less in combined control design than vector control design. For the position servo control, only need four parameters while in the common controller using vector control need seven parameters [15]. Therefore the control method proposed is more easily to design and to achieve in engineering than the vector control. (2) The combined control method proposed can give reference to the parameters selection to the controller, which can be seen with equation (30).

(3) The combined control method proposed with good static characteristics and dynamic characteristics can track quickly the location of a given signal.

(4) The combined control method proposed with large robustness has the ability to inhibit a number of load disturbances under the outside world and parameters change internal.

5. Conclusions

In order to meet with the worldwide demands for energy-saving emission, in many areas permanent magnet synchronous motor is gradually replacing the traditional three-phase asynchronous motor. The PMSM is a complex multi-variable, nonlinear, strong coupling of the multiple-input multiple-output system. It is difficult to design the controller, epecally the control performance of the PM synchronous motor drive system is still influenced by the uncertainties of the plant, which usually are composed of unpredictable plant parameter variations, external load disturbances.

This paper try to settle the control problem of the permanent magnet synchronous motor position servo control in the situation where there exist unpredictable plant parameter variations and external load disturbances. According to the characteristics of permanent magnet synchronous motor, the combined control method which consists of the sliding mode control method, recursive control method as well as adaptive control method is proposed, which integrates the virtues of these three kinds of controller. The Lyapunov stability theory is used to testify the control theory and design the tracking controller of PMSM, and with this theory, the learning algorithm of the undecided parameters has been derived and the reference to the control parameters has been given. The position servo controller has been tested in simulation experiment on the PMSM model with internal interference and external load disturbance. The theoretical and simulation experiments have validated

the effectiveness and correctness of the combined control strategy.

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